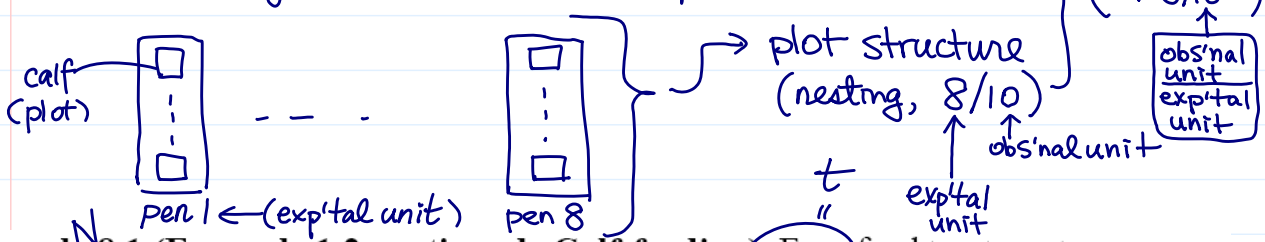
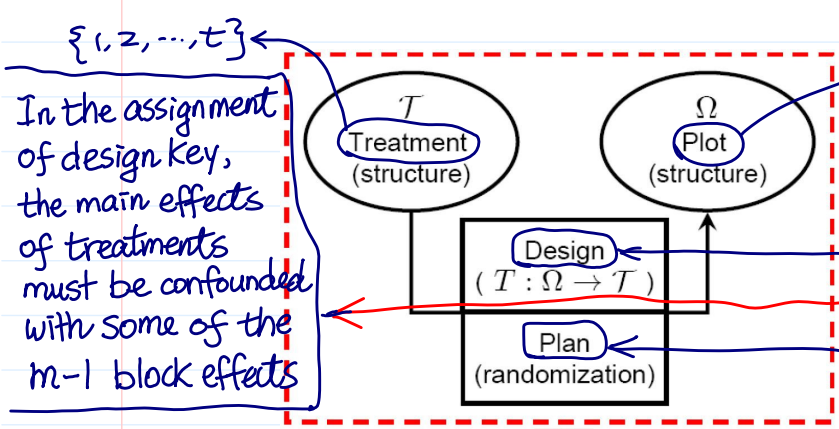
(Bailey) Chapter 8. Small units inside large units

Recall: experimental units
observational units (plot)
LNp.1-4, 1-8.

- an exp'tal unit = a Obs'nal unit
  ⇒ no restriction on the assignment of treatments to plots
  (Note: design: $T:\Omega\to\mathcal{T}$ is the assignment)
- an exp'tal unit ⊃ several obs'nal units (due to practical reasons)
  ⇒ assignment of treatment to plots is restricted



Example 8.1 (Example 1.2 continued: Calf-feeding) Four feed treatments are compared on 80 calves. The calves are kept in eight pens, with 10 calves per pen. Each feed is applied to two whole pens, with calves feeding ad lib. Even though every calf is weighed individually, differences between feeds should be compared with differences between pens, not with differences between calves. So should we say that the replication is 20 or 2?



m blocks
nesting (m/k)
exp'tal unit
obs'nal unit (plot)
restricted, because exp'tal unit ⊃ obs'nal units

① restricted randomization due to the plot structure.
⇒ cause cov(Y) = $\begin{bmatrix} \sigma^2+\sigma_p^2 & \sigma_p^2 \\ \sigma_p^2 & \sigma^2+\sigma_p^2 \end{bmatrix}$
② no need to do within-block randomization

Q: what happens if use fix-effect approach for the m blocks?
⇒ cannot estimate treatment effects

Example 1.8 is similar: treatments are applied to whole classes but individual children are measured. So is one version of Example 1.11: pullets are fed but eggs are weighed. So are Examples 7.7 and 7.8: treatments are applied to whole doctors' practices or to villages, but it is individual people that are measured.

- Now, suppose that ① there are m exp'tal units, each consisting of k obs'nal units ② there are t treatments, each of which is applied to s exp'tal units (⇒ m = t·s)

plot structure

Stratum	source	df	SS	EMS	VR
mean	mean	1	$\frac{\text{sum}^2}{N}$	$\|\|\tau_0\|\|^2+\xi_0$	–
pens	feed	$t-1$	$\frac{\sum \text{sum}_{T=i}^2}{sk}-\frac{\text{sum}^2}{N}$	$\frac{\|\|\tau_T\|\|^2}{t-1}+\xi_1$	$\frac{\text{MS(feed)}}{\text{MS(residual)}}$
	residual	$(m-1)-(t-1)$	... by subtraction ...	ξ_1	–
	total	$m-1$	$\frac{\sum P_i^2}{k}-\frac{\text{sum}^2}{N}$		
calves	calves	$m(k-1)$	$\sum y_\omega^2-\frac{\sum P_i^2}{k}$	ξ_2	–
Total		N	$\sum_\omega y_\omega^2$		

$8\times(10-1)=72$, $72/4=18$

MS(calves)

- If $s=1 \Rightarrow m=t \Rightarrow$ no residual in pen stratum left for F-test
  - Sometimes, people wrongly compare MS(feed) to MS(calves)
  - If $\rho_1>\rho_2 \Rightarrow \xi_{\rho_1}>\xi_{\rho_2} \Rightarrow$ likely to detect differences that do not really exists $\Rightarrow$ known as false-replication.
  - In the designing stage, try not to cause $s=1$ situation.
- even when $s\neq 1$, some people mistakenly combine SS for residuals in pen stratum with SS for calves Stratum, then.

$$E(MS)=\frac{(m-t)\xi_{\rho_1}+m(k-1)\xi_{\rho_2}}{N-t}<\xi_{\rho_1} \text{ if } \rho_1>\rho_2$$

7-3

(Bailey) 8.2.

4 treatments $=2^2$ hay cake

Example 8.1 revisited (Calf-feeding) Suppose that the four feeds consist of all combinations of two types of hay, which is put directly into the pen, with two types of cake, which is fed to calves individually. If all calves in the same pen have the same type of cake then the design and analysis are just as in Section 8.1, except that the treatments line in the analysis of variance is split into three, giving the skeleton analysis of variance in Table 8.5.

"exp'tal unit for cake" ← calf (plot)

plot ← "exp'tal unit for hay" ← pen

pen 8

⇒ different exp'tal units for different treatment factors

large exp'tal unit

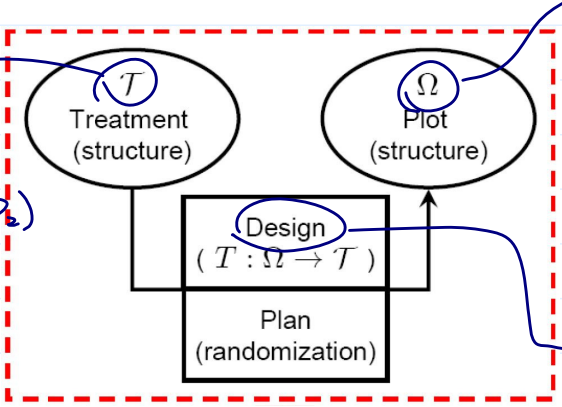
small exp'tal unit

Stratum	source	degrees of freedom
mean	mean	1
pens	hay	1
	cake	1
	hay ∧ cake	1
	residual	4
	total	7
calves	calves	72
Total		80

Stratum	source	degrees of freedom
mean	mean	1
pens	hay	1
	residual	6
	total	7
calves	cake	1
	hay ∧ cake	1
	residual	70
	total	72
Total		80

It might be better to give five calves in each pen one type of cake and the other five calves the other type of cake.

7-4



$\{1, \cdots, t\}$

say, $\sum (n_1 - p_1) + (n_2 - p_2)$

n_1 factors with larger exp'tal units

n_2 factors with smaller exp'tal units

Nesting (m/k) — m blocks

larger exp'tal unit | smaller exp'tal unit | obs'nal unit.

restricted, restriction imposed on the n_1 factors.

In the assignment of design key, the n_1 factors must be confounded with some of the $(m-1)$ block effects, but the n_2 factors can be confounded with any plot effects

Suppose that

① there are m large units, each of which consists of k small units

② there are n_H levels of factor H, each of which applied to r_H large units

③ there are n_C levels of treatment factor C, each of which applied to r_C small units per large unit

Then, (i) $n_C \cdot r_C = k$ (ii) # of treatments $= n_H n_C$, each of which has $r_H r_C$ replicates (iii) $N = m \cdot k = n_H n_C r_H r_C$.

7-5