$N = t$

A quarter replicate design for a crop experiment

$n_1/n_2/n_3$

$N(n_1, N(n_2, n_3))$

$= 1 + (n_1-1) + n_1(N(n_2,n_3)-1)$

$\quad + (n_2-1) + n_2(n_3-1)$

$= 1 + (n_1-1) + n_1(n_2-1) + n_1 n_2(n_3-1)$

$\quad\; \underbrace{}_{S_0} \quad \underbrace{}_{S_1} \qquad \underbrace{}_{S_2} \qquad \underbrace{}_{S_3}$

Treatment structure	Block / plot structure
$B \times C \times D \times E$	$U/V/W$
4　2　2　4	2　4　2 = 16
$B_1 B_2$　　　E_1	V_1　V_2

64 → ¼ → 16 → 2^4 → 2^{6-2}

Design key

B_1	B_2	C	D	E_1　E_2
Q_1	Q_3	Q_3	Q_4	$Q_1 Q_2 Q_3$　$Q_2 Q_4$
W	$V_2 W$	$UV_1 V_2 W$	V_1	$UV_1 W$　$V_1 V_2 W$

Inverse key

U	V_1	V_2	W	I_1	I_2
$B_1 DE_1$	D	$B_1 B_2$	B_1	$B_1 B_2 CE_1$	$B_2 DE_2$

I_3 ↙ CDE_1E_2
$B_1CDE_1E_2$

construct design matrix
$T: \Omega \to \Upsilon$

$B_1B_2DE_2 = B_2CE_1 = B_1$
$\qquad\qquad\qquad = B_2$
$\qquad\qquad\qquad = B_1 B_2$
$\qquad\qquad\qquad = C$
$\qquad\qquad\qquad = B_1 C$
$\qquad\qquad\qquad = B_2 C$
$\qquad\qquad\qquad B_1 B_2 C$
$\qquad\qquad\qquad D$
$\qquad\qquad\qquad B_1 D$
$\qquad\qquad\qquad B_2 D$
$\qquad\qquad\qquad BB_2 D$
$\qquad\qquad\qquad CD$
$\qquad\qquad\qquad B_1 CD$
$\qquad\qquad\qquad B_2 CD$
$\qquad\qquad\qquad B_1 B_2 CD$

treatment structure ∇ 　 plot structure drm
$\nabla_0 = S_0$
⊕　　　⊕
W_T　　S_1　　$1 - U$
⊕　　　⊕
$\cancel{\nabla_T}$ 1 　S_2　6　→ $\overline{V_1}$, $\overline{V_2}$, $\overline{V_1V_2}$, $\overline{UV_1}$, $\overline{UV_2}$, $\overline{UV_1V_2}$
⊕
　　　S_3　8　→ $\overline{W}$, $\overline{UW}$, $\overline{V_1W}$, $\overline{V_2W}$, $\overline{V_1V_2W}$, ⋮

$N = 2t$

<u>Recall</u>: in Bailey ch2, ch4, ch6, ∇_T lies in a stratum.

⇒ Some plot effects not confounded treatment effects.

Q: why does it happen?

A multiple replicate design

$n_1 \times (n_2/n_3)$

$C(n_1, N(n_2, n_3))$

$= 1 + (n_1-1) + (N(n_2,n_3)-1)$
$\quad + (n_1-1)(N(n_2,n_3)-1)$

$\quad + (n_2-1) + n_2(n_3-1)$

$= 1 + \underbrace{(n_1-1)}_{S_1} + \underbrace{(n_2-1)}_{S_2} + \underbrace{n_2(n_3-1)}_{S_4}$
$\underbrace{}_{S_0}$
$\quad + \underbrace{(n_1-1)(n_2-1)}_{S_3} + \underbrace{(n_1-1)n_2(n_3-1)}_{S_5}$

Treatment structure	Block / plot structure
$A \times B \times C \times D$	$U \times (V/W)$
2　2　2　2	4　4　2 = 32
	$U_1 U_2$　$V_1 V_2$

16 =

Design key

A	B	C	D
$U_1 W$	$V_1 W$	$V_2 W$	$V_1 V_2 W$

drm:

∇_0　⊕	S_0 1	⊕
W_T　⊕	S_1 3 { $\overline{U_1}$, $\overline{U_2}$, $\overline{U_1U_2}$ }	⊕
$\nabla_T^\perp$　↑	S_2 3 { $\overline{V_1}$, $\overline{V_2}$, $\overline{V_1V_2}$ }	⊕
	S_3 9 { $\overline{U_1V_1}$, $\overline{U_2V_1}$, $\overline{U_1U_2V_1}$; $\overline{U_1V_2}$ ⋮ ; $\overline{U_1V_1V_2}$ ⋮ }	⊕
$\nabla_T^\perp$ not an empty set	S_4 4 { $\overline{W}$—BCD, $\overline{V_2 W}$—C ; $\overline{V_1 W}$—B, $\overline{V_1V_2W}$—D }	⊕
	S_5 12 { $\overline{U_1 W}$, $\overline{U_1 V_1 W}$, $\overline{U_1V_2W}$, $\overline{U_1V_1V_2W}$; $\overline{U_2W}$ ⋮ ; $\overline{U_1U_2W}$ ⋮ }	

← no residuals left for F-test (within stratum)

– alternative: assume BCD not important, there would be one degree of freedom left for testing.

jointly made by Ching-Shui Cheng (Berkeley) and Shao-Wei Cheng (NTHU)

For asymmetric design:

① design key takes the same form as before, i.e, it's a list of plot aliases of main effects of treatment factors

An asymmetric design

$\boxed{N=2t}$

Treatment structure	Block structure	②

$A \times B \times C$　　　　$X \times Y \times Z$

$36 = (\,2\quad 3\quad 6\,)$　　$(\,6\quad 6\quad 2\,) = 72$

choice of plot aliases is limited, i.e., cannot use P_j as a plot alias for T_i if their # of levels are different.

$C_1\ C_2$ Design key　$X_1\ X_2\ Y_1\ Y_2$
　2　3　　　　　　　2　3　2　3

A	B	C_1	C_2
$X_1\,Y_1$	$X_2\,Y_2$	$X_1\,Z$	$X_2\,Y_2^2$
2	3	2	3

Stratum contents

Stratum	d.f.	Treatment effects
X	5	BC_2
Y	5	BC_2^2
Z	1	—
XY	25	$A, B, C_2, A.B, A.C_2, A.BC_2, A.BC_2^2$ ⟹ 25-13=12 d.f. left for residuals.
XZ	5	$C_1, C_1.BC_2$
YZ	5	$AC_1, AC_1.BC_2^2$
XYZ	25	$B.C_1, C_1.C_2, C_1.BC_2^2, AC_1.B, AC_1.C_2, AC_1.BC_2$

Rules of construction

$q(A) = q(X_1)+q(Y_1) \quad \text{mod } 2$
$q(B) = q(X_2)+q(Y_2) \quad \text{mod } 3$
$q(C_1) = q(X_1)+q(Z) \quad \text{mod } 2$
$q(C_2) = q(X_2)+2q(Y_2) \quad \text{mod } 3$

B: 3-level
C_2: 3-level
$\begin{bmatrix} BC_2 : \text{3-level} \\ BC_2^2 : \text{3-level} \end{bmatrix}$

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6　6　2
$n_1 \times n_2 \times n_3$

$C(n_1, C(n_2, n_3))$

$= 1 + (n_1-1) + (C(n_2,n_3)-1) + (n_1-1)\cdot(C(n_2,n_3)-1)$

↳ $1 + (n_2-1) + (n_3-1) + (n_2-1)(n_3-1)$

$= 1 + \underbrace{(n_1-1)}_{S_1} + \underbrace{(n_2-1)}_{S_2} + \underbrace{(n_3-1)}_{S_3} + \underbrace{(n_1-1)(n_2-1)}_{S_4} + \underbrace{(n_1-1)(n_3-1)}_{S_5} + \underbrace{(n_2-1)(n_3-1)}_{S_6}$
$+ \underbrace{(n_1-1)(n_2-1)(n_3-1)}_{S_7}$

$V \sim$　dim

S_0	1	
⊕		
S_1	5	$\begin{bmatrix} X_1 - 1 \\ X_2 - 2 \\ X_1X_2 - 2 \end{bmatrix}$
⊕		
S_2	5	$\begin{bmatrix} Y_1 - 1 \\ Y_2 - 2 \\ Y_1Y_2 - 2 \end{bmatrix}$
⊕		
S_3	1	$- Z - 1$
⊕		
S_4	25	$\begin{bmatrix} X_1Y_1 - 1 & X_2Y_1 - 2 & X_1X_2Y_1 - 1 \\ X_1Y_2 - 2 & X_2Y_2 - 2 & X_1X_2Y_2 - 2 \\ X_1Y_1Y_2 - 2 & X_2Y_2^2 - 2 & X_1X_2Y_2^2 - 2 \\ & X_2(Y_1Y_2) - 2 & X_1X_2Y_1Y_2 - 2 \\ & X_2(Y_1Y_2)^2 - 2 & X_1X_2(Y_1Y_2)^2 - 2 \end{bmatrix}$
⊕		
S_5	5	
⊕		
S_6	5	
⊕		
S_7	25	

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