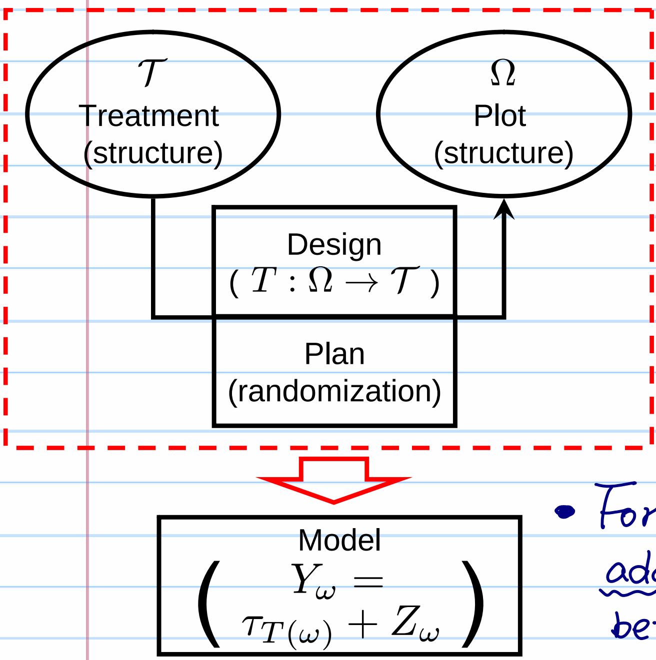


Bailey, Sec. 1.5

- The response on plot ω is Y_ω: a random variable, whose value denoted by y_ω
- collected data vector $\mathbf{y} = \{y_\omega \mid \omega \in \Omega\}$ is a realization of the r.v. $\mathbf{Y}$

- For plot ω, assume treatment-plot additivity (no interaction effects between treatments & plots)

$$Y_\omega = \tau_{T(\omega)} + Z_\omega$$

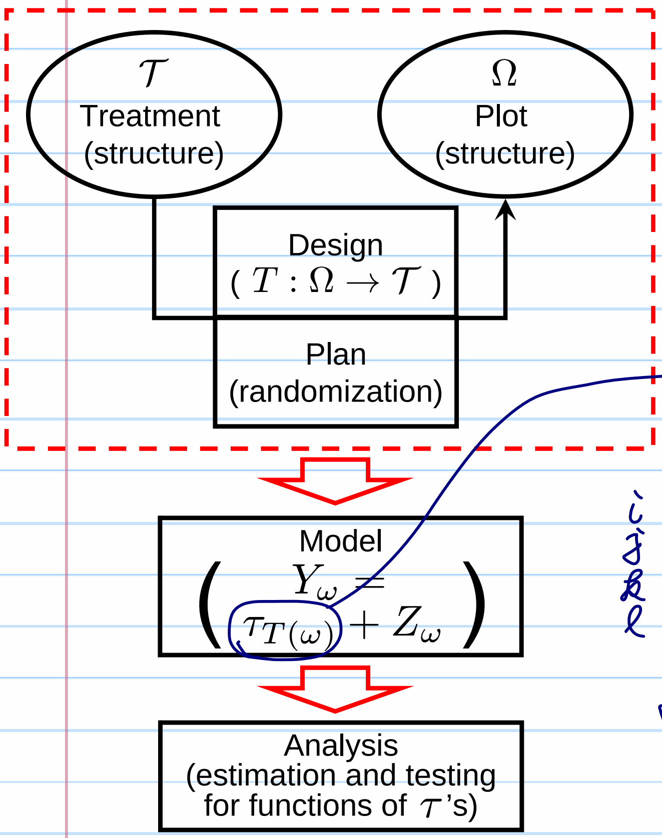
- $\tau_{T(\omega)}$: (1) a constant (2) depend on the treatment $T(\omega)$ (3) can be thought of the contribution of treatment $T(\omega)$
- Z_ω: (1) random variable (2) depend on ω (3) can be thought of the contribution of plot ω

2.34

- Z_ω is a r.v. $\Rightarrow$ Z_ω must have probability space $\Rightarrow$ the set of occasions & uncontrolled conditions.
- Z_α & Z_β are different random variables for different plots α & β $\Rightarrow$ Z_α & Z_β must have a joint distribution, including a correlation (may be zero)
- Q: What should be the joint dist. of $\{Z_\omega \mid \omega \in \Omega\} \equiv \mathbf{Z}_\Omega$

1. simple textbook model
   $Z_\omega \overset{\text{i.i.d.}}{\sim} N(0, \sigma^2) \Rightarrow \mathbf{Z}_\Omega \sim N(\underset{\sim}{0}, \sigma^2 I)$ $\to$ often seen in completely randomized design
2. fixed effect model
   Z_ω are indep. Normal with variance σ^2 & mean μ_ω $\Rightarrow$ $\mathbf{Z}_\Omega \sim N(\mu_\Omega, \sigma^2 I)$ $\to$ often seen in block design.
3. random effect model
   Z_ω have identical (marginal) distribution, $\mathrm{cov}(Z_\alpha, Z_\beta)$ depends on how α, β related, $\to$ often seen in block design, split-plot design.
4. randomization model $\Rightarrow \mathbf{Z}_\Omega \sim N(\underset{\sim}{0}, \Sigma)$
   Z_ω have identical (marginal) distribution, $\mathrm{cov}(Z_\alpha, Z_\beta)$ depends on the method of randomization. $\to$ non-parametric methods.

2.35



Analysis: estimation & testing for function of τ's

$\Downarrow$

$\boxed{\text{linear combinations}}$

$\Uparrow$

$\boxed{\text{depend on the treatment structure}}$

Example: $(E(\underline{Y}) = X\beta)$

1. treatment structure: no structure

e.g. 4 treatments.

$$
\begin{array}{ll}
i & \tau_i \\
j & \tau_j \\
k & \tau_k \\
l & \tau_l
\end{array}
\qquad
\beta_0=\tau_i,\ \beta_1=\tau_j,\ \beta_2=\tau_k,\ \beta_3=\tau_l
$$

$$
X = \begin{array}{c} \\ i \\ j \\ k \\ l \end{array}
\begin{array}{c}
\begin{array}{cccc} \beta_0 & \beta_1 & \beta_2 & \beta_3 \end{array} \\
\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{array}
$$

$\beta_0 = (\tau_i+\tau_j+\tau_k+\tau_l)/4 \equiv \bar{\tau}$

$\beta_1 = \tau_j - \bar{\tau}$

$\beta_2 = \tau_k - \bar{\tau}$

$\beta_3 = \tau_l - \bar{\tau}$

$$
X = \begin{array}{c} \\ i \\ j \\ k \\ l \end{array}
\begin{array}{c}
\begin{array}{cccc} \beta_0 & \beta_1 & \beta_2 & \beta_3 \end{array} \\
\begin{bmatrix} 1 & -1 & -1 & -1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}
\end{array}
$$

2. several treatments v.s. a control

$$
\begin{array}{lll}
\text{control} \leftarrow & i & \tau_i \\
 & j & \tau_j \\
 & k & \tau_k \\
 & l & \tau_l
\end{array}
\qquad
\begin{array}{l}
\beta_0 = \tau_i \\
\beta_1 = \tau_j - \tau_i \\
\beta_2 = \tau_k - \tau_i \\
\beta_3 = \tau_l - \tau_i
\end{array}
$$

$$
X = \begin{array}{c} \\ i \\ j \\ k \\ l \end{array}
\begin{array}{c}
\begin{array}{cccc} \beta_0 & \beta_1 & \beta_2 & \beta_3 \end{array} \\
\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}
\end{array}
$$

3. factorial structure

		A	B	AB
i	τ_i	+	+	+
j	τ_j	+	−	−
k	τ_k	−	+	−
l	τ_l	−	−	+

$\beta_0 = (\tau_i+\tau_j+\tau_k+\tau_l)/4$

$\beta_A = [(\tau_i+\tau_j)-(\tau_k+\tau_l)]/c$

$\beta_B = [(\tau_i+\tau_k)-(\tau_j+\tau_l)]/c$

$\beta_{AB} = [(\tau_i+\tau_l)-(\tau_j+\tau_k)]/c$

$$
X = \begin{array}{c} \\ i \\ j \\ k \\ l \end{array}
\begin{array}{c}
\begin{array}{cccc} \beta_0 & \beta_A & \beta_B & \beta_{AB} \end{array} \\
\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}
\end{array}
$$

Note: all the X-matrices have same column space, i.e., the space spanned by the columns of X, which will be referred to as treatment space.

(Bailey) the data analysis should give

- minimum variance unbiased est'or of τ_i's & linear combinations such as $\tau_i - \tau_j$
- estimate the variance of these est'ors
- inference about present/absent of effects $\Rightarrow$ test $H_0: \beta_i = 0$.