Split-Lot Designs: Experiments for Multistage Batch Processes

Final report —Design and Analysis of Multi-stratum Randomized Experiments

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Outline

- Introduction
  - two different designs
- Constructing 16-wafer experiments
- Symmetric split-lot designs for 64-wafer experiments
- Discussions

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- Introduction
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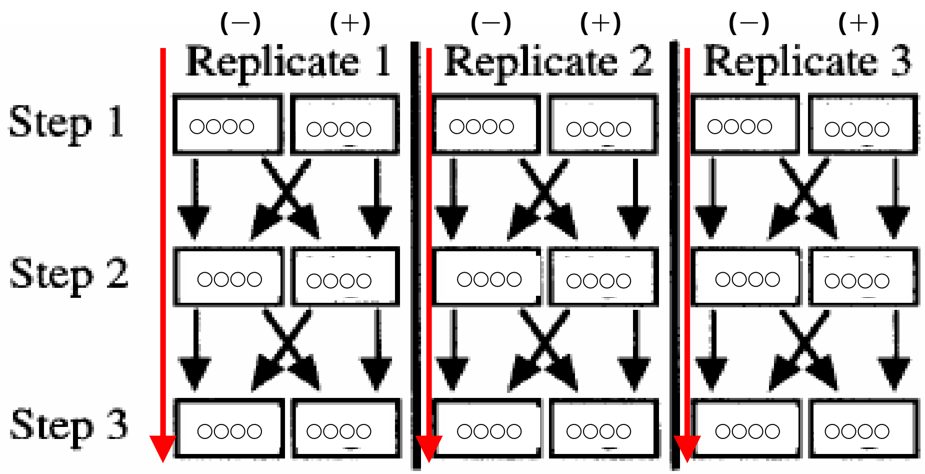
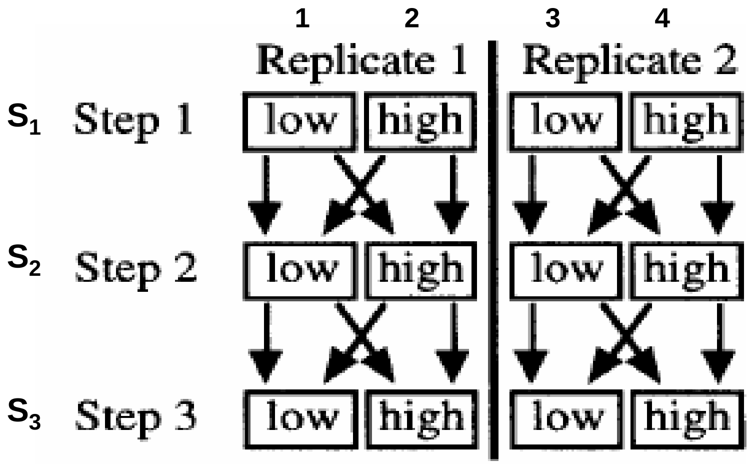
Common Split-lot Design: 3 separate replicates of a full 2^3 factorial



16-wafer split-lot design:

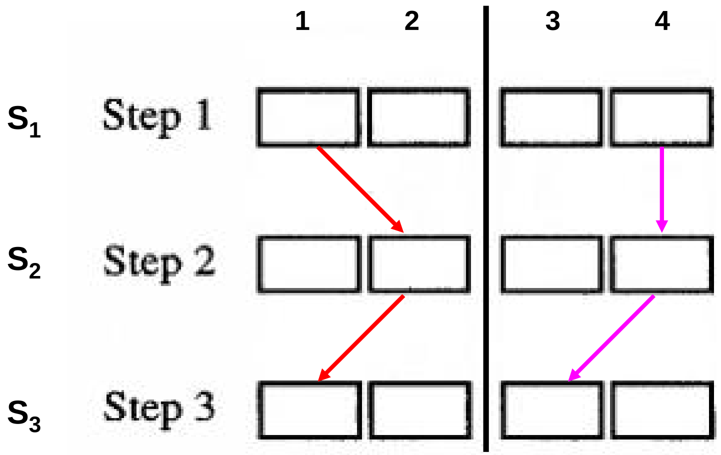


3 factors, 16-wafer design I

Two separate replicates

	S_1			
S_2	1	2	3	4
1	1, 2	1, 2		
2	1, 2	1, 2		
3			3, 4	3, 4
4			3, 4	3, 4

(cont.)



A reasonable model

$$Y_S = \mu + \sum_{i=1}^{T}\left(a_{S_i(i)} + \alpha_{S_i(i)}\right) + \sum_{i<j} a_{S_iS_j(ij)} + \varepsilon_S,$$

where $a_{S_i(i)}$ is the effect of the ith stage factor for the S_ith subplot,

$\alpha_{S_i(i)}$ is the error term associated with the S_ith subplot,

$a_{S_iS_j(ij)}$ is the interaction effect between the ith stage factor and the jth stage factor,

& $\alpha_{S_i(i)} \sim N\left(0,\sigma_i^2\right)$, $\varepsilon_S \sim N\left(0,\sigma^2\right)$.

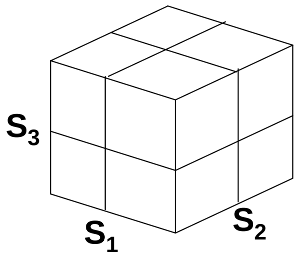
Treatment and plot structure

- Treatment structure

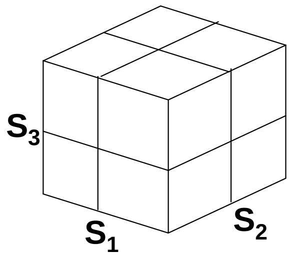
$$X_1 \times X_2 \times X_3$$
$$2 \quad 2 \quad 2$$

- Plot structure

$$U/(Y \times Z \times W)$$
$$2 \quad 2 \quad 2 \quad 2$$



Replicate 1



Replicate 2

Treatment strata and dimensions

Treatment structure		
df	effect	strata
1		V_0
7	X_1 X_2 X_1X_2 X_3 X_1X_3 X_2X_3 $X_1X_2X_3$	W_T
8		$V_T^{\perp}$

Plot strata and dimensions (？)

Plot structure		
strata	effect	df
S_0		1
S_1	U	1
S_2	Y, UY	2
S_3	Z, UZ	2
S_4	W, UW	2
S_5	YZ, UYZ	2
S_6	YW, UYW	2
S_7	ZW, UZW	2
S_8	$YZW, UYZW$	2

Plot strata and dimensions

Plot structure		
strata	effect	df
S_0		1
S_1	U	1
S_2	Y, UY	2
S_3	Z, UZ	2
S_4	W, UW	2
S_5	YZ, UYZ YW, UYW ZW, UZW $YZW, UYZW$	8

Experimental unit & restrictions

- observational unit & experimental unit

$$U / (Y \times Z \times W)$$

$$EU_{X_1} \; EU_{X_2} \, EU_{X_3}$$

- restrictions on allocations of treatments to plots

X_1 can be confounded with plot effects involving U, Y, UY
X_2 U, Z, UZ
X_3 U, W, UW

Design and design key

- Design:
  2 separate replicates of a full 2^3 factorial

- Design key:

$$X_1 = Y, X_2 = Z, X_3 = W$$

Restrictions on randomization

- no pair of wafers is processed together more than once in a sublot for the experimental factors

- all three processing steps are performed on the first eight wafers before any processing is done on the second eight wafers

How treatment and plot effects are confounded

$$\begin{cases} X_1 = Y \\ X_2 = Z \\ X_3 = W \end{cases} \quad \begin{cases} X_1 X_2 = YZ \\ X_1 X_3 = YW \\ X_2 X_3 = ZW \\ X_1 X_2 X_3 = YZW \end{cases}$$

ANOVA table

Source	df	EMS
Block	1	ξ_{Block}
X_1	1	$\xi_1 + \lVert\tau_{X_1}\rVert^2$
Residual	1	ξ_1
X_2	1	$\xi_2 + \lVert\tau_{X_2}\rVert^2$
Residual	1	ξ_2
X_3	1	$\xi_3 + \lVert\tau_{X_3}\rVert^2$
Residual	1	ξ_3
X_1X_2	1	$\xi_4 + \lVert\tau_{X_1X_2}\rVert^2$
X_2X_3	1	$\xi_4 + \lVert\tau_{X_2X_3}\rVert^2$
X_1X_3	1	$\xi_4 + \lVert\tau_{X_1X_3}\rVert^2$
$X_1X_2X_3$	1	$\xi_4 + \lVert\tau_{X_1X_2X_3}\rVert^2$
Residual	4	ξ_4

Concluding remark I

γ separate replicates of a full 2^3 factorial:

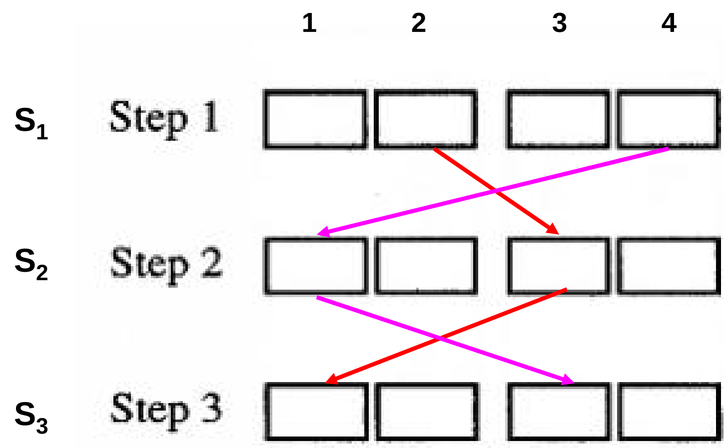
- The γ estimates for the main effect X_i provides $\gamma - 1$ df for estimating the subplot variation
- According to the reasonable model, it provides 4($\gamma - 1$) df for estimating σ^2

3 factors, 16-wafer design II

Alternative design

S_2	S_1: 1	2	3	4
1	1	4	2	3
2	3	2	4	1
3	4	1	3	2
4	2	3	1	4

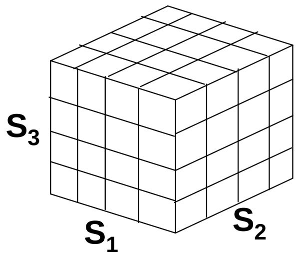
(cont.)



Treatment and plot structure

- Treatment structure

$$X_1 \times X_2 \times X_3$$
$$2 \quad 2 \quad 2$$



Replicate 1

- Plot structure

$$U \times Y \times Z$$
$$4 \quad 4 \quad 4 \quad ?$$

$$\overbrace{U_1 \ U_2} \quad \overbrace{Y_1 \ Y_2} \quad \overbrace{Z_1 \ Z_2}$$

⇒ ¼ Block design

Treatment strata and dimensions

Treatment structure		
df	effect	strata
1		V_0
7	X_1 X_2 X_1X_2 X_3 X_1X_3 X_2X_3 $X_1X_2X_3$	W_T
8		$V_T^{\perp}$

Plot strata and dimensions

Plot structure		
strata	effect	df
S_0		1
S_1	$U_1 = U_2Y_1Z_2 = U_1U_2Y_1Y_2Z_1 = Y_2Z_1Z_2$ $U_2 = U_1Y_1Z_2 = Y_1Y_2Z_1 = U_1U_2Y_2Z_1Z_2$ $U_1U_2 = Y_1Z_2 = U_1Y_1Y_2Z_1 = U_2Y_2Z_1Z_2$	3
S_2	$Y_1 = U_1U_2Z_2 = U_2Y_2Z_1 = U_1Y_1Y_2Z_1Z_2$ $Y_2 = U_1U_2Y_1Y_2Z_2 = U_2Y_1Z_1 = U_1Z_1Z_2$ $Y_1Y_2 = U_1U_2Y_2Z_2 = U_2Z_1 = U_1Y_1Z_1Z_2$	3
S_3	$Z_1 = U_1U_2Y_1Z_1Z_2 = U_2Y_1Y_2 = U_1Y_2Z_2$ $Z_2 = U_1X_2Y_1 = U_2Y_1Y_2Z_1Z_2 = U_1Y_2Z_1$ $Z_1Z_2 = U_1U_2Y_1Z_1 = U_2Y_1Y_2Z_2 = U_1Y_2$	3
S_4	$U_1Z_1 = U_2Y_1Z_1Z_2 = U_1U_2Y_1Y_2 = Y_2Z_2$ $U_1Z_2 = U_2Y_1 = U_1U_2Y_1Y_2Z_1Z_2 = Y_2Z_1$ $U_2Z_2 = U_1Y_1 = Y_1Y_2Z_1Z_2 = U_1U_2Y_2Z_1$ $U_2Z_1Z_2 = U_1Y_1Z_1 = Y_1Y_2Z_2 = U_1U_2Y_2$ $U_1U_2Z_1 = Y_1Z_1Z_2 = U_1Y_1Y_2 = U_2Y_2Z_2$ $U_1U_2Z_1Z_2 = Y_1Z_1 = U_1Y_1Y_2Z_2 = U_2Y_2$	6

$$I = U_1U_2Y_1Z_2 = U_2Y_1Y_2Z_1 = U_1Y_2Z_1Z_2$$

Experimental unit & restrictions

- observational unit & experimental unit

$$(U \times Y \times Z)$$
$$EU_{X_1} \ EU_{X_2} EU_{X_3}$$

- restrictions on allocations of treatments to plots

X_1 can be confounded with plot effects involving U

X_2 Y

X_3 Z

Design and design key

- Design:

  The authors called "Alternative design"

  Taguchi (1989) referred as "Multiway split-unit design"

- Design key:

$$X_1 = U_1, X_2 = Y_1, X_3 = Z_1$$

25

Restrictions on randomization

- no pair of wafers appears together in more than one sublot

26

How treatment and plot effects are confounded

$$\begin{cases} X_1 = U_1 \\ X_2 = Y_1 \\ X_3 = Z_1 \end{cases} \quad \begin{cases} X_1X_2 = U_1Y_1 \\ X_1X_3 = U_1Z_1 \\ X_2X_3 = Y_1Z_1 \\ X_1X_2X_3 = U_1Y_1Z_1 \end{cases}$$

27

ANOVA table

Source	df	EMS
X_1	1	$\xi_1 + \lVert\tau_{X_1}\rVert^2$
Residual	2	ξ_1
X_2	1	$\xi_2 + \lVert\tau_{X_2}\rVert^2$
Residual	2	ξ_2
X_3	1	$\xi_3 + \lVert\tau_{X_3}\rVert^2$
Residual	2	ξ_3
X_1X_2	1	$\xi_4 + \lVert\tau_{X_1X_2}\rVert^2$
X_2X_3	1	$\xi_4 + \lVert\tau_{X_2X_3}\rVert^2$
X_1X_3	1	$\xi_4 + \lVert\tau_{X_1X_3}\rVert^2$
$X_1X_2X_3$	1	$\xi_4 + \lVert\tau_{X_1X_2X_3}\rVert^2$
Residual	2	ξ_4

28

Concluding remark II

Alternative design of a full 2^3 factorial:

- The estimates for the main effect X_i provides 2 df for estimating the subplot variation
- According to the reasonable model, it provides 2 df for estimating σ^2

Comparisons

Design I

Source	df	EMS
Block	1	ξ_{Block}
X_1	1	$\xi_1+\|\tau_{X_1}\|^2$
Residual	1	ξ_1
X_2	1	$\xi_2+\|\tau_{X_2}\|^2$
Residual	1	ξ_2
X_3	1	$\xi_3+\|\tau_{X_3}\|^2$
Residual	1	ξ_3
X_1X_2	1	$\xi_4+\|\tau_{X_1X_2}\|^2$
X_2X_3	1	$\xi_4+\|\tau_{X_2X_3}\|^2$
X_1X_3	1	$\xi_4+\|\tau_{X_1X_3}\|^2$
$X_1X_2X_3$	1	$\xi_4+\|\tau_{X_1X_2X_3}\|^2$
Residual	4	ξ_4

Design II

Source	df	EMS
X_1	1	$\xi_1+\|\tau_{X_1}\|^2$
Residual	2	ξ_1
X_2	1	$\xi_2+\|\tau_{X_2}\|^2$
Residual	2	ξ_2
X_3	1	$\xi_3+\|\tau_{X_3}\|^2$
Residual	2	ξ_3
X_1X_2	1	$\xi_4+\|\tau_{X_1X_2}\|^2$
X_2X_3	1	$\xi_4+\|\tau_{X_2X_3}\|^2$
X_1X_3	1	$\xi_4+\|\tau_{X_1X_3}\|^2$
$X_1X_2X_3$	1	$\xi_4+\|\tau_{X_1X_2X_3}\|^2$
Residual	2	ξ_4

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A plan of Design II

Table 1. The Alternative Replicated 2^3 Design Written With Wafer Codes

Wafer 1: 111	Wafer 2: 214	Wafer 3: 312	Wafer 4: 413
Wafer 5: 123	Wafer 6: 222	Wafer 7: 324	Wafer 8: 421
Wafer 9: 134	Wafer 10: 231	Wafer 11: 333	Wafer 12: 432
Wafer 13: 142	Wafer 14: 243	Wafer 15: 341	Wafer 16: 444

2-level designs of size 16:
Orthogonal array given in Yates order

Table 4. Sixteen-Run Orthogonal Array With Contrasts in Standard Order

Wafer	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}
1	−1	−1	1	−1	1	1	−1	−1	1	1	−1	1	−1	−1	1
2	1	−1	−1	−1	−1	1	1	−1	−1	1	1	1	1	−1	−1
3	−1	1	−1	−1	1	−1	1	−1	1	−1	1	1	−1	1	−1
4	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	1	1	1	1
5	−1	−1	1	1	−1	−1	1	−1	1	1	−1	−1	1	1	−1
6	1	−1	−1	1	1	−1	−1	−1	−1	1	1	−1	−1	1	1
7	−1	1	−1	1	−1	1	−1	−1	1	−1	1	−1	1	−1	1
8	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1
9	−1	−1	1	−1	1	1	−1	1	−1	−1	1	−1	1	1	−1
10	1	−1	−1	−1	−1	1	1	1	1	−1	−1	−1	−1	1	1
11	−1	1	−1	−1	1	−1	1	1	−1	1	−1	−1	1	−1	1
12	1	1	1	−1	−1	−1	−1	1	1	1	1	−1	−1	−1	−1
13	−1	−1	1	1	−1	−1	1	1	−1	−1	1	1	−1	−1	1
14	1	−1	−1	1	1	−1	−1	1	1	−1	−1	1	1	−1	−1
15	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1
16	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Procedres

- First, choose two independent contrasts and the third is the product of these two.
- Choose C_1 and C_2

Wafers	C_1	C_2	$S_1 = 2.5 + .5C_1 + C_2$
1, 5, 9, 13	−1	−1	1
2, 6, 10, 14	1	−1	2
3, 7, 11, 15	−1	1	3
4, 8, 12, 16	1	1	4

(cont.)

- This definition for S_1 and "odd=low, even= high" coding rule imply that $X_1=C_1$. Note that $C_3=C_1C_2$ is the third between-subplot contrast for step 1.
- Step 2 subplot was defined by
  $S_2 = 2.5+0.5C_4+C_8$
- Step 3 subplot was defined by
  $S_3 = 2.5+0.5C_{11}+C_6$

(cont.)

- $X_1=C_1$; C_2 and $C_3=C_1C_2$ are affected by step 1 subplot variation.
- $X_2=C_4$; C_8 and $C_{12}=C_4C_8$ are affected by step 2 subplot variation.
- $X_3=C_{11}$; C_6 and $C_{13}=C_{11}C_6$ are affected by step 3 subplot variation.
- $X_1X_2=C_1$, $X_1X_3=C_{10}$, $X_2X_3=C_{15}$, $X_1X_2X_3=C_{14}$.
- Remaining C_7 and C_9 are used to est. σ^2.

How to generate disjoint groups？

Table 5. Five Disjoint Groups of Three Contrasts for a 2^4

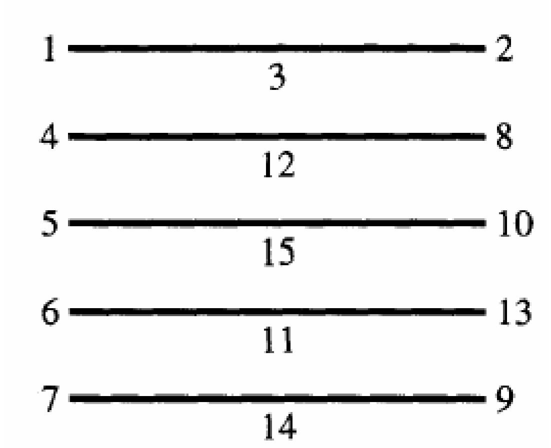
Group	Two independent contrasts		Generalized interaction
1	X_1	X_2X_3	$X_1X_2X_3$
2	X_2	X_3X_4	$X_2X_3X_4$
3	X_3	X_1X_4	$X_1X_3X_4$
4	X_4	X_1X_2	$X_1X_2X_4$
5	X_1X_3	X_2X_4	$X_1X_2X_3X_4$

Correspond to previous example

$$(C_1, C_2, C_3)$$
$$(C_4, C_8, C_{12})$$
$$(C_{11}, C_6, C_{13})$$
$$(C_5, C_{10}, C_{15})$$
$$(C_7, C_9, C_{14})$$

➡ We can experiment in as many as 5 different steps

Linear graph (Taguchi, 1987)



Four factors in four steps

Source	df	EMS
X_1	1	$\xi_1 + \|\tau_{X_1}\|^2$
X_2X_3	1	$\xi_1 + \|\tau_{X_2X_3}\|^2$
Residual	1	ξ_1
X_2	1	$\xi_2 + \|\tau_{X_2}\|^2$
X_3X_4	1	$\xi_2 + \|\tau_{X_3X_4}\|^2$
Residual	1	ξ_2
X_3	1	$\xi_3 + \|\tau_{X_3}\|^2$
X_1X_4	1	$\xi_3 + \|\tau_{X_1X_4}\|^2$
Residual	1	ξ_3
X_4	1	$\xi_4 + \|\tau_{X_4}\|^2$
X_1X_2	1	$\xi_4 + \|\tau_{X_1X_2}\|^2$
Residual	1	ξ_4
X_1X_3	1	$\xi_5 + \|\tau_{X_1X_3}\|^2$
X_2X_4	1	$\xi_5 + \|\tau_{X_2X_4}\|^2$
Residual	1	ξ_5

More factors in 4 steps

Table 6. Aliasing Structure for 2^{8-4} Design (using Table 5 grouping)

Design generators	*Effects with the same standard error as* X_3
$X_5 = X_1X_2X_3$ $X_6 = X_2X_3X_4$ $X_7 = X_1X_3X_4$ $X_8 = X_1X_2X_4$	X_7 $X_3X_7 = X_1X_4 = X_2X_8 = X_5X_6$
Effects with the same standard error as X_1	*Effects with the same standard error as* X_4
X_5 $X_1X_5 = X_2X_3 = X_4X_6 = X_7X_8$	X_8 $X_1X_2 = X_3X_5 = X_4X_8 = X_6X_7$
Effects with the same standard error as X_2	*Effects orthogonal to sublots*
X_6 $X_2X_6 = X_3X_4 = X_1X_7 = X_5X_8$	$X_1X_3 = X_2X_5 = X_4X_7 = X_6X_8$ $X_1X_6 = X_2X_7 = X_3X_8 = X_4X_5$ $X_1X_8 = X_2X_4 = X_3X_6 = X_5X_7$

- If we would like to accommodate two factors in one or more steps
- $2^{5-1}, 2^{6-2}, 2^{7-3}, 2^{8-4}$ fractional factorial designs

Five factors in five steps

- 2^{5-1} fractional factorial design

$$X_5 = X_1X_2X_3X_4$$

Table 5. Five Disjoint Groups of Three Contrasts for a 2^4

Group	Two independent contrasts		Generalized interaction
1	X_1	X_2X_3	$X_1X_2X_3$
2	X_2	X_3X_4	$X_2X_3X_4$
3	X_3	X_1X_4	$X_1X_3X_4$
4	X_4	X_1X_2	$X_1X_2X_4$
5	X_1X_3	X_2X_4	$X_1X_2X_3X_4$

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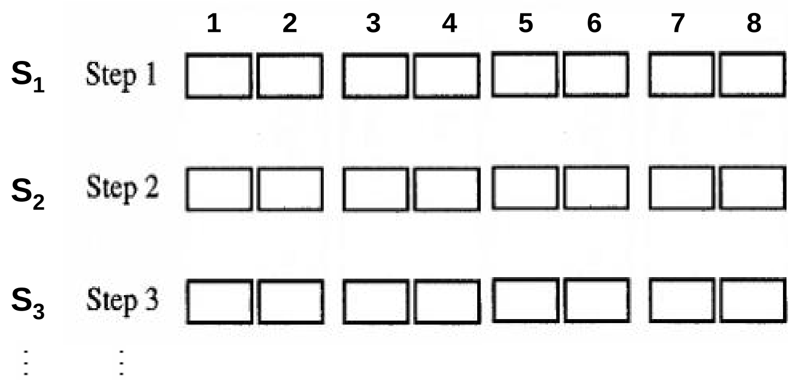
64-wafer split-lot design:



Two 9-disjoint groups

Table 7. Nine Disjoint Groups of Seven Contrasts for a 2^6 = 64-Wafer Experiment

Group	Three independent contrasts			Four generalized interactions			
				Solution A			
1	X_1	X_5X_6	$X_2X_3X_5$	$X_1X_5X_6$	$X_2X_3X_6$	$X_1X_2X_3X_5$	$X_1X_2X_3X_6$
2	X_2	X_1X_6	$X_3X_4X_6$	$X_1X_2X_6$	$X_1X_3X_4$	$X_2X_3X_4X_6$	$X_1X_2X_3X_4$
3	X_3	X_1X_2	$X_1X_4X_5$	$X_1X_2X_3$	$X_2X_4X_5$	$X_1X_3X_4X_5$	$X_2X_3X_4X_5$
4	X_4	X_2X_3	$X_2X_5X_6$	$X_2X_3X_4$	$X_3X_5X_6$	$X_2X_4X_5X_6$	$X_3X_4X_5X_6$
5	X_5	X_3X_4	$X_1X_3X_6$	$X_3X_4X_5$	$X_1X_4X_6$	$X_1X_3X_5X_6$	$X_1X_4X_5X_6$
6	X_6	X_4X_5	$X_1X_2X_4$	$X_4X_5X_6$	$X_1X_2X_5$	$X_1X_2X_4X_6$	$X_1X_2X_5X_6$
7	X_2X_4	X_2X_6	$X_1X_3X_5$	X_4X_6	$X_1X_2X_3X_5X_6$	$X_1X_2X_3X_4X_5$	$X_1X_3X_4X_5X_6$
8	X_1X_3	X_3X_5	$X_2X_4X_6$	X_1X_5	$X_2X_3X_4X_5X_6$	$X_1X_2X_3X_4X_6$	$X_1X_2X_4X_5X_6$
9	X_1X_4	X_2X_5	X_3X_6	$X_1X_2X_4X_5$	$X_2X_3X_5X_6$	$X_1X_3X_4X_6$	$X_1X_2X_3X_4X_5X_6$
				Solution B			
1	X_1	X_2X_4	X_3X_6	$X_1X_2X_4$	$X_1X_3X_6$	$X_2X_3X_4X_6$	$X_1X_2X_3X_4X_6$
2	X_2	X_1X_6	X_3X_5	$X_1X_2X_6$	$X_2X_3X_5$	$X_1X_3X_5X_6$	$X_1X_2X_3X_5X_6$
3	X_3	X_2X_6	X_4X_5	$X_2X_3X_6$	$X_3X_4X_5$	$X_2X_4X_5X_6$	$X_2X_3X_4X_5X_6$
4	X_4	X_1X_3	X_2X_5	$X_1X_3X_4$	$X_2X_4X_5$	$X_1X_2X_3X_5$	$X_1X_2X_3X_4X_5$
5	X_5	X_1X_2	X_4X_6	$X_1X_2X_5$	$X_4X_5X_6$	$X_1X_2X_4X_6$	$X_1X_2X_4X_5X_6$
6	X_6	X_1X_5	X_3X_4	$X_1X_5X_6$	$X_3X_4X_6$	$X_1X_3X_4X_5$	$X_1X_3X_4X_5X_6$
7	X_1X_4	X_2X_3	X_5X_6	$X_1X_2X_3X_4$	$X_1X_4X_5X_6$	$X_2X_3X_5X_6$	$X_1X_2X_3X_4X_5X_6$
8	$X_1X_3X_5$	$X_1X_4X_6$	$X_2X_5X_6$	$X_2X_3X_4$	$X_1X_2X_3X_6$	$X_1X_2X_4X_5$	$X_3X_4X_5X_6$
9	$X_1X_2X_3$	$X_1X_4X_5$	$X_3X_5X_6$	$X_2X_4X_6$	$X_1X_3X_4X_6$	$X_2X_3X_4X_5$	$X_1X_2X_5X_6$

JMP software (1997)

Solution B: S_i decision

$$
\begin{aligned}
S_1 &= 4.5 + .5X_1 + X_2X_4 + 2X_3X_6 \\
S_2 &= 4.5 + .5X_2 + X_1X_6 + 2X_3X_5 \\
S_3 &= 4.5 + .5X_3 + X_2X_6 + 2X_4X_5 \\
S_4 &= 4.5 + .5X_4 + X_1X_3 + 2X_2X_5 \\
S_5 &= 4.5 + .5X_5 + X_1X_2 + 2X_4X_6 \\
S_6 &= 4.5 + .5X_6 + X_1X_5 + 2X_3X_4 \\
S_7 &= 4.5 + .5X_7 + X_1X_4 + 2X_2X_3 \\
S_8 &= 4.5 + .5X_8 + X_1X_3X_5 + 2X_2X_5X_6 \\
S_9 &= 4.5 + .5X_9 + X_1X_2X_3 + 2X_1X_4X_5.
\end{aligned}
$$

9 factors in 9 steps (2^{9-3} design)

Table 9. Aliasing Structure for JMP's 2^{9-3} Design (with Solution B contrast grouping from Table 7)

Defining relation: $I = X_1X_2X_7X_8$
$= X_3X_4X_7X_9$
$= X_1X_2X_5X_6X_9$
$= X_3X_4X_5X_6X_8$
$= X_5X_6X_7X_8X_9$
$= X_1X_2X_3X_4X_8X_9$
$= X_1X_2X_3X_4X_5X_6X_7$

Effects with the same standard error as $X_1 = X_2X_7X_8$
X_2X_4
$X_3X_6 = X_4X_5X_8$
$X_5X_7 = X_6X_8X_9$
$X_1X_2X_4 = X_3X_8X_9 = X_4X_7X_8$
$X_1X_3X_6$
$X_1X_5X_7 = X_2X_5X_8$

Effects with the same standard error as $X_2 = X_1X_7X_8$
$X_1X_6 = X_2X_5X_9$
$X_3X_5 = X_4X_6X_8$
$X_3X_9 = X_4X_7$
$X_5X_9 = X_1X_2X_6 = X_6X_7X_8$
$X_1X_4X_8 = X_2X_3X_9 = X_2X_4X_7$
$X_2X_3X_5$

Effects with the same standard error as $X_3 = X_4X_7X_9$
$X_1X_7 = X_2X_8$
$X_2X_6 = X_1X_5X_9$
$X_4X_5 = X_3X_6X_8$
$X_6X_8 = X_3X_4X_5 = X_5X_7X_9$
$X_1X_3X_7 = X_1X_4X_9 = X_2X_3X_8$
$X_2X_3X_6$

Effects with the same standard error as $X_4 = X_3X_7X_9$
X_1X_3
$X_2X_5 = X_1X_6X_9$
$X_6X_7 = X_5X_8X_9$
$X_1X_3X_4 = X_1X_7X_9 = X_2X_8X_9$
$X_2X_4X_5$
$X_3X_6X_9 = X_4X_6X_7$

Effects with the same standard error as X_5
$X_1X_2 = X_7X_8 = X_5X_6X_9$
$X_3X_7 = X_4X_9$
$X_3X_8 = X_4X_5X_6$
$X_4X_6 = X_3X_5X_8$
$X_6X_9 = X_1X_2X_5 = X_5X_7X_8$
$X_3X_5X_7 = X_4X_5X_9$

Effects with the same standard error as X_6
$X_1X_5 = X_2X_6X_9$
$X_1X_8 = X_2X_7$
$X_2X_9 = X_1X_5X_6$
$X_3X_4 = X_7X_9 = X_5X_6X_8$
$X_5X_8 = X_3X_4X_6 = X_6X_7X_9$
$X_1X_6X_8 = X_2X_6X_7$

Effects with the same standard error as $X_7 = X_1X_2X_8 = X_3X_4X_9$
X_1X_4
X_2X_3
$X_5X_6 = X_1X_2X_9 = X_3X_4X_8 = X_7X_8X_9$
$X_8X_9 = X_5X_6X_7$
$X_1X_3X_8 = X_2X_3X_7 = X_2X_4X_9$
$X_1X_3X_9 = X_1X_4X_7 = X_2X_4X_8$

Effects with the same standard error as $X_8 = X_1X_2X_7$
$X_1X_9 = X_2X_5X_6$
$X_1X_3X_5$
$X_1X_4X_6$
$X_1X_8X_9 = X_2X_3X_4 = X_2X_7X_9$
$X_3X_5X_9 = X_4X_5X_7$
$X_3X_6X_7 = X_4X_6X_9$

Effects with the same standard error as $X_9 = X_3X_4X_7$
$X_4X_8 = X_3X_5X_6$
$X_1X_2X_3 = X_3X_7X_8 = X_4X_8X_9$
$X_1X_4X_5$
$X_1X_5X_8 = X_2X_5X_7$
$X_1X_6X_7 = X_2X_6X_8$
$X_2X_4X_6$

Table 10. Symmetric Split-Lot Design Based on Table 9

Wafer	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9
1	1	1	5	4	8	2	5	3	2
2	1	2	3	7	2	6	3	8	7
3	1	3	4	6	3	7	2	4	8
4	1	4	6	1	5	3	8	7	1
5	1	5	1	8	7	4	6	6	5
6	1	6	7	3	1	8	4	1	4
7	1	7	8	2	4	5	1	5	3
8	1	8	2	5	6	1	7	2	6
9	2	1	4	8	1	5	3	2	1
10	2	2	6	3	7	1	5	5	8
11	2	3	5	2	6	4	8	1	7
12	2	4	3	5	4	8	2	6	2
13	2	5	8	4	2	7	4	7	6
14	2	6	2	7	8	3	6	4	3
15	2	7	1	6	5	2	7	8	4
16	2	8	7	1	3	6	1	3	5
17	3	1	1	3	4	6	8	4	6
18	3	2	7	8	6	2	2	7	3
19	3	3	8	5	7	3	3	3	4
20	3	4	2	2	1	7	5	8	5
21	3	5	5	7	3	8	7	5	1
22	3	6	3	4	5	4	1	2	8
23	3	7	4	1	8	1	4	6	7
24	3	8	6	6	2	5	6	1	2
25	4	1	8	7	5	1	2	1	5
26	4	2	2	4	3	5	8	6	4
27	4	3	1	1	2	8	5	2	3
28	4	4	7	6	8	4	3	5	6
29	4	5	4	3	6	3	1	8	2
30	4	6	6	8	4	7	7	3	7
31	4	7	5	5	1	6	6	7	8
32	4	8	3	2	7	2	4	4	1
33	5	1	2	6	7	8	1	7	7
34	5	2	8	1	1	4	7	4	2
35	5	3	7	4	4	1	6	8	1
36	5	4	1	7	6	5	4	3	8
37	5	5	6	2	8	6	2	2	4
38	5	6	4	5	2	2	8	5	5
39	5	7	3	8	3	3	5	1	6
40	5	8	5	3	5	7	3	6	3
41	6	1	7	2	2	3	7	6	8
42	6	2	1	5	8	7	1	1	1
43	6	3	2	8	5	6	4	5	2
44	6	4	8	3	3	2	6	2	7
45	6	5	3	6	1	1	8	3	3
46	6	6	5	1	7	5	2	8	6
47	6	7	6	4	6	8	3	4	5
48	6	8	4	7	4	4	5	7	4
49	7	1	6	5	3	4	4	8	3
50	7	2	4	2	5	8	6	3	6
51	7	3	3	3	8	5	7	7	5
52	7	4	5	8	2	1	1	4	4
53	7	5	2	1	4	2	3	1	8
54	7	6	8	6	6	6	5	6	1
55	7	7	7	7	7	7	8	2	2
56	7	8	1	4	1	3	2	5	7
57	8	1	3	1	6	7	6	5	4
58	8	2	5	6	4	3	4	2	5
59	8	3	6	7	1	2	1	6	6
60	8	4	4	4	7	6	7	1	3
61	8	5	7	5	5	5	5	4	7
62	8	6	1	2	3	1	3	7	2
63	8	7	2	3	2	4	2	3	1
64	8	8	8	8	8	8	8	8	8

(cont.)

Table 8. Aliasing Structure for Minimum Aberration 2^{9-3} Design (with Solution A grouping from Table 7)

Defining relation: $I = X_5X_6X_7X_8$
$= X_2X_3X_5X_6X_9$
$= X_1X_4X_6X_7X_9$
$= X_1X_4X_5X_8X_9$
$= X_2X_3X_7X_8X_9$
$= X_1X_2X_3X_4X_5X_7$
$= X_1X_2X_3X_4X_6X_8$

Effects with the same standard error as X_1
$X_5X_6 = X_7X_8 = X_2X_3X_9$
$X_6X_9 = X_2X_3X_5 = X_1X_4X_7$
$X_5X_9 = X_2X_3X_6 = X_1X_4X_8$
$X_4X_7 = X_1X_6X_9$
$X_4X_8 = X_1X_5X_9$
$X_1X_5X_6 = X_1X_7X_8$

Effects with the same standard error as X_2
$X_1X_6 = X_4X_7X_9$
$X_1X_8 = X_4X_5X_9$
$X_6X_8 = X_5X_7$
$X_1X_2X_6 = X_3X_4X_8$
$X_3X_4X_6 = X_1X_2X_8$
$X_1X_3X_4 = X_2X_5X_7 = X_2X_6X_8$

Effects with the same standard error as X_3
X_1X_2
$X_1X_7 = X_4X_6X_9$
$X_2X_7 = X_3X_8X_9$
$X_8X_9 = X_1X_4X_5 = X_2X_3X_7$
$X_1X_2X_3 = X_4X_5X_7 = X_4X_6X_8$
$X_2X_4X_5 = X_1X_3X_7$

Effects with the same standard error as X_4
$X_2X_3 = X_5X_6X_9 = X_7X_8X_9$
$X_2X_9 = X_3X_5X_6 = X_3X_7X_8$
$X_3X_9 = X_2X_5X_6 = X_2X_7X_8$
$X_2X_3X_4 = X_1X_5X_7 = X_1X_6X_8$
$X_3X_4X_9$
$X_2X_4X_9$

Effects with the same standard error as $X_5 = X_6X_7X_8$
X_3X_4
$X_7X_9 = X_1X_4X_6 = X_2X_3X_8$
$X_3X_4X_5 = X_1X_2X_7$
$X_1X_3X_6 = X_2X_4X_8$
$X_5X_7X_9 = X_6X_8X_9$
$X_1X_2X_9$

Effects with the same standard error as $X_6 = X_5X_7X_8$
$X_4X_5 = X_1X_8X_9$
$X_3X_8 = X_2X_7X_9$
$X_4X_5X_6 = X_4X_7X_8$
$X_1X_2X_4 = X_3X_5X_7 = X_3X_6X_8$
$X_1X_2X_5 = X_3X_4X_7$
$X_1X_3X_9$

Effects with the same standard error as $X_7 = X_5X_6X_8$
X_2X_4
$X_2X_6 = X_3X_5X_9$
$X_4X_6 = X_1X_7X_9$
$X_1X_9 = X_4X_6X_7 = X_4X_5X_8$
$X_1X_3X_5 = X_2X_4X_7$
$X_2X_6X_7 = X_2X_5X_8$

Effects with the same standard error as $X_8 = X_5X_6X_7$
X_1X_3
$X_3X_5 = X_2X_6X_9$
$X_1X_5 = X_4X_8X_9$
$X_4X_9 = X_1X_6X_7 = X_1X_5X_8$
$X_2X_4X_6 = X_1X_3X_8$
$X_3X_6X_7 = X_3X_5X_8$

Effects with the same standard error as X_9
$X_6X_7 = X_5X_8 = X_1X_4X_9$
$X_1X_4 = X_6X_7X_9 = X_5X_8X_9$
$X_2X_5 = X_3X_6X_9$
$X_3X_6 = X_2X_5X_9$
$X_3X_7 = X_2X_8X_9$
$X_2X_8 = X_3X_7X_9$

Extensions (64 wafers)

- 9 factors in 9 steps (2^{9-3} design)
- 8 factors in 8 steps (2^{8-2} design)
- 7 factors in 7 steps (2^{7-1} design)
- 6 factors in 6 steps (2^{6} design)
- Other possibilities

Outline

- Introduction
  - two different designs
- Constructing 16-wafer experiments
- Symmetric split-lot designs for 64-wafer experiments
- Discussions

Other important split-plot design

- Split-plot design for 3-level factors
  3^4 of 3^{k-p} factorials in 81-wafers
- Split-plot designs of size 27
- Split-plot designs of size 32

Future research

- three or four experimental stages, with many factors per stage [the case of two stages was addressed by Miller (1997)]
- asymmetric split-lot designs-that is, those in which the sublot size differs from one experimental step to the next

(cont.)

- mixed-level factorials and designs for fitting second order models
- designs that can check for lack of fit of our assumed model-for example, that could detect existence of additional variance components associated with wafers having two sublots in common

Thank you all