Exploitation of Nonlinearity

not a straight line (not a linear ME model)

- Nonlinearity between y and $\mathbf{x}$ can be exploited for robustness if $\mathbf{x_0}$, nominal values of $\mathbf{x}$, are control factors and deviations of $\mathbf{x}$ around $\mathbf{x_0}$ are viewed as noise factors (called *internal noise*). Expand $y = f(\mathbf{x})$ around $\mathbf{x_0}$, ↳ one variable x_i

$$y \approx f(\mathbf{x_0}) + \sum_i \frac{\partial f}{\partial x_i}\bigg|_{x_{i0}} \underbrace{(x_i - x_{i0})}_{N_i}. \quad (1)$$

$C = x_0$ — one control factor $\underset{\sim}{C}$; one noise factor. $\underset{\sim}{N} = \underset{\sim}{x} - \underset{\sim}{x_0}$

δ-method

This leads to

$$\mathrm{Var}_{\underset{\sim}{N}}(y \mid \underset{\sim}{x_0}) = \sigma^2 \approx \sum_i \left(\frac{\partial f}{\partial x_i}\bigg|_{x_{i0}}\right)^2 \sigma_i^2$$

variation of x_i around x_{i0}

function of C

where $\sigma^2 = var(y)$, $\sigma_i^2 = var(x_i)$, each component x_i has mean x_{i0} and variance σ_i^2.

- From (1), it can be seen that σ^2 can be reduced by choosing x_{i0} with a smaller slope $\frac{\partial f}{\partial x_i}\Big|_{x_{i0}}$. This is demonstrated in Figure 1. Moving the nominal value a to b can reduce $var(y)$ because the slope at b is more flat. This is a **parameter design** step. On the other hand, reducing the variation of x around a can also reduce $var(y)$. This is a **tolerance design** step.

Exploitation of Nonlinearity to Reduce Variation

But, their means ($E_N(y_c)$) also change from $f(a)$ to $f(b)$.

Hint: use other control factor (if exist) that can influence $E(y_c)$ but not $\mathrm{Var}(y_c)$ to adjust mean

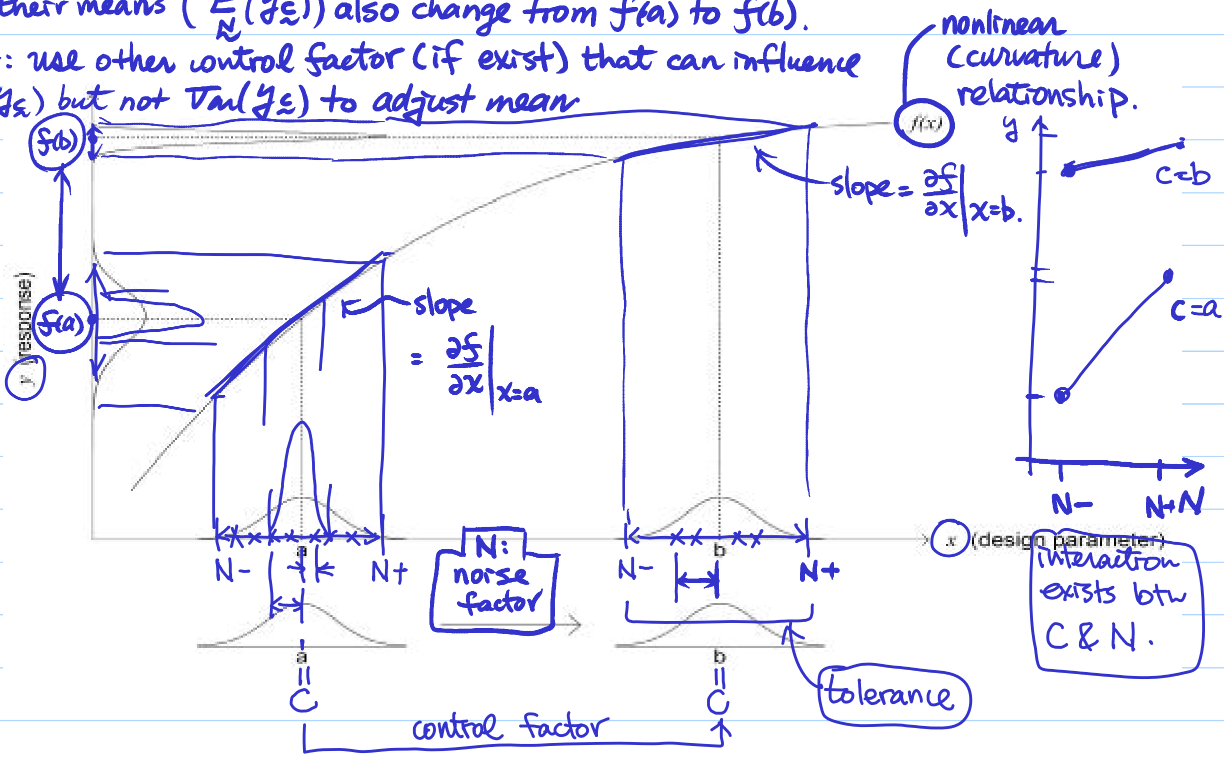


Figure 1: Exploiting the Nonlinearity of $f(x)$ to Reduce Variation

v **Reading**: textbook, 11.4

Cross Array and Location-Dispersion Modeling

the whole design matrix ← $n_c \times n_N$ runs.

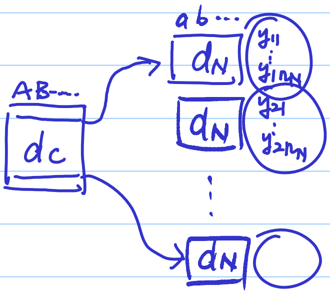
d_C ← n_c runs d_N ← n_N runs

- Cross array = control array × noise array, ← Note: for each level combination of d_C, it corresponds to same level combinations for noise factors.

  control array = array for control factors,
  inner array
  noise array = array for noise factors.
  outer array
  single array (c.f.)

- Location-dispersion modeling
  - compute $\bar{y}_i, s_i^2$ based on the noise settings for the i^{th} control setting,
    use noise array
  - analyze $\bar{y}_i$ (location), and $\ln s_i^2$ (dispersion), identify significant location and dispersion effects. build two models — mean. $E(y|\underset{\sim}{c})$; var. $\mathrm{Var}(y|\underset{\sim}{c})$
    use control array

same analysis strategy used in CH4 & CH5 for data with replicates



Two-step Procedures for Parameter Design Optimization

- **Two-Step Procedure for Nominal-the-Best Problem**

  (i) *select the levels of the dispersion factors to minimize dispersion,*
  (ii) *select the level of the adjustment factor to bring the location on target.* (2)

- **Two-Step Procedure for Larger-the-Better and Smaller-the-Better Problems**

  (i) *select the levels of the location factors to maximize (or minimize) the location,*
  (ii) *select the levels of the dispersion factors that are not location factors to minimize dispersion.* (3)

Note that the two steps in (3) are in reverse order from those in (2).
Reason: It is usually harder to increase or decrease the response y in the latter problem, so this step should be the first to perform.

Analysis of Layer Growth Experiment

- From the $\bar{y}_i$ and $\ln s_i^2$ columns of Table 5, compute the factorial effects for location and dispersion respectively. (These numbers are not given in the book.) From the half-normal plots of these effects (Figure 2), D is significant for location and H, A for dispersion.

Note. there is no significant 2fi's in the two model (resolution = IV), 2fi exist → de-aliasing.

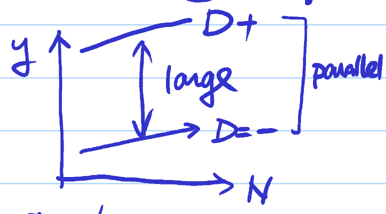
$$\begin{cases} \hat{y} = 14.352 + 0.402 x_D, \\ \hat{z} = -1.822 + 0.619 x_A - 0.982 x_H. \end{cases}$$

D: adjustment factor

- Two-step procedure:

  (i) choose A at the "−" level (continuous rotation) and H at the "+" level (nozzle position 6).

  (ii) By solving



$$\hat{y} = 14.352 + 0.402 x_D = 14.5,$$

target value

choose x_D= 0.368. −1 ≤ ≤ 1 interpolation.

Layer Growth Experiment: Analysis Results

regular design (no complex aliasing)

2^{8-4}_{IV} ← 16 d.f. available

Table 5: Means, Log Variances and SN Ratios, Layer Growth Experiment

Control Factor											
A	B	C	D	E	F	G	H	$\bar{y}_i$	$\ln s_i^2$	$\ln \bar{y}_i^2$	$\hat{\eta}_i$
−	−	−	+	−	−	−	−	14.79	-1.018	5.389	6.41
−	−	−	+	+	+	+	+	14.86	-3.879	5.397	9.28
−	−	+	−	−	−	+	+	14.00	-4.205	5.278	9.48
−	−	+	−	+	+	−	−	13.91	-1.623	5.265	6.89
−	+	−	−	−	+	−	+	14.15	-5.306	5.299	10.60
−	+	−	−	+	−	+	−	13.80	-1.236	5.250	6.49
−	+	+	+	−	+	+	−	14.73	-0.760	5.380	6.14
−	+	+	+	+	−	−	+	14.89	-1.503	5.401	6.90
+	−	−	−	−	+	+	−	13.93	-0.383	5.268	5.65
+	−	−	−	+	−	−	+	14.09	-2.180	5.291	7.47
+	−	+	+	−	+	−	+	14.79	-1.238	5.388	6.63
+	−	+	+	+	−	+	−	14.33	-0.868	5.324	6.19
+	+	−	+	−	−	+	+	14.77	-1.483	5.386	6.87
+	+	−	+	+	+	−	−	14.88	-0.418	5.400	5.82
+	+	+	−	−	−	−	−	13.76	-0.418	5.243	5.66
+	+	+	−	+	+	+	+	13.97	-2.636	5.274	7.91

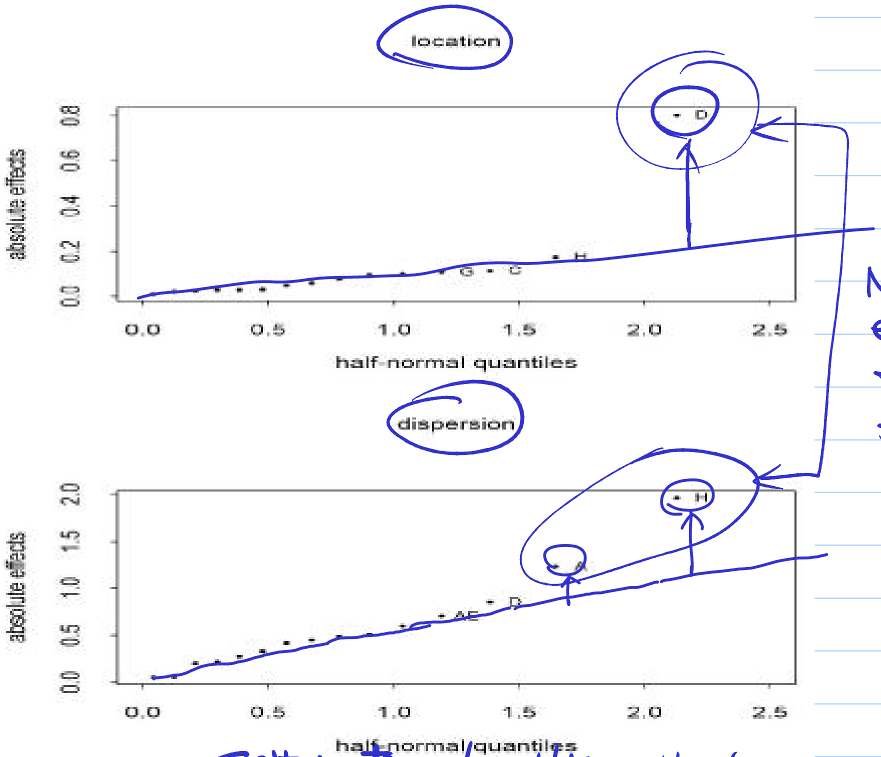
← use 8 observations.

I = −ABCD
= ABEF = −CDEF
= ACEG = −BDEG
= BCFG = −ADFG
= BCEH = −ADEH
= ACFH = −BDFH
= ABGH = −CDGH
= EFGH = −ABCDEFGH.

modeling: $\bar{y}$ or $\ln(s^2)$ → $z \sim \sum \beta_i (\text{factorial effects})_i + \epsilon$ (no replicates)

an alias set can contribute an effect ⇒ 16 effects → no d.f. for residuals.

Layer Growth Experiment: Plots



Note: significant effects from different factors.

alternative: Lenth's method.

Figure 2: Half-Normal Plots of Location and Dispersion Effects, Layer Growth Experiment

Analysis of Leaf Spring Experiment

- Based on the half-normal plots in Figure 3, B, C and E are significant for location, C is significant for dispersion:

adjustment factors

$$\hat{y} = 7.6360 + 0.1106 x_B + 0.0881 x_C + 0.0519 x_E,$$

$$\ell(s_i^2) = \hat{z} = -3.6886 + 1.0901 x_C$$

$\hat{\sigma}^2 = \exp(\ \ )$

$E(y_x - \mu_0)^2 = \hat{\sigma}^2 + (\text{bias})^2 = \text{MSE}$

	B	C	E	$\hat{y}$	$\hat{\sigma}^2$	MSE
①	+	−	+	7.71	0.0084	0.0925
②	2.78	−	2.78	8	0.0084	0.0084
③	+	+	+	7.89	0.0744	0.0865

- Two-step procedure:
  (i) choose C at $-$.
  (ii) With $x_C = -1$, $\hat{y} = 7.5479 + 0.1106 x_B + 0.0519 x_E$.

extrapolation. Note: review the discussion about exp'tal region

To achieve $\hat{y} = 8.0$, x_B and x_E must be chosen beyond $+1$, i.e., $x_B = x_E = 2.78$. This is too drastic, and not validated by current data. An alternative is to select $x_B = x_E = x_C = +1$ (not to follow the two-step procedure), then $\hat{y}$=7.89 is closer to 8. (Note that $\hat{y} = 7.71$ with $B_+C_-E_+$.)

do confirm exp't.

Reason for the breakdown of the 2-step procedure: its second step cannot achieve the target 8.0

move the 2nd step into 1st step.

Leaf Spring Experiment: Analysis Results

Table 6: Means and Log Variances, Leaf Spring Experiment

2^{4-1}_{IV} (8 runs)

I=BCDE

Control Factor					
B	*C*	*D*	*E*	$\bar{y}_i$	$\ln s_i^2$
−	+	+	−	7.540	-2.4075
+	+	+	+	7.902	-2.6488
−	−	+	+	7.520	-6.9486
+	−	+	−	7.640	-4.8384
−	+	−	+	7.670	-2.3987
+	+	−	−	7.785	-2.9392
−	−	−	−	7.372	-3.2697
+	−	−	+	7.660	-4.0582

modeling: same as what in LNp.2-14

Leaf Spring Experiment: Plots

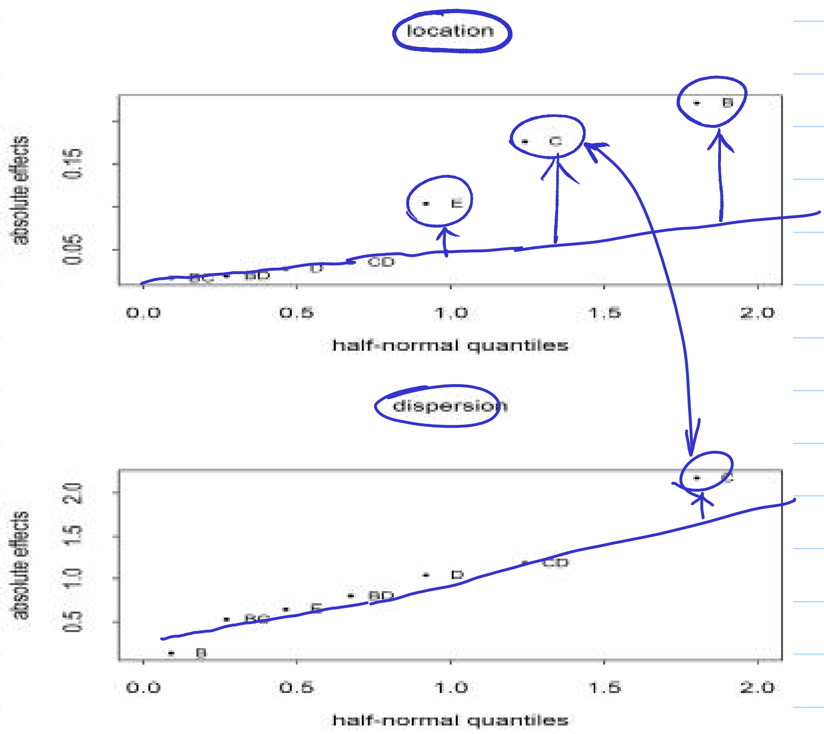


Figure 3: Half-Normal Plots of Location and Dispersion Effects, Leaf Spring Experiment

Location-dispersion modeling

Response Modeling and Control-by-Noise Interaction Plots

① mask some important relationship between C & N factors
② modeling ⟷ approximation.
Q: Is it appropriate to build ~~model~~ dispersion model based on $\ln(s^2)$?

constant var error

Note: In regression analysis, C, N treated as variables with fixed values. (DOE step)

- Response Model: model y_{ij} directly in terms of control, noise effects and control-by-noise interactions.
  ① $y_{C,N} \sim f(C,N)+\varepsilon$ $E[y|C,N]$
  ② treat N as r.v. (use condition) $E_N(y|C) \to$ mean model, $Var_N(y|C) \to$ var model — function of C only.
  - half normal plot of various effects.
  - regression model fitting, obtaining $\hat{y}$.

graphical method (intuitive perception)

- Make control-by-noise interaction plots for significant effects in $\hat{y}$, choose **robust** control settings at which y has a flatter relationship with noise.

numerical method (easier to do interpolation)

- Compute $Var(\hat{y})$ with respect to variation in the noise factors. Call $Var(\hat{y})$ the **transmitted variance model**. Use it to identify control factor settings with small transmitted variance.

objective response ~~modeling~~ variable
modeling response variable
← no need to be identical.
Q: what variable is more suitable to be the response in the fitted model? ← apply approximation viewpoint.

Half-normal Plot, Layer Growth Experiment

similar to the coding of 2 2-level factors & their interaction

the whole design matrix (C & N) can be treated as $2^{(8+3)-4}_{IV}$ regular design with the same defining contrast subgroup given in LNp.2-14.

10/21

- Define

$$M_l = (M_1+M_2)-(M_3+M_4),$$
$$M_q = (M_1+M_4)-(M_2+M_3),$$
$$M_c = (M_1+M_3)-(M_2+M_4),$$

M	M_l (A)	M_q (B)	M_c (AB)
1	+1	+	+
2	+1	−	−
3	−	−	+
4	−	+	−

coding; $M_c = M_l \times M_q$; benefit: orthogonality

model: $y \sim \sum \beta_i$(all factorial effects formed by C & N)$_i + \varepsilon$ ← no replicates

- From Figure 4, select D, L, HL as the most significant effects.
- How to deal with the next cluster of effects in Figure 4? Use **step-down multiple comparisons**.
- After removing the top three points in Figure 4, make a half-normal plot (Figure 5) on the remaining points. The cluster of next four effects (M_l, H, CM_l, AHM_q) appear to be significant.