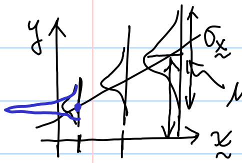

parameter version $\hookrightarrow \eta_{\underset{\sim}{x}} = \ln\left(\frac{\mu_{\underset{\sim}{x}}^2}{\sigma_{\underset{\sim}{x}}^2}\right)$

coefficient of variation: $\sigma_{\underset{\sim}{x}} / \mu_{\underset{\sim}{x}}$ ← dimensionless

Signal-to-Noise Ratio

- Taguchi's SN ratio $\hat{\eta}_{\underset{\sim}{x}} = \ln \frac{\bar{y}_{\underset{\sim}{x}}^2}{s_{\underset{\sim}{x}}^2} = \ln \bar{y}_{\underset{\sim}{x}}^2 - \ln s_{\underset{\sim}{x}}^2$

  t^2 for testing $\mu_{\underset{\sim}{x}} = 0$

  2 extreme cases: $\bar{y}_{\underset{\sim}{x}}^2$ dominate $\hat{\eta}_{\underset{\sim}{x}}$ — ①; $s_{\underset{\sim}{x}}^2$ dominate $\hat{\eta}_{\underset{\sim}{x}}$ — ②

- Two-step procedure:

  replace minimize $\ln s_{\underset{\sim}{x}}^2$

  $\eta_{\underset{\sim}{x}} \sim f(\underset{\sim}{x}) + \varepsilon$

  1. Select control factor levels to maximize SN ratio.

     for ①, maximizing $\bar{y}_{\underset{\sim}{x}}^2$: ②, minimizing $s_{\underset{\sim}{x}}^2$

     $\bar{y}_{\underset{\sim}{x}} \sim g(\underset{\sim}{x}) + \varepsilon$

  2. Use an adjustment factor to move mean on target.

  model: $y_{\underset{\sim}{x}} \sim \mu(\underset{\sim}{x}_1, \underset{\sim}{x}_2) \cdot \varepsilon(\underset{\sim}{x}_1)$ ← r.v.; $\underset{\sim}{x}_1$: noise factor, $\underset{\sim}{x}_2$: control factor

  $E(\varepsilon) = 1$, $Var(\varepsilon) = P(\underset{\sim}{x}_1)$

  $E(y_{\underset{\sim}{x}}) = \mu(\underset{\sim}{x}_1, \underset{\sim}{x}_2)$

  $Var(y_{\underset{\sim}{x}}) = \mu^2(\underset{\sim}{x}_1, \underset{\sim}{x}_2) \cdot P(\underset{\sim}{x}_1)$

  $\eta = \ln\left(\mu^2 / \mu^2 P(\underset{\sim}{x}_1)\right) = -\ln(P(\underset{\sim}{x}_1))$

- **Limitations**

  For ①, why get an $\underset{\sim}{x}$ with large $|\bar{y}_{\underset{\sim}{x}}|$ in step 1, then adjust $\bar{y}_{\underset{\sim}{x}}$ to target in step 2?

  – maximizing $\bar{y}^2$ not always desired.

  – little justification outside linear circuitry.

  – statistically justifiable only when $Var(y)$ is proportional to $E(y)^2$

  – $\eta_{\underset{\sim}{x}} \approx -\ln(Var(\ln y_{\underset{\sim}{x}}))$ ← see LN p. 2-33.

- **Recommendation**: Use SN ratio sparingly. Better to use the location-dispersion modeling or the response modeling. The latter strategies can do whatever SN ratio analysis can achieve.

  especially, when $y_{\underset{\sim}{x}} \underset{\text{assumed}}{\sim} N(\mu_{\underset{\sim}{x}}, \sigma_{\underset{\sim}{x}}^2)$

Half-normal Plot for S/N Ratio Analysis

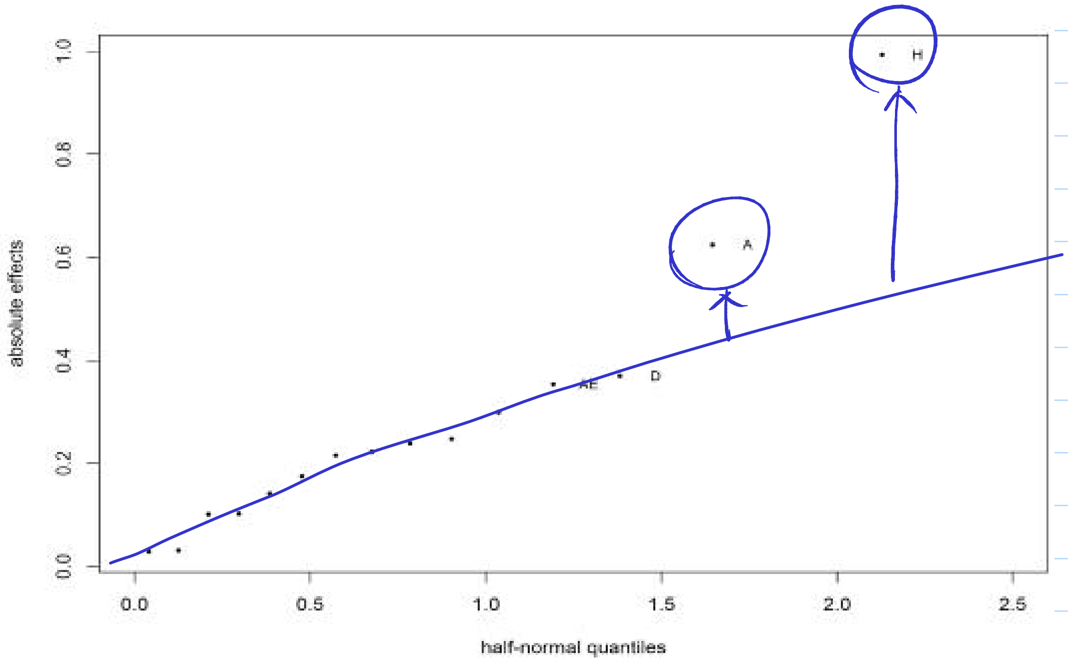

Figure 8: Half-Normal Plots of Effects Based on SN Ratio, Layer Growth Experiment

S/N Ratio Analysis for Layer Growth Experiment

- Based on the $\hat{\eta}_i$ column in Table 5, compute the factorial effects using SN ratio. From Figure 7, the $\boxed{\text{conclusion}}$ is similar to location-dispersion analysis. Why? Using $\boxed{H+A-}$ ← from the fitted model.

$$\hat{\eta}_i = \ln \bar{y}_i^2 - \ln s_i^2,$$

and from Table 5, the variation among $\ln s_i^2$ is much larger than the variation among $\ln \bar{y}_i^2$; thus maximizing SN ratio is equivalent to minimizing $\ln s_i^2$ in this case.

✲ If $y_{\underset{\sim}{x}} \sim$ log-normal$(\mu_{\underset{\sim}{x}}, \sigma_{\underset{\sim}{x}}^2) \Rightarrow z_{\underset{\sim}{x}} = \ln(y_{\underset{\sim}{x}}) \sim N(\mu_{\underset{\sim}{x}}, \sigma_{\underset{\sim}{x}}^2)$ — $\left[\begin{array}{l} E(z_{\underset{\sim}{x}}) = \mu_{\underset{\sim}{x}} \\ Var(z_{\underset{\sim}{x}}) = \sigma_{\underset{\sim}{x}}^2 \end{array}\right.$

$Var(y_{\underset{\sim}{x}}) = E(y_{\underset{\sim}{x}}^2) - [E(y_{\underset{\sim}{x}})]^2 = \exp(2\mu_{\underset{\sim}{x}} + 2\sigma_{\underset{\sim}{x}}^2) - [\exp(\mu_{\underset{\sim}{x}} + \frac{1}{2}\sigma_{\underset{\sim}{x}}^2)]^2$

$= \exp(2\mu_{\underset{\sim}{x}} + \sigma_{\underset{\sim}{x}}^2)[\exp(\sigma_{\underset{\sim}{x}}^2) - 1] = [E(y_{\underset{\sim}{x}})]^2(\exp(\sigma_{\underset{\sim}{x}}^2) - 1)$

$\Rightarrow E(y_{\underset{\sim}{x}})^2 / Var(y_{\underset{\sim}{x}}) = [\exp(\sigma_{\underset{\sim}{x}}^2) - 1]^{-1} \propto \sigma_{\underset{\sim}{x}}^{-2} = [Var(z_{\underset{\sim}{x}})]^{-1}$ locally.

$\Rightarrow \eta_{\underset{\sim}{x}} = -\ln(Var(z_{\underset{\sim}{x}}))$

v **Reading**: textbook, 11.9