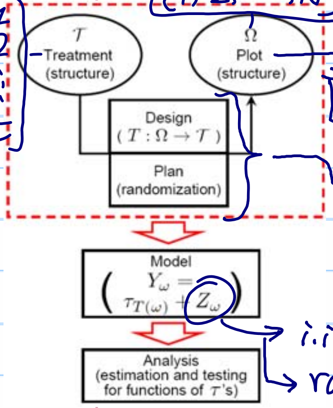
Bailey, Section 2.1. completely randomized design

$\{1, 2, \cdots, N\}$



1, 2, $\vdots$, t

no structure

completely randomized design

$\to$ i.i.d $N(0, \sigma^2 I)$

$\to$ randomization

- Ω is called unstructured if no reason to group the plots into blocks. (In other words, all plots are homogeneous $\Rightarrow$ no systematic difference)
- Suppose that treatment i applied to r_i plots

$$\sum_{i=1}^{t} r_i = |\Omega| = N.$$

a completely randomized design:

design
(1) number the plots $1, 2, \cdots, N$
(2) apply treatment 1 to $1 \sim r_1$ plots, treatment 2 to $r_1+1, \cdots, r_1+r_2$ plots, and so on

plan
(3) choose a random permutation of $\{1, \cdots, N\}$ and apply it to actual plots.

actual plot: $\nearrow 10 \quad \nearrow 8 \quad \nearrow 5 \; \cdots$
plot label: $\underbrace{1, 2, 3,}_{A} \; \underbrace{\cdots}_{B} \; , N$ $\Rightarrow$ generate a probability space ($N!$ element)

Start with an initial design

F	F	F	F	F	F	F	F	F	F	F	F
F	F	F	F	F	F	F	F	F	F	F	F
F	F	F	F	F	F	F	F	F	F	F	F

Randomly permute (labels of) the experimental units

Complete randomization: Pick one of the 72! Permutations randomly

4 treatments

1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	2	2	2	2	2	2
2	2	2	2	2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3	3	3	3	3
3	3	3	3	3	3	4	4	4	4	4	4
4	4	4	4	4	4	4	4	4	4	4	4

Pick one of the 72! Permutations randomly

Completely randomized design

Sec 2.2. why & how to randomize (reading assignment)

Sec 2.3. treatment space

$|\Omega| = N$, $\mathbb{Y} = \begin{bmatrix} y_1 \\ \vdots \\ y_\omega \\ \vdots \\ y_N \end{bmatrix} \in \mathbb{R}^{|\Omega|} \equiv V$

Def: The treatment space of V consists of those vectors in V which are constant on each treatment, denoted by V_T.

Def: A vector v in V is a treatment vector if $v \in V_T$.
It's a treatment contrast if $v \in V_T$ and $\sum_{\omega \in \Omega} v_\omega = 0$.
$\sum_{\omega \in \Omega} v_\omega = \mathbb{1} \cdot v$

Def: 1. scalar product: $u, v \in V$.

$$u \cdot v = \sum_{\omega \in \Omega} u_\omega \cdot v_\omega$$

2. $v \cdot v = \sum v_\omega^2 = \|v\|^2$ is called sum of square or squared length of v.

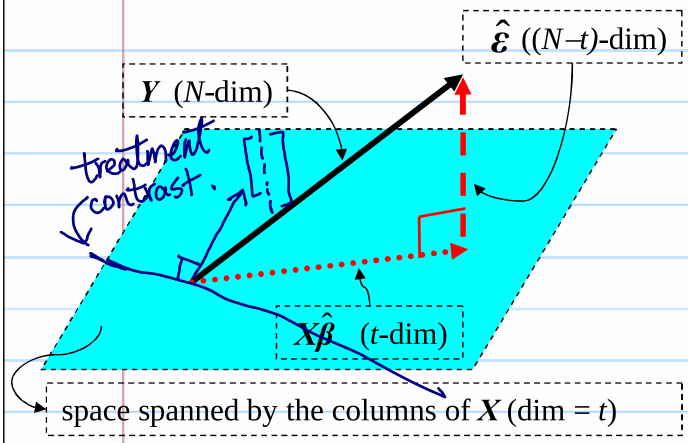
3. v & w are orthogonal (denoted by $v \perp w$) if $v \cdot w = 0$

- observed data $(y_i, x_{1i}, \ldots, x_{mi})$, $i=1,\ldots,N$.

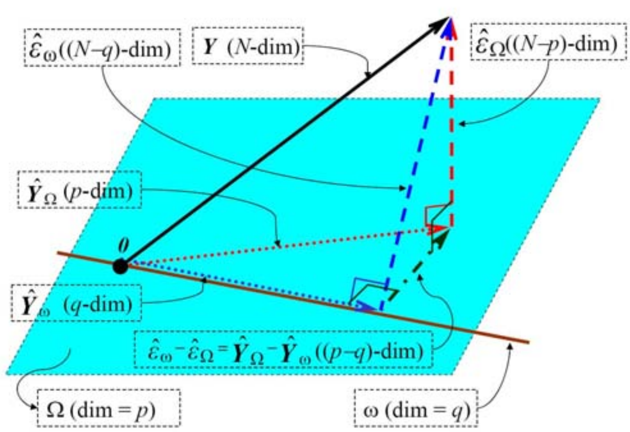
$$Y_i = \sum_{j=1}^{t} \beta_j \cdot g_j(x_{1i}, \ldots, x_{mi}) + \epsilon_i,$$

$$E(\epsilon_i) = 0,\ Var(\epsilon_i) = \sigma^2,\ Cor(\epsilon_{i_1}, \epsilon_{i_2}) = 0,\ \text{for } i_1 \neq i_2$$

$$\begin{bmatrix} y_1 \\ y_2 \\ \cdots \\ y_N \end{bmatrix} = \boldsymbol{Y} = \boldsymbol{X\beta} + \boldsymbol{\varepsilon} = \beta_0 \begin{bmatrix} 1 \\ 1 \\ \cdots \\ 1 \end{bmatrix} + \beta_1 \begin{bmatrix} g_{11} \\ g_{21} \\ \cdots \\ g_{N1} \end{bmatrix} + \cdots + \beta_{t-1} \begin{bmatrix} g_{1\,t-1} \\ g_{2\,t-1} \\ \cdots \\ g_{N\,t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \cdots \\ \epsilon_N \end{bmatrix}$$



- a geometric view of H_0: ω v.s. H_1: Ω\ω



Regression course website: http://www.stat.nthu.edu.tw/~swcheng/Teaching/stat5040/

Prop 2.1. For each treatment i, let u_i be the vector whose value on plot w is equal to

$$\begin{cases} 1 & \text{if } T(w) = i \\ 0 & \text{o.w.} \end{cases}$$

Then $\{u_i \mid i \in \mathcal{J}\}$ is an orthogonal basis for V_T.

Cor 2.2. If there are t treatments, $\dim(V_T) = t$.

Def: 1. $\tau = \{\tau_{T(w)} \mid w \in \Omega\}$ is called unknown treatment parameters.
$\quad\ \ = X\beta$

2. $E[Y] = \tau$

3. $\hat{\tau}$ is called fitted value of τ
$\quad\ \ = X\hat{\beta}$

4. $\tau, \hat{\tau} \in V_T$

$13 = \lvert\Omega\rvert$	Ω	1	2	3	4	5	6	7	8	9	10	11	12	13		
	T	B	A	C	C	B	A	B	A	C	C	A	B	A		$t=3$
	typical $\mathbf{v}$	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9	v_{10}	v_{11}	v_{12}	v_{13}		
	data $\mathbf{y}$	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}	y_{12}	y_{13}		
treatment vectors	some	1	0	3	3	1	0	1	0	3	3	0	1	0	$= \mathbf{u}_B + 3\mathbf{u}_C$	
	vectors	0	$\frac{1}{5}$	0	0	0	$\frac{1}{5}$	0	$\frac{1}{5}$	0	0	$\frac{1}{5}$	0	$\frac{1}{5}$	$= (\frac{1}{5})\mathbf{u}_A$	treatment contrast
	in V_T	$-\frac{1}{4}$	$\frac{1}{5}$	0	0	$-\frac{1}{4}$	$\frac{1}{5}$	$-\frac{1}{4}$	$\frac{1}{5}$	0	0	$\frac{1}{5}$	$-\frac{1}{4}$	$\frac{1}{5}$	$= (\frac{1}{5})\mathbf{u}_A - (\frac{1}{4})\mathbf{u}_B$	
	orthogonal	0	1	0	0	0	1	0	1	0	0	1	0	1	$= \mathbf{u}_A$	
$X' =$	basis	1	0	0	0	1	0	1	0	0	0	0	1	0	$= \mathbf{u}_B$	$\dim(V_T) = 3$
	for V_T	0	0	1	1	0	0	0	0	1	1	0	0	0	$= \mathbf{u}_C$	
$X\beta$	unknown treatment parameters	τ_B	τ_A	τ_C	τ_C	τ_B	τ_A	τ_B	τ_A	τ_C	τ_C	τ_A	τ_B	τ_A	$= \tau$	
$X\hat{\beta}$	fitted values	$\hat\tau_B$	$\hat\tau_A$	$\hat\tau_C$	$\hat\tau_C$	$\hat\tau_B$	$\hat\tau_A$	$\hat\tau_B$	$\hat\tau_A$	$\hat\tau_C$	$\hat\tau_C$	$\hat\tau_A$	$\hat\tau_B$	$\hat\tau_A$	$= \hat\tau$	