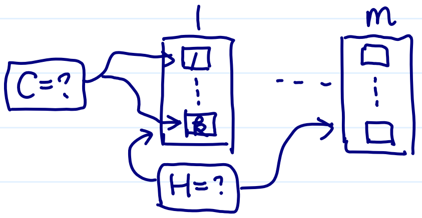
treatment $\quad$ plot structure

$V_0 \oplus W_T \oplus V_T^{\perp}$ (H, C, HC) $\quad$ $S_0 \oplus S_1 \oplus S_2$

Stratum	source	df	SS	EMS	VR
mean	mean	1	$\frac{\text{sum}^2}{N}$	$\lVert\tau_0\rVert^2+\xi_0$	–
large units	H	n_H-1	SS(H)	$\frac{\lVert\tau_H\rVert^2}{n_H-1}+\xi_1$	$\frac{\text{MS}(H)}{\text{MS(large units residual)}}$
	residual	← . . . . by subtraction . . . . →		ξ_1	–
	total	$m-1$	$\text{CSS}(L)-\frac{\text{sum}^2}{N}$		
small units	C	n_C-1	SS(C)	$\frac{\lVert\tau_C\rVert^2}{n_C-1}+\xi_2$	$\frac{\text{MS}(C)}{\text{MS(small units residual)}}$
	$H\wedge C$	d_{HC}	SS$(H\wedge C)$	$\frac{\lVert\tau_{HC}\rVert^2}{d_{HC}}+\xi_2$	$\frac{\text{MS}(H\wedge C)}{\text{MS(small units residual)}}$
	residual	← . . . . by subtraction . . . . →		ξ_2	–
	total	$m(k-1)$	$\sum y_\omega^2-\text{CSS}(L)$		
Total		N	$\sum_\omega y_\omega^2$		

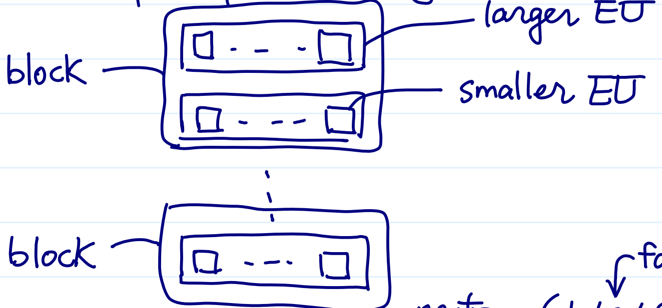


plot structure: m/k $= b\cdot s$ (later slides)

d.f. identity: $\underbrace{1}_{S_0}+\underbrace{(m-1)}_{S_1}+\underbrace{m(k-1)}_{S_2}$

- usually, expect $\xi_2 < \xi_1$ (c.f. ANOVA table in LNp.7-3)
  $\Rightarrow$ the design give more precision on the estimation & testing of $\begin{cases}\text{main effects of } C \\ \text{2-fi } HC\end{cases}$

(Bailey) 8.3 Split-plot design

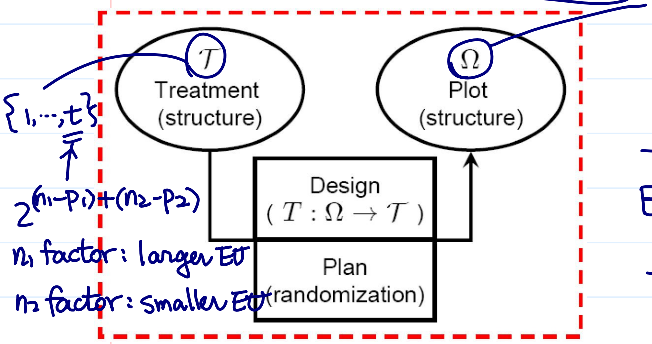


without block
plot structure: m/k $\quad$ ($b\cdot s = m$)

with block
plot structure: $b/s/k$

nesting $(b/s/k)$ — factor H, factor C

	larger exp'tal unit	smaller exp'tal units	obs'nal units
Bailey	called larger blocks	called smaller blocks	plots
	whole plot		subplot



Example 8.2 (Example 1.5 continued: Rye-grass) Here the large blocks are the fields and the small blocks are the strips. Thus $b=2$, $s=3$ and $k=4$.

Example 8.3 (Animal-breeding) In animal breeding it is typical to mate each *sire* (male parent) with several *dams* (female parents), each of which may then produce several offspring. Then the large blocks are the sires, the small blocks are the dams, and the plots are the offspring.

restricted randomization

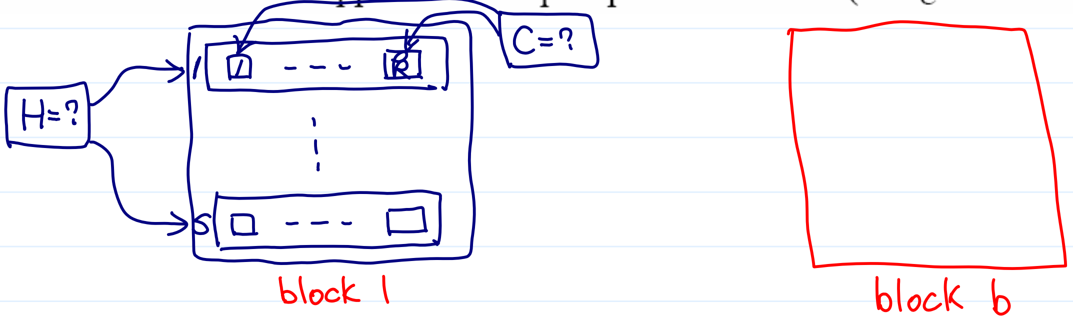
Construction and randomization

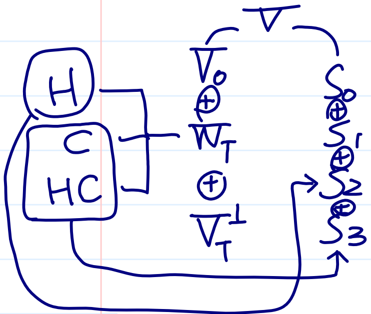
(i) Apply levels of H to small blocks just as in a complete-block design.
→ larger EU

(ii) Within each small block independently, apply levels of C just as in a completely randomized design.

For simplicity, I shall describe only the classic *split-plot* design. This is like the second design in Section 8.2 except that

- the large units (small blocks) are grouped into b large blocks of size s;
- each level of H is applied to one small block per large block (so $n_H = s$ and $r_H = b$);
- each level of C is applied to one plot per small block (so $n_C = k$ and $r_C = 1$).





Stratum		df
W_0	mean	1
W_B	large blocks	$b-1$
W_S	small blocks	$b(s-1)$
$V_S^\perp$	plots	$bs(k-1)$
	Total	bsk

plot structure

$b/s/k$

larger EU ↕ H

smaller EU ↕ C

d.f. identity

$\underbrace{1}_{S_0}+\underbrace{(b-1)}_{S_1}+\underbrace{b(s-1)}_{S_2}+\underbrace{bs(k-1)}_{S_3}$

Stratum	source	df	SS	EMS	VR
mean	mean	1	$\frac{\text{sum}^2}{bsk}$	$\Vert\tau_0\Vert^2+\xi_0$	–
large blocks	large blocks	$b-1$	$\text{CSS}(B)-\text{SS(mean)}$	ξ_B	–
small blocks	H	$s-1$	$\text{SS}(H)$	$\frac{\Vert\tau_H\Vert^2}{s-1}+\xi_S$	$\frac{\text{MS}(H)}{\text{MS(small blocks residual)}}$
	residual	$(b-1)(s-1)$	by subtraction	ξ_S	–
	total	$b(s-1)$	$\text{CSS}(S)-\text{CSS}(B)$		
plots	C	$k-1$	$\text{SS}(C)$	$\frac{\Vert\tau_C\Vert^2}{k-1}+\xi$	$\frac{\text{MS}(C)}{\text{MS(plots residual)}}$
	$H\wedge C$	$(s-1)(k-1)$	$\text{SS}(H\wedge C)$	$\frac{\Vert\tau_{HC}\Vert^2}{(s-1)(k-1)}+\xi$	$\frac{\text{MS}(H\wedge C)}{\text{MS(plots residual)}}$
	residual	$(b-1)s(k-1)$	by subtraction	ξ	–
	total	$bs(k-1)$	$\Sigma y_\omega^2-\text{CSS}(S)$		
Total		bsk	$\Sigma_\omega y_\omega^2$		

large units

LNp.7-6

$= bs-1 = m-1$

usually, $\xi < \xi_S < \xi_B$

(c.f. ANOVA table in LNp.7-6)

plot factor $\mathcal{V}_1 \;\; \mathcal{V}_2 \;\; \mathcal{V}_3 \; \cdots \; \mathcal{V}_k$

In general, plot structure : $n_1/n_2/n_3/\cdots/n_k$

treatment factors

$A_1, \cdots, A_{t_1}$ $\qquad EU_A \qquad EU_B$

$B_1, \cdots, B_{t_1}$

↳ cause restriction on design : $T: \Omega \to \mathcal{T}$

⋮

In the assignment of design key, i.e.,

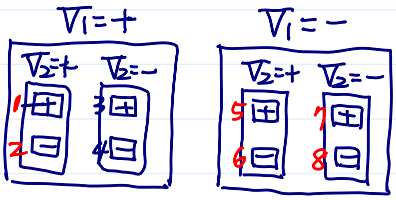
$A_1 = \mathcal{V}_1 \cdots \mathcal{V} \qquad B_1 = \mathcal{V} \cdots \mathcal{V}$

⋮

$A_{t_1} = \mathcal{V} \cdots \mathcal{V}. \qquad B_{t_2} = \mathcal{V} \cdots \mathcal{V}.$

A cannot confound with plot effects involving $\mathcal{V}_3, \mathcal{V}_4, \cdots, \mathcal{V}_k$

A must be confounded with plot effects formed by $\mathcal{V}_1$ or $\mathcal{V}_2$ or both.



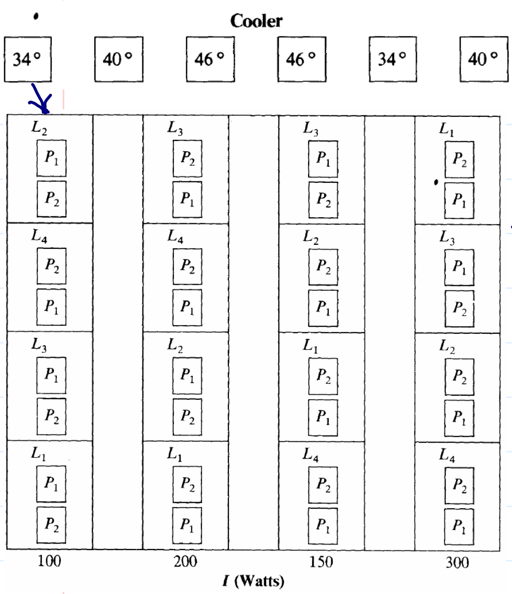
	$\mathcal{V}_1$	$\mathcal{V}_2$	$\mathcal{V}_3$	$A = \mathcal{V}_1\mathcal{V}_2$	$A = \mathcal{V}_1\mathcal{V}_2\mathcal{V}_3$
1	+	+	+	+	+
2	+	+	−	+	−
3	+	−	+	−	−
4	+	−	−	−	+
5	−	+	+	−	−
6	−	+	−	−	+
7	−	−	+	+	+
8	−	−	−	+	−

$A = \mathcal{V}_1$ ← not a good design

$A = \mathcal{V}_2$ → eligible.

(reading material 03)

A meat scientist wants to study the effect of temperature (T) with three levels, types of packaging (P) with two levels, types of lighting (L) with four levels, and intensity of light (I) with four levels on the color of meat stored in a meat cooler for seven days. Six coolers are available for the experiment, and the three temperatures (34°F, 40°F, and 46°F) are each assigned at random to two coolers,



$N = 2t$

$EU_T \quad EU_I \quad EU_P$

$6/4/4/2 \Rightarrow 192$ plots

EU_L

$\{1, \cdots, 3\times2\times4\times4\} = 96$

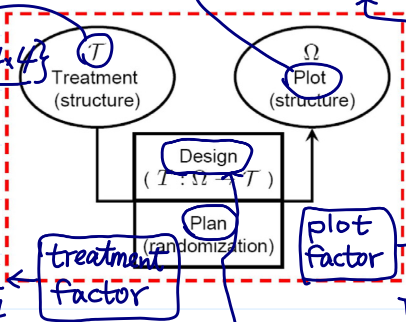
factorial structure

$T \times P \times L \times I$

3 2 4 4

$L_1 \; L_2 \; I_1 \; I_2$

2 2 2 2



Q: What are their replicates?

$6/4/4/2$

$\mathcal{V} \;\; \mathcal{W} \;\; \mathcal{X} \;\; \mathcal{Y}$

$\mathcal{V}_1 \; \mathcal{V}_2 \;\; \mathcal{W}_1 \mathcal{W}_2 \;\; \mathcal{X}_1 \; \mathcal{X}_2$

2 3 2 2 2 2

design key

$T = \mathcal{V}_2$

$I_1 = \mathcal{W}_1$
$I_2 = \mathcal{W}_2$ ← $\mathcal{V}, \mathcal{W}$

$L = \mathcal{X}$ ← $\mathcal{V}, \mathcal{W}, \mathcal{X}$

$P = \mathcal{Y}$ ← $\mathcal{V}, \mathcal{W}, \mathcal{X}, \mathcal{Y}$

jointly made by Ching-Shui Cheng (Berkeley) and Shao-Wei Cheng (NTHU)

Source of Variation	df	
Cooler Analysis		
S_1 T	2	5
Error(Cooler)	3	
Column Analysis		
Block	5	
S_2 I	3	18
I * T	6	
Error(Column)	9	
Partition Analysis		
Block	23	
S_3 L	3	72
L * T	6	
L * I	9	
L * I * T	18	
Error(Partition)	36	
Half-Partition Analysis		
Block	95	
S_4 P	1	96
P * T	2	
P * I	3	
P * I * T	6	
P * L	3	
P * L * T	6	
P * L * I	9	
P * L * I * T	18	
Error(Half-Partition)	48	

strata:

6/4/4/2

d.f. identity:

$$\underbrace{1}_{S_0}+\underbrace{(6-1)}_{S_1}+\underbrace{6\cdot(4-1)}_{S_2}+\underbrace{6\cdot4(4-1)}_{S_3}+\underbrace{6\cdot4\cdot4\cdot(2-1)}_{S_4}$$

$T=V_2$ $T=V_2$

$I=W$ $I=W$

$L=X$ $TI: V_2W$

$P=Y$ $L=X$

$LT=XV_2$

$LI=XW$

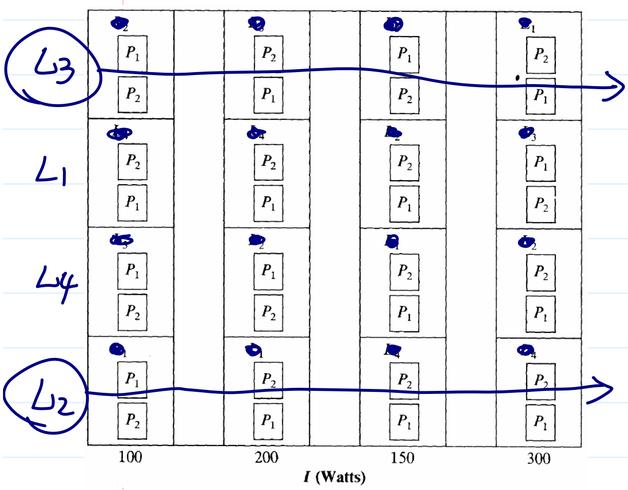
$LTI=XV_2W$

⋮

If V, W, X, Y are handled using fixed effect approach:

T cannot be estimated.

S_2 $\begin{cases} 3 - I \\ 6 - I{\times}T \\ 3 - W \end{cases}$ 6



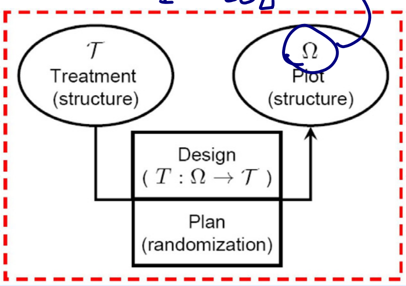
T

I

L

P

$$\overset{EU_T}{6}/(\underset{EU_I}{4}\times\underset{EU_L}{4})/\underset{EU_P,\ OU}{2}$$



d.f. identity

$N(n_1, N(C(n_2,n_3),n_4))$

6/(4x4)/2

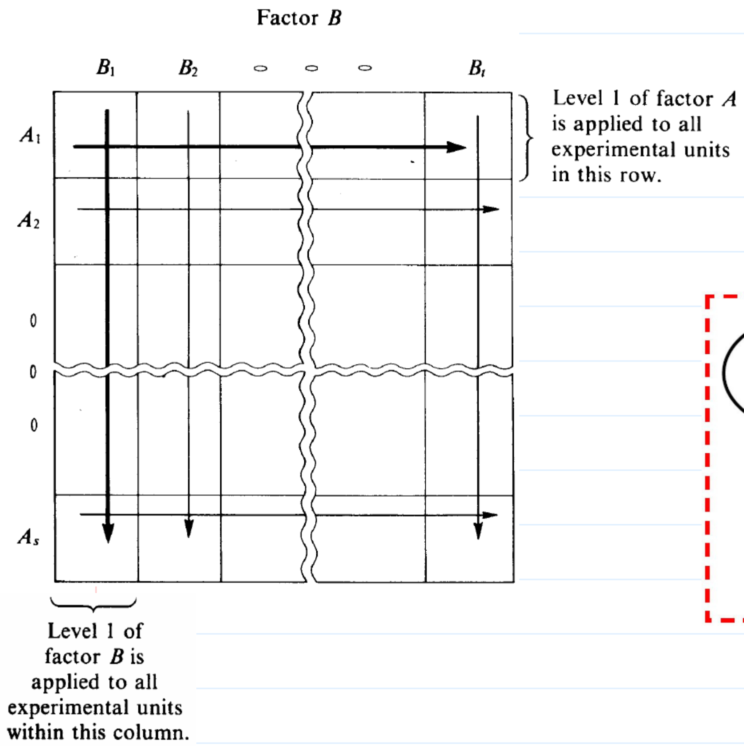
V W X Y

On the choice of design key:

I can be confounded with plot effects involving V or W or VW

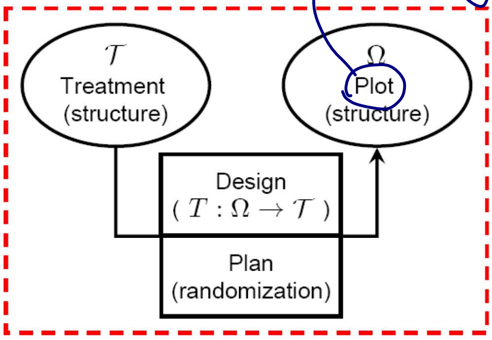
L : : : V or X or VX

(reading material 4) strip-plot design



EU_A EU_B
$\downarrow$ $\downarrow$
$n_1 \times n_2$

crossing



		Irrigation Method I_1	Irrigation Method I_2
Nitrogen Level	N_1	55	71
	N_3	69	78
	N_2	62	77

		Irrigation Method I_1	Irrigation Method I_2
Nitrogen Level	N_2	70	78
	N_3	79	80
	N_1	63	77

		Irrigation Method I_2	Irrigation Method I_1
Nitrogen Level	N_3	81	77
	N_1	77	63
	N_2	79	66

		Irrigation Method I_1	Irrigation Method I_2
Nitrogen Level	N_3	76	79
	N_2	66	76
	N_1	65	75

d.f. identity

$N(n_1, c(n_2,n_3))$

$= 1+(n_1-1)+n_1\cdot(c(n_2,n_3)\cancel{-1})$

$\cancel{1}+(n_2-1)+(n_3-1)+(n_2-1)(n_3-1)$

$= \underbrace{1}_{S_0}+\underbrace{(n_1-1)}_{S_1}+\underbrace{n_1(n_2-1)}_{S_2}+\underbrace{n_1(n_3-1)}_{S_3}+\underbrace{n_1(n_2-1)(n_3-1)}_{S_4}$

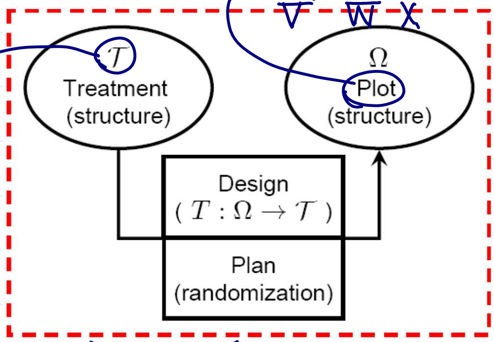
	Source of Variation	df
S_0 1	Replication or Set	3
S_1 3 — V	N	2
	Error(Row)	6
S_2 8 — W, VW	I	1
	Error(Column)	3
S_3 4 — X, XV	I*N	2
S_4 8 — XW, XWV	Error(Unit)	6

$\{1, \ldots, 3\times 2\}$ $= 6$

factorial structure

$N \times I$
3 2

EU_N EU_I
$\downarrow$ $\downarrow$
$\underset{V}{4}/(\underset{W}{3}\times\underset{X}{2})$



$\boxed{N = W \quad I = X \quad NI = WX}$ design key