# Strip-Plot Configurations of Fractional Factorials

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## Outline

- STRIP-PLOT DESIGN (Laundry Experiment)
- STRIP-PLOT ARRANGEMENTS FOR FRACTIONS OF TWO-LEVEL FACTORIAL DESIGNS
  1. A Simple Example
  2. A More Complex Example#
  3. The General Procedure for Two-Level Designs
- The Analysis of the Laundry Experiment

## 主題一

STRIP-PLOT DESIGN

## Laundry example

Ex: Investigated methods of reducing the wrinkling of clothes being laundered

Factors: washing machine
dryer

Each factors has four level

$W_j$ : washer effects $D_k$ : dryer effects

$[WD]_{jk}$ : the washer by dryer interactions

# Laundry example

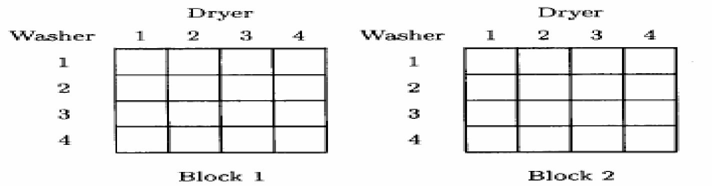


*Figure 1. Strip-plot Configuration of the Laundry Experiment.*

- Model : $y_{ijk} = \mu + W_j + D_k + [WD]_{jk} + \beta_i + \rho_{ij} +, \kappa_{ik} + \varepsilon_{ijk}$

  where $i$ 1 to 2, $j$ 1 to 4, $k$ 1 to 4,

  $\beta_i \sim \text{iid } N(0, \sigma_B^2), \rho_{ij} \sim \text{iid } N(0, \sigma_R^2), \kappa_{ik} \sim \text{iid } N(0, \sigma_C^2)$, and $\varepsilon_{ijk} \sim \text{iid } N(0, \sigma^2)$

# ANOVA Table

*Table 1 ANOVA Table for a Strip-plot Design*

| *Strata* | *Source* | *df* | *EMS* |
|---|---|---|---|
| Block | Blocks | 1 | $\sigma^2 + 4\sigma_R^2 + 4\sigma_C^2 + 16\sigma_B^2$ |
| Row | $W$ | 3 | $\sigma^2 + 4\sigma_R^2 + (8/3)\sum_{j=1}^{4} W_j^2$ |
| | Row residual | 3 | $\sigma^2 + 4\sigma_R^2$ |
| Column | $D$ | 3 | $\sigma^2 + 4\sigma_C^2 + (8/3)\sum_{k=1}^{4} D_k^2$ |
| | Column residual | 3 | $\sigma^2 + 4\sigma_C^2$ |
| Unit | $W \times D$ | 9 | $\sigma^2 + (1/9)\sum_{j=1}^{4}\sum_{k=1}^{4}[WD]_{jk}^2$ |
| | Unit residual | 9 | $\sigma^2$ |

# Treatment structure applied to rows and columns represented $2^p$ design Box & Jones(1992)

- Washer configurations can be represented by a $2^2$ design in factors A and B
- Dryer configurations can be represented by a $2^2$ design in factors a and b
- W could be split into the standard contrasts A, B,and AB
- D could be split into the standard contrasts a, b, and ab
- W x D could be split into Aa, Ab, Aab, Ba, Bb, Bab, ABa, ABb, and ABab

# In situations in which the strip-plot configuration was not replicated

- 發生的問題：no residual degrees of freedom available in the row, column, or unit strata to estimate variation

- 解決的辦法：

  1.identifying significant effects would be to assume effect sparsity and construct a separate normal plot for each stratum

  2. assume that specific (usually high-order) interactions in a particular stratum were negligible and pool these to provide an estimate of experimental error for that stratum

## How to construct Strip-Plot Configurations of Fractional Factorials

The procedure consists of three steps:

(1) Identify a suitable row design

(2) Identify a suitable column design

(3) Select a latin-square fraction of the product of the designs in (1) and (2)

## 主題二

STRIP-PLOT ARRANGEMENTS FOR FRACTIONS OF TWO-LEVEL FACTORIAL DESIGNS

## A Simple Example

- Three row factors (A, B, C) ; Three column factors (a, b, c) using two 4x4 blocks investigated (2/ 4x4). $2^{6-1}$ Design is required
- Consider the full $2^6$ design as the product array of a $2^3$ row design and a $2^3$ column design

| | | | | abc = −1 | | | | abc = +1 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | −1 | +1 | −1 | +1 | −1 | +1 | −1 | +1 | a |
| | | | | −1 | −1 | +1 | +1 | −1 | −1 | +1 | +1 | b |
| | A | B | C | −1 | +1 | +1 | −1 | +1 | −1 | −1 | +1 | c |
| | −1 | −1 | −1 | • | • | • | • | • | • | • | • | |
| ABC | +1 | −1 | +1 | • | • | • | • | • | • | • | • | |
| = −1 | −1 | +1 | +1 | • | • | • | • | • | • | • | • | |
| | +1 | +1 | −1 | • | • | • | • | • | • | • | • | |
| | −1 | −1 | +1 | • | • | • | • | • | • | • | • | |
| ABC | +1 | −1 | −1 | • | • | • | • | • | • | • | • | |
| = +1 | −1 | +1 | −1 | • | • | • | • | • | • | • | • | |
| | +1 | +1 | +1 | • | • | • | • | • | • | • | • | |

*Figure 2. Product-Array Representation of a $2^6$ Design.*

## Analysis(I=ABCabc)

- Row(column)effect: all main effect and interactions that only involve row(column) factors EX: A,B,AC……
- Mixed effect: interactions that involve both row and column factors EX:Aa,ABc…….
- Notice the following:
  1. The contrasts used to block the row and column designs ABC and abc, end up confounded with blocks.
  2. All row effects end up in the row stratum, as do those mixed effects that contain the string abc.
  3. All column effects end up in the column stratum as do those mixed effects that contain the string ABC.
  4. All remaining mixed effects end up in the unit stratum

# A More Complex Example

- Six row factors A, B, C, D, E, F ; Five column factors a, b; c, d, e in four 8 x 8 blocks. (4/ 8×8)
- This requires identifying a $2^{11-3}$ design in four blocks such that each block is a product array of eight row-factor combinations and eight column-factor combinations.

Row design:

treatment defining contrast subgroup I=ABCDEF

block defining contrast subgroup B1=ABC ; B2=CDE ; B1B2=ABDE

multiplying defining word B1=ABC=DEF ; B2=CDE=ABF ;

B1B2=CF=ABDF

Column design :

block effect : b1=abc ; b2=ade ; b1b2=bcde

# The product of these designs

| | abc = −1, ade = −1, bcde = +1 | abc = −1, ade = +1, bcde = −1 | abc = +1, ade = −1, bcde = −1 | abc = +1, ade = +1, bcde = +1 |
|---|---|---|---|---|
| ABCDEF = +1, ABC = DEF = −1, CDE = ABF = −1, CF = ABDE = +1 | 1 | 2 | 3 | 4 |
| ABCDEF = +1, ABC = DEF = −1, CDE = ABF = +1, CF = ABDE = −1 | 5 | 6 | 7 | 8 |
| ABCDEF = +1, ABC = DEF = +1, CDE = ABF = −1, CF = ABDE = −1 | 9 | 10 | 11 | 12 |
| ABCDEF = +1, ABC = DEF = +1, CDE = ABF = +1, CF = ABDE = +1 | 13 | 14 | 15 | 16 |

*Figure 3. Product of a $2^{6-1}$ Row Design and a $2^4$ Column Design in Four Blocks.*

# Conclusion

- Latin-square fraction : taking blocks corresponding to one letter in a 4 x 4 latin square
- The blocks numbered 1, 6, 12, and 15 are chose . These correspond to a $2^{11-3}$ design with defining relation

  I = ABCabc = -CDEbcde = -ABDEade = ABCDEF= DEFabc

  = -ABFbcde = -CFabe

  Choosing blocks 1, 6, 11, and 16

  I = ABCabc = CDEade = ABDEbcde = ABCDEF= DEFabc

  = ABFade = CFbcde,

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- **Treatment structure**

  **A × B × C × D × E × F × a × b × c × d × e**

  **2 2 2 2 2 2 2 2 2 2 2**

  **Plot structure**

  **U / V×W**

  **4/ 8×8**

  **U = U1 , U2**

  **4 2 2**

  **V = V1 , V2 , V3**

  8 2 2 2

  **W = W1 , W2 , W3**

  **8 2 2 2**

**Design Key**

| A | B | C | D | E | F | a | b | c | d | e |
|---|---|---|---|---|---|---|---|---|---|---|
| V1 | V2 | V3 | V1V2 | V2V3 | U1V1 | W1 | W2 | W3 | W1W2 | U2V1W1 |

**Inverse Key**

| V1 | V2 | V3 | U1 | W1 | W2 | W3 | U2 |
|---|---|---|---|---|---|---|---|
| A | B | C | AF | a | b | c | Abe |

**I=ABD=BCE=abd=ACDE=ABDabd=BCEabd=ACDEabd**

# Plot structure

$$\mathcal{N}(n_1, \mathcal{C}(n_2, n_3)) = 1 + \nu_1 + n_1(1 + \nu_2 + \nu_3 + \nu_2\nu_3 - 1)$$
$$= 1 + \nu_1 + n_1\nu_2 + n_1\nu_3 + n_1\nu_2\nu_3$$

d.f. identity:

$$n_1 n_2 n_3 = 1 + (n_1 - 1) + n_1(n_2 - 1) + n_1(n_3 - 1) + n_1(n_2 - 1)(n_3 - 1)$$

| | S0 | S1 | S2 | S3 | S4 |
|---|---|---|---|---|---|
| Dim | 1 | 3 | 28 | 28 | 196 |

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S1 = U1 , U2 , U1U2

S2 = V1 , V2 , V3 , V1V2 , V1V3 , V2V3 , V1V2V3
V1U1 , V2U1 , V3U1 , V1V2U1 , V1V3U1 , V2V3U1 , V1V2V3U1
V1U2 , V2U2 , V3U2 , V1V2U2 , V1V3U2 , V2V3U2 , V1V2V3U2
V1U1U2,V2U1U2,V3U1U2,V1V2U1U2,V1V3U1U2,V2V3U1U2,V1V2V3U1U2

S3 = W1 , W2 , W3 , W1W2 , W1W3 , W2W3 , W1W2W3
W1U1 , W2U1 , W3U1 , W1W2U1 , W1W3U1 , W2W3U1 , W1W2W3U1
W1U2 , W2U2 , W3U2 , W1W2U2 , W1W3U2 , W2W3U2 , W1W2W3U2
W1U1U2,W2U1U2,W3U1U2,W1W2U1U2,W1W3U1U2,W2W3U1U2, W1W2W3U1U2

S4 =

| U1 U2 | | V1 V2 V3 | | W1W2W3 |
|---|---|---|---|---|
| $C^2_0$ | | $C^3_1$ | | $C^3_1$ |
| $C^2_1$ | × | $C^3_2$ | × | $C^3_2$ |
| $C^2_2$ | | $C^3_3$ | | $C^3_3$ |

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# Treatment structure

**Treatment structure**

**V0 ⊕ $\overline{W}_T$ ( N=T )**

| Treatment effect | DF | Plot alias | Stratum |
|---|---|---|---|
| BD=ABCE=Aabd=CDE=BDabd=ABCEabd=CDEabd=A | 1 | V1 | S2 |
| AD=CE=Babd=ABCDE=ADabd=CEabd=ABCDEabd=B | 1 | V2 | S2 |
| . | | | |
| . | | | |
| . | | | |
| . | | | |
| . | | | |
| Total 256 | | | |

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## Plan and the restrictions on randomization

Each block :

row factors randomization

column factors randomization

# ANOVA Table

| Strata | Source | df |
|---|---|---|
| Block | Blocks | 3 |
| Row | R | 7 |
| | Row residual | 21 |
| Column | C | 7 |
| | Column residual | 21 |
| Unit | R×C | 49 |
| | Unit residual | 147 |

# The General Procedure for Two-Level Designs

b= $2^w$ blocks
each block: r= $2^M$ row , c= $2^m$ column
K : number of row factors ; k : number of column factors
Define: Q = K-(w+M) ; q = k-(w+m)

Then the procedure is as follows:

1. Select a row design that consists of a $2^{K-Q}$ design in b blocks.
2. Select a column design that consists of a $2^{k-q}$ design in b blocks.
3. Consider the product of the designs in steps 1 and 2 and select a latin-square fraction of this product.

# How to construct a latin-square fraction

- Suppose that both the row and the column designs are complete $2^p$ designs in b = $2^w$ blocks.
- Let $\{E_1, E_2, E_{12}, E_3, E_{13}, E_{23}, E_{123}, \ldots, E_{12\ldots w}\}$ be the set of b - 1 row effects that are confounded with blocks in which $E_{ij} = E_i E_j$ is the generalized interaction between $E_i$ and $E_j$. Similarly let $\{e_1, e_2, e_{12}, e_3, \ldots, e_{12\ldots w}\}$ be the set of column effects that are confounded with blocks. Then selecting the blocks corresponding to
$$I = E_1 e_1 = E_2 e_2 = \ldots = E_{12\ldots w} e_{12\ldots w}$$
results in a proper fraction that is also a latin-square fraction.

# 主題三

The Analysis of the Laundry Experiment

# Laundry Experiment

- Six factors representing washing conditions (e.g., wash temperature) ; Four factors representing drying conditions were identified

  Two levels for each of these 10 factors were selected
- The six washer factors will be designated as A, B , C, D , E and F ; the four dryer factors as a. b, c, and d.
- A $2^{6-3}$ design in two blocks was required for washers(rows)

  I=ABC=BDF=ABDE=ACDF=CDE=AEF=BCEF

  string AD was used to define blocks
- A $2^{4-1}$ design in two blocks was required for dryers(columns)

  I=abcd ; string ab=cd was used to define blocks

# Laundry Experiment

Table 4. Confounding Patterns for the Row and Column Designs

Row design

$I = ABC = BDF = ABDE = ACDF = CDE = AEF = BCEF$
$A = BC = ABDF = BDE = CDF = ACDE = EF = ABCEF$
$B = AC = DF = ADE = ABCDF = BCDE = ABEF = CEF$
$C = AB = BCDF = ABCDE = ADF = DE = ACEF = BEF$
$D = ABCD = BF = ABE = ACF = CE = ADEF = BCDEF$
$E = ABCE = BDEF = ABD = ACDEF = CD = AF = BCF$
$F = ABCF = BD = ABDEF = ACD = CDEF = AE = BCE$
blocks $= AD = BCD = ABF = BE = CF = ACE$
$= DEF = ABCDEF$

Column design

$I = abcd$ $b = acd$ $d = abc$ $ac = bd$
$a = bcd$ $c = abd$ $ad = bc$ blocks $= ab = cd$

Table 3. Factor Settings for the Laundry Experiment

| Machine | Washer settings A | B | C | D | E | F | Machine | Dryer settings a | b | c | d |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | Block 1 | | | | | | |
| 3 | [illegible] | [illegible] | +1 | +1 | +1 | 1 | 3 | −1 | −1 | −1 | −1 |
| 4 | +1 | −1 | −1 | −1 | +1 | +1 | 1 | +1 | +1 | +1 | −1 |
| 1 | [illegible] | +1 | 1 | +1 | −1 | +1 | 2 | −1 | −1 | +1 | +1 |
| 2 | +1 | +1 | +1 | −1 | −1 | −1 | 4 | +1 | +1 | +1 | +1 |
| | | | | | Block 2 | | | | | | |
| 1 | −1 | − | +1 | [illegible] | [illegible] | +1 | 4 | 1 | +1 | 1 | +1 |
| 4 | +1 | −1 | −1 | +1 | −1 | −1 | 2 | +1 | −1 | +1 | +1 |
| 2 | −1 | +1 | −1 | −1 | +1 | −1 | 3 | −1 | +1 | +1 | −1 |
| 3 | +1 | +1 | +1 | +1 | +1 | +1 | 1 | +1 | 1 | +1 | [illegible] |

Table 5. Results for the Laundry Experiment

| Washer | Dryer 3 | 1 | 2 | 4 |
|---|---|---|---|---|
| | | Block 1 | | |
| 1 | 3.19 | 2.75 | 3.02 | 2.63 |
| 4 | 4.01 | 3.33 | 3.79 | 2.82 |
| 2 | 3.77 | 3.36 | 3.47 | 3.08 |
| 3 | 3.83 | 3.48 | 4.25 | 3.94 |
| | | Block 2 | | |
| 3 | 2.28 | 1.88 | 2.91 | 2.37 |
| 4 | 2.95 | 3.25 | 3.11 | 2.85 |
| 1 | 2.40 | 1.89 | 3.51 | 2.38 |
| 2 | 4.05 | 3.68 | 3.24 | 3.31 |

Table 6. Confounding and Estimates for the Laundry Experiment

Row stratum

$A = BC = EF \rightarrow .68$   $B = AC = DF \rightarrow .40$   $C = AB = DE \rightarrow .05$
$D = BF = CE \rightarrow -.16$   $E = CD = AF \rightarrow .05$   $F = BD = AE \rightarrow -.12$

Column stratum

$a \rightarrow -.42$   $b \rightarrow .07$   $c \rightarrow .04$
$d \rightarrow .12$   $ac = bd \rightarrow -.07$   $ad = bc \rightarrow -.04$

Unit stratum

$Aa = -Db \rightarrow .10$   $Ab = -Da \rightarrow .19$   $Ac = -Dd \rightarrow -.19$
$Ad = -Dc \rightarrow -.32$   $Ba = -Eb \rightarrow .00$   $Bb = -Ea \rightarrow -.13$
$Bc = -Ed \rightarrow .05$   $Bd = -Ec \rightarrow -.11$   $Ca = -Fb \rightarrow .08$
$Cb = -Fa \rightarrow -.04$   $Cc = -Fd \rightarrow .03$   $Cd = -Fc \rightarrow -.20$

Strings containing $\geq 3$ – factor interactions → .02, .05, .11, −.06, −.01, −.07

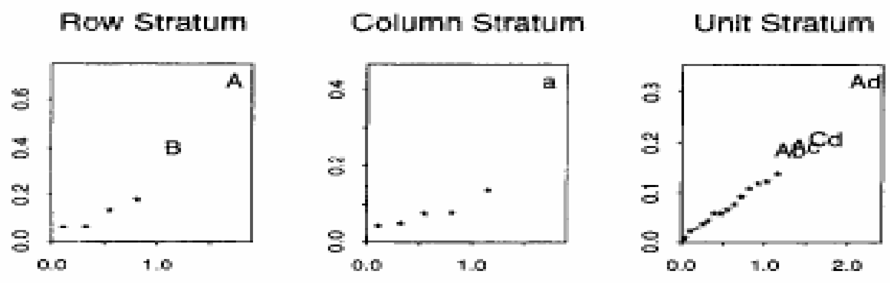


Figure 4. Half-Normal Plots for the Laundry Experiment