# Blocked Fractional Factorial Split-Plot Experiments for Robust Parameter Design

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## 架構

- 介紹Robust parameter design(RPD)
- We provide an example(A Chrome-Plating Experiment ) and discuss methods of blocking FFSP designs.( the MA design & the RPD scenario )
- We then discuss optimality criteria for ranking BFFSP designs for RPD and present catalogs of optimal BFFSP RPDs for the two situations of interest, with control factors as SP factors & with control factors as WP factors.
- We conclude the article with a brief discussion.

## 前言：

- Fractional factorial experiments is commonly used for robust parameter design and for ease of use,such experiments are often run as split-plot design.
- If the control factors are at the subplot level and the noise factors are at the whole-plot level, this also results in gains in efficiency.
- If all runs of fractional factorial split-plot design cannot be run under homogeneous conditions, such designs are frequently blocked.

## Robust parameter design(RPD)

Robust parameter design(RPD)是對產品實現活動的研究，它強調在選擇製程或者產品的可控因子的水準，達到兩個目的：

- 保證產出反應值的平均數達到所要求的水準跟目標
- 保證產出反應值的變異數儘可能的小

## Factors

- Noise factor：一個會造成系統反應的變數，通常是環境條件，像是溫度或者是相對濕度形成的，它也可能是製程中，不同批原料性質的變化，也可能是製程變數難以控制或者維持的目標值。
- Noise factor在研發的時候是可以控制的，但是在生產或使用時卻不能控制。
- Control factor：在製程或生產過程中可以控制的變數。

## Noise factors

- Ex：以速食蛋糕配方爲例，速食蛋糕配方的生產機器要確定蛋糕的材料及份量，像是麵粉、糖、奶油等，在製造時都可以控制這些變數，但消費者使用這個產品時將這些成份加上水，攪拌成蛋糕糊，再放進烤箱，以指定的時間烘烤，產品的設計者不知道消費者會加入多少水，攪拌蛋糕糊的狀態及正確的烤箱溫度與烤箱時間。產品設計者面對一個穩健設計的問題，他的目的就是不管noise factor造成多大的變異，蛋糕配方會達成甚至超過消費者的期望。

## Robust parameter design(RPD) 的發展

- 田口玄一(Genichi Taguchi)發展一般的RPD問題，並於1980年代介紹到美國。田口氏建議以設計的實驗與一些特別的數據分析方法來處理RPD的問題，他的想法與做法除了廣泛引起工程師與統計學家的注意外，也在1980年代應用於如AT&T貝爾實驗室、福特汽車及全錄等諸多大公司。因田口方法簡單易學、應用方便，所以深受缺乏統計基礎訓練的工程人員所喜愛。

## Robust parameter design(RPD) 的發展

- 在統計或工程圈內，田口的方法引起一些爭辯，爭辯不在於RPD問題本身，而在田口氏倡導的實驗與數據的分析方法，諸多分析顯示田口方法是沒有效率的，有些情形甚至是無效的。隨後有一段時間是對RPD問題新方法的研發時期，其中反應曲面方法(RSM)成爲RPD問題的一種新手法，它包含田口參數設計的觀念，但提供更嚴謹與更有效的設計與分析方法。

## Crossed array

- It consists of an inner array(for control factors) and an outer array(for the noise factors) and , during the conduct of the experiment, all combinations of the inner array and outer array setting are run.
- 即control factors 設計的每一種處理組合，對應noise factors設計的每一種處理組合，都進行實驗，這種實驗稱爲crossed array design(交叉陣列設計)。

## Single array

- Single array：it involve both control and noise factors as a run-saving alternative.
- In running a fractional factorial experiment—based on either a crossed array or a single array—randomization restrictions often result in a split-plot.

## Whole-plot error term and split-plot error term

- In a split-plot design, there are two error terms—the whole-plot(WP)-error term and the split-plot(SP)-erroe term.
- Effects involoving only WP factors, and aliases of these effects, are tested against the WP-error term, other effects are tested against the SP-error term.
- Effects tested against the WP-error term are estimated with less precision than effects tested against the SP-error term.

- In an RPD experiment, we have little interest in noise effects; rather, we are primarily interested in control effects and control-by-noise interactions.
- Thus, if we have a choice, it is natural to run an RPD experiment as a split-plot, with the control factors at the SP level and the noise factors at the WP level.
- Because experimental conditions may not remain homogeneous over all runs of an FFSP experiment, it is often desirable to run an FFSP experiment in blocks.

# A Chrome-Plating Experiment

- In this case study, a company wanted to identify the factors affecting the quality of one of its chrome-plating processes. Six factors, each at two levels, were examined in the experiment.

- *A* = chrome(鉻) concentration
  *B* = chrome-to-sulfate ratio(鉻用硫酸鹽處理的比例)
  *C* = bath temperature(bath的溫度)
  *p* = etching current density(蝕刻電流的密度)
  *q* = plating current density(繪製電流plating的密度)
  *r* = part geometry

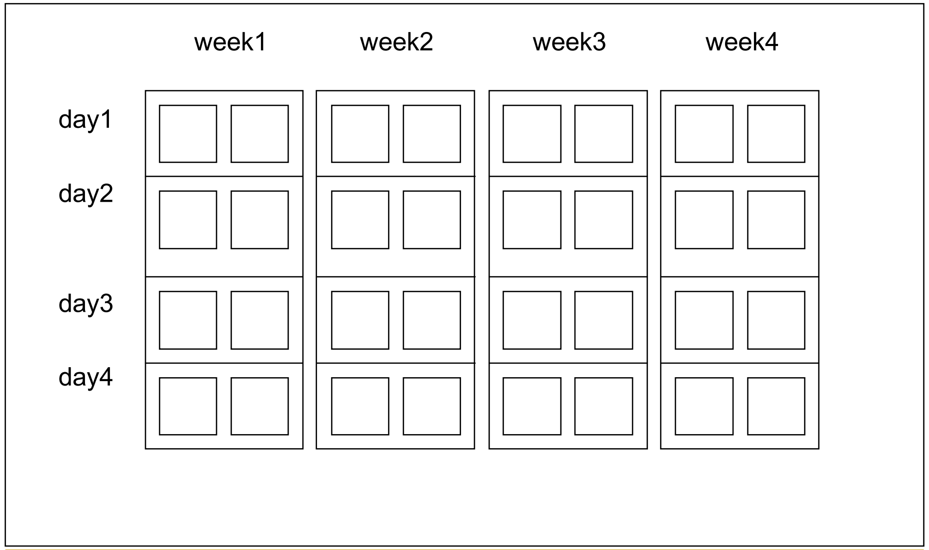
- the experiment could be run for 16 days.With only 32 parts being plated in all, rather than $2^6 = 64$, this resulted in a fractional factorial design.

- Finally, it was desirable to divide the 16 days into four 4-day weeks, with each week being regarded as a block.

- This situation called for a BFFSP design having 3 WP factors, 3 SP factors, and a 4/4/2 *structure*, where structure is a characteristic of any BFFSP design.

- In the chrome-plating experiment, the design required 4 blocks, 4 WPs per block, and 2 SPs per WP.

# A Chrome-Plating Experiment




# The Screening Scenario Revisited

- In McLeod and Brewster (2004), a BFFSP designis represented by $2^{(n_1+n_2)-(k_1+k_2)\pm(b_1+b_2)}$

- n1= the number of WP factors
  n2= the number of SP factors
  k1= the level of fractionation at the WP levels
  k2= the level of fractionation at the SP levels
  b1= the number of pure WP blocking generators( involve only WP factors )
  b2= the number of separators( involve both WP and SP factors, or SP factors alone, and are not aliased with effects involving only WP factors.)

## the MA design

- A, B ,C are three noise factors, p, q, r are three control factors, $\beta_1$ is a pure WP blocking generator, $\delta_1$ is a separator
- r = ABq, $\beta_1$ = ABC, and $\delta_1$= ACpq,
  ∴ n1: n2: k1: k2: b1: b2 = 3: 3: 0: 1: 1: 1
- It can be shown that a minimum aberration (MA) design for screening purposes has generators r = ABq, $\beta_1$ = ABC, and $\delta_1$ = ACpq, using the optimality criteria and tables of optimal designs in McLeod and Brewster (2004)

## the MA design

- → The MA design for the chrome-plating experiment is a $2^{(3+3)-(0+1)\pm(1+1)}$ BFFSP design.
- The defining relation is given by
  I = ABqr = ABC $\beta_1$ = ACpq $\delta_1$ = Cqr $\beta_1$ = BCpr $\delta_1$ = Bpq $\beta_1\delta_1$ = Apr $\beta_1\delta_1$.
- If we focus only on main effects and 2-factor interactions,then we see that all of the main effects are clear, in that they are not aliased with other main effects or 2-factor interactions, or confounded with blocks. Many of the 2-factor interactions are also clear, but some are not, namely: AB, Aq, Ar, Bq, Br, and qr.

## the MA design

- Note that, in using this definition of a clear effect, we are implicitly making the usual assumption that treatment-by-block interactions are negligible. For this reason, AC is clear despite being confounded with B $\beta_1$. However, AB is not clear because it is aliased with qr.

## the RPD scenario

- The question now is whether the MA screening design is also optimal for RPD purposes.In RPD, we are primarily interested in control effects and control-by-noise interactions.With the screening design above, however, some of these effects are not clear, namely: *Aq*, *Ar*, *Bq*, *Br*, and *qr*. We can resolve this problem if we are prepared to sacrifice some of the noise effects.
- A, B ,C are three noise factors, p, q, r are three control factors, $\delta_1$、$\delta_2$ are separators. C= AB, $\delta_1$= Apq, $\delta_2$ = Bpr
  ∴ n1: n2: k1: k2: b1: b2 = 3: 3: 1: 0: 0: 2

## the RPD scenario

- → The RPD scenario for the chrome-plating experiment is a $2^{(3+3)-(1+0)\pm(0+2)}$ BFFSP design.
- The defining relation is given by
  $I = ABC = Apq\,\delta_1 = Bpr\,\delta_2 = BCpq\,\delta_1 = ACpr\,\delta_2 = ABqr\,\delta_1\,\delta_2 = Cqr\,\delta_1\,\delta_2$.
- With this design, none of the noise main effects or 2-factor noise interactions are clear—but all control main effects and 2-factor control interactions are clear, as are all 2-factor control-by-noise interactions. This is a better design for RPD purposes.

## comparison

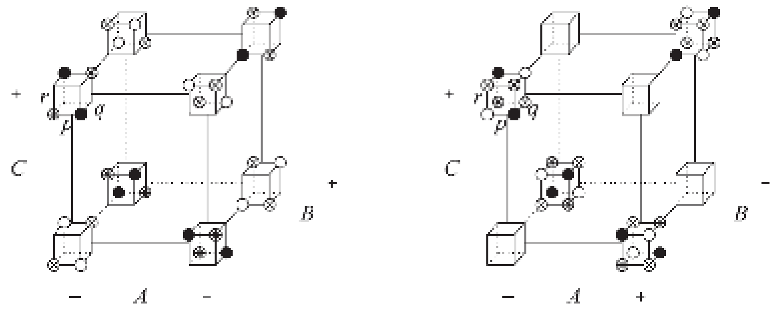



FIGURE 1. A Comparison of Optimal BFFSP Designs for Screening and Robust Parameter Design in the Chrome-Plating Experiment. The MA Screening Design, on the Left, Is a $2^{(3+3)-(0+1)-(1+1)}$ Design; the Optimal RPD, on the Right, Is a $2^{(3+3)-(1+0)-(0+2)}$ Design.

## comparison

- Circles of the same form signify treatment combinations belonging to the same block.
- the RPD design is a crossed design—a $2^{3-1}$ design at the WP (noise) level crossed with a $2^3$ design at the SP (control) level.
- the RPD design is somewhat sparse at the WP level, in that it utilizes only 4 of the 8 WP treatment combinations, represented by the corners of the large cube. This is because the focus here is on control and control-by-noise effects, and the control factors are at the SP level, represented by the corners of the smaller cubes.

## comparison

- This is in contrast with the MA screening design, which utilizes all 8 WP treatment combinations, but is sparser at the SP level, using only 4 of the 8 possible SP treatment combinations, for a given WP treatment combination.

# A Ranking Scheme for BFFSP RPDs

- For each situation, the first step is to provide a definition of word length. MA designs are then found by sequentially minimizing the number of words of each length in a design's defining contrast subgroup (DCS) in ascending order. The intent is to insure that low-order effects are free of aliasing.
- Bingham and Sitter (2003) redefine the notions of design resolution and word length for use in RPD situations and, using their new wordlength definition, employ the MA criterion to form catalogs of MA FF RPDs and MA FFSP RPDs.

TABLE 2. Sequential Optimization Procedure for Ranking $2^{(n_1+n_2)-(k_1+k_2)+(k_1+k_2)}$ BFFSP RPDs. Optimal Designs Are Chosen by Maximizing Criteria *a*–*c*, in Order, and then Minimizing Criteria *d*–*f*, in Order

| Criterion | Clear effects/Effect estimation precision |
|---|---|
| *a* | number of clear control main effects |
| *b* | number of clear control-by-noise 2-factor interactions |
| *c* | number of clear control-by-control 2-factor interactions |
| *d* | number of clear control main effects tested against WP error |
| *e* | number of clear control-by-noise 2-factor interactions tested against WP error |
| *f* | number of clear control-by-control 2-factor interactions tested against WP error |

# A Ranking Scheme for BFFSP RPDs

- In RPD, the control main effects and control-by-noise 2-factor interactions are both of high interest. Although control-by-control interactions may also be used to adjust the mean of the process. We therefore rank them below the control-by-noise effects.
- 所以我們可以透過 d, e, f 來了解到哪些control main effects、control-by-noise 2-factor interactions或control-by-control interactions和 WP effect aliase 在一起。

# Control factors at the SP level & control factors at the WP level.

- If control factors at the WP level，then d and f become not important，because control main effects and control-by-control interactions are automatically tested against WP error.

## Why use a Ranking Scheme for BFFSP RPDs ?

- There is some ambiguity about the appropriate definition of word length to use in RPD settings, there is some ambiguity about the manner in which wordlength definitions should be altered to take account of blocking.
- the MA criterion is not able to distinguish between effects of varying precision, which is an important issue in SP designs

## Comparison

TABLE 3 A Further Comparison of the Optimal BFFSP Designs for Screening (Labeled MA) and Robust Parameter Design (Labeled RPD) in the Chrome-Plating Experiment

| $n_1, n_2$ | Structure | Design | Design Generators | $a$ | $b$ | $c$ | $d$ | $e$ | $f$ |
|---|---|---|---|---|---|---|---|---|---|
| MA: 3, 3 | 4:4:2 | 3,3;0,1;1,1 | $ABC\delta_1$, $ABqr$, $ACpq\delta_1$ | 3 | 5 | 2 | 0 | 0 | 2 |
| RPD: 3, 3 | 4:4:2 | 3,3;1,0;0,2 | $ABC$, $Apq\delta_1$, $Bpr\delta_2$ | 3 | 9 | 3 | 0 | 0 | 3 |

- Note ：

  The given RPD design is better than the MA screening design under our sequential optimization scheme because it has a larger number of clear 2-factor control-by-noise interactions (criterion b), and this is the first criterion for which the designs differ.

## Alternative Ranking Schemes

- Because control main effects and control-by-noise 2-factor interactions should be given equal weight in RPD settings, but more weight than control-by-control 2-factor interactions.
- The first step would be to maximize the total number of clear control main effects and control-by-noise 2-factor interactions, and the sequential ranking scheme would involve only four criteria, namely: (a + b), c, (d + e), f. In our tables of optimal designs, we also consider this alternative scheme, which we call the control~control-by-noise ranking scheme.

## Table 4和 Table 5

- Table 4 displays those designs having control factors at the SP level, while Table 5 displays those designs having control factors at the WP level.
- All designs have between 7 and 10 treatment and blocking variables combined and consist of 32 runs in either 2 or 4 blocks.
- We have presented two designs—a design that is optimal with respect to our primary scheme and a design that is optimal with respect to our alternative scheme. The latter design is denoted by a “*” in the table and follows immediately after the former design.

- We see Table 5

Structure 2:4:4  Design 4,4；1,2；1,0

Our primary ranking scheme

(1)D=ABC, r=BCp, s=BCpq, $\beta_1$=BC

| a | b | c | e |
|---|---|---|---|
| 4 | 8 | 0 | 0 |

The alternative control~control-by-noise ranking scheme

(2)D=ABC, r=Ap, s=Aq, $\beta_1$=BC

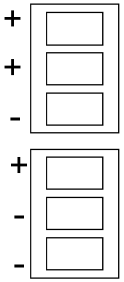
| a | b | c | e |
|---|---|---|---|
| 3 | 12 | 0 | 0 |

## Discussion

- We have focused exclusively on BFFSP RPDs in which the block sizes and the numbers of blocks are powers of 2. For one thing,if block sizes or numbers of blocks are not powers of 2, then the lack of balance and orthogonality complicates the analysis, something that most experimenters would prefer to avoid.

- Ex：treatment factor
  2×2 (two level,+-)
  block factor
  2/3

| | |
|---|---|
| + | + |
| + | + |
| + | - |
| - | + |
| - | - |
| - | - |



## Discussion

- Finally, in the chrome-plating experiment, a natural week (or block) consisted of 4 days, because of the timing of the plating process; yet, in other situations,a 5-day week may have been more appropriate.Dealing with block sizes that are not a power of 2 is another challenging, but important problem.

Thanks for your attention!!