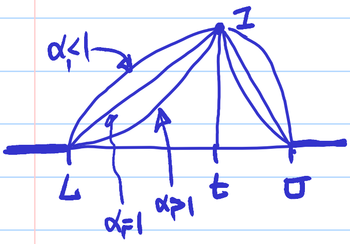
Analysis Strategies for Multiple Responses

- Desirability function
  - For nominal-the-best problem: $\hat{f}(\underset{\sim}{x})$

lower bound of acceptable $\hat{y}$ value



target value

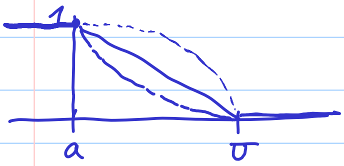
$$d = \begin{cases} \left|\dfrac{\hat{y}-L}{t-L}\right|^{\alpha_1}, & L \le \hat{y} \le t \\ \left|\dfrac{\hat{y}-U}{t-U}\right|^{\alpha_2}, & t \le \hat{y} \le U \\ 0, & \text{otherwise} \end{cases}$$

upper bound

larger α: importance of being close to t

smaller α: do not have to be very close to the target t.

  - For smaller-the-better problem

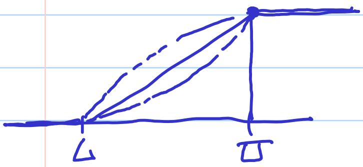


upper bound

$$d = \begin{cases} \left|\dfrac{\hat{y}-U}{a-U}\right|^{\alpha}, & a \le \hat{y} \le U \\ 0, & \hat{y} > U \end{cases}$$

smallest possible value of $\hat{y}$

  - For larger-the-better problem



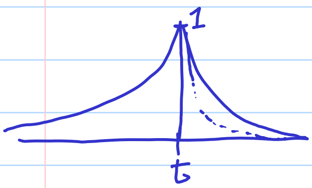
upper bound

$$d = \begin{cases} \left|\dfrac{\hat{y}-L}{U-L}\right|^{\alpha}, & L \le \hat{y} \le U \\ 1, & \hat{y} > U \\ 0, & \hat{y} < L \end{cases}$$

lower bound

Analysis Strategies for Multiple Responses

- Desirability function II
  - In some practical situations, the L and U values in the desirability function I cannot be properly chosen.
  - For nominal-the-best problem:

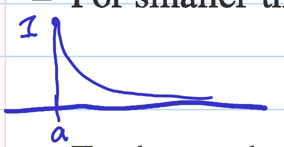


$$d = \begin{cases} \exp\{-c_1|\hat{y}-t|^{\alpha_1}\}, & -\infty < \hat{y} \le t \\ \exp\{-c_2|\hat{y}-t|^{\alpha_2}\}, & t \le \hat{y} < \infty \end{cases}$$

smaller α: make the desirability function drop more slowly from its peak.

smaller c: increase the spread of the desirability function by lowering the whole exponential curve between 0 & 1.

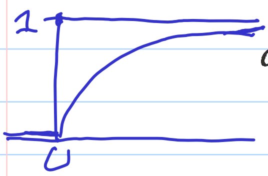
  - For smaller-the-better problem



$$d = \exp\{-c|\hat{y}-a|^{\alpha}\}, \quad a \le \hat{y} < \infty.$$

smallest possible value.

  - For larger-the-better problem



$$d = \begin{cases} 1-\exp\{-c\hat{y}^{\alpha}\}/\exp\{-cL^{\alpha}\}, & L \le \hat{y} < \infty \\ 0, & \hat{y} < L \end{cases}$$

lower bound

Analysis Strategies for Multiple Responses

- Overall desirability function D

  transform multiple response problem into single response problem. (why reasonable?)

  suggestion: better to understand the relationship between $\hat{y}_1, \cdots, \hat{y}_m$ before perform desirability analysis.

  e.g. do PCA on $y_1, \cdots, y_m$

  - Weighted geometric mean

    weight

    $$D \overset{(\times)}{=} d_1^{w_1} d_2^{w_2} \cdots d_m^{w_m},$$

    where the weights w_i's satisfy $0 < w_i < 1$ and $w_1 + \cdots + w_m = 1$.

    * $\ln D = \sum_{i=1}^{m} w_i \ln d_i$ weighted average of $\ln(d_i)$'s
    * If any $d_i = 0$ then $D = 0$ $\hat{y}_i$ undesirable/unacceptable
    * When $w_1 = \cdots = w_m = 1/m$, D is the geometric mean

  - Weighted arithmetic mean

    $$D \overset{(\times)}{=} w_1 d_1 + w_2 d_2 + \cdots + w_m d_m$$

- Any setting for the input factors that maximizes the D value is chosen to be one which achieves an optimal balance over the m responses.
  - After finding such a setting, it should be verified whether all constraints on the responses are satisfied can do confirm expt some constraint may not be explicitly spelled out in the beginning.

❖ **Reading**: textbook, 10.6