Robust Parameter Design

- Statistical/engineering method for product/process improvement (G. Taguchi).
- Two types of factors in a system (product/process):
  - control factors: once chosen, values remain fixed.
  - noise factors: hard-to-control during normal process or usage.
- **Robust Parameter design** (**RPD** or **PD**): choose control factor settings to make response less sensitive (i.e.more robust) to noise variation; exploiting control-by-noise interactions.

A Robust Design Perspective of Layer-growth and Leaf Spring Experiments

- The original AT & T layer growth experiment had 8 control factors, 2 noise factors (location and facet). Goal was to achieve *uniform* thickness around 14.5 μm over the noise factors. See Tables 1 and 2.
- The original leaf spring experiment had 4 control factors, 1 noise factor (quench oil temperature). The quench oil temperature is not controllable; with efforts it can be set in two ranges of values 130-150, 150-170. Goal is to achieve *uniform* free height around 8 inches over the range of quench oil temperature. See Tables 3 and 4.
- Must understand the role of *noise factors* in achieveing *robustness*.

Layer Growth Experiment: Factors and Levels

Table 1: Factors and Levels, Layer Growth Experiment

Control Factor		Level	
		−	+
A.	susceptor-rotation method	continuous	oscillating
B.	code of wafers	668G4	678D4
C.	deposition temperature(°C)	1210	1220
D.	deposition time	short	long
E.	arsenic flow rate(%)	55	59
F.	hydrochloric acid etch temperature(°C)	1180	1215
G.	hydrochloric acid flow rate(%)	10	14
H.	nozzle position	2	6
Noise Factor		**Level**	
		−	+
L.	location	bottom	top
M.	facet	1 2	3 4

Layer Growth Experiment: Thickness Data

Table 2: Cross Array and Thickness Data, Layer Growth Experiment

Control Factor								Noise Factor							
								L-Bottom				L-Top			
A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*M*-1	*M*-2	*M*-3	*M*-4	*M*-1	*M*-2	*M*-3	*M*-4
−	−	−	+	−	−	−	−	14.2908	14.1924	14.2714	14.1876	15.3182	15.4279	15.2657	15.4056
−	−	−	+	+	+	+	+	14.8030	14.7193	14.6960	14.7635	14.9306	14.8954	14.9210	15.1349
−	−	+	−	−	−	+	+	13.8793	13.9213	13.8532	14.0849	14.0121	13.9386	14.2118	14.0789
−	−	+	−	+	+	−	−	13.4054	13.4788	13.5878	13.5167	14.2444	14.2573	14.3951	14.3724
−	+	−	−	−	+	−	+	14.1736	14.0306	14.1398	14.0796	14.1492	14.1654	14.1487	14.2765
−	+	−	−	+	−	+	−	13.2539	13.3338	13.1920	13.4430	14.2204	14.3028	14.2689	14.4104
−	+	+	+	−	+	+	−	14.0623	14.0888	14.1766	14.0528	15.2969	15.5209	15.4200	15.2077
−	+	+	+	+	−	−	+	14.3068	14.4055	14.6780	14.5811	15.0100	15.0618	15.5724	15.4668
+	−	−	−	−	+	+	−	13.7259	13.2934	12.6502	13.2666	14.9039	14.7952	14.1886	14.6254
+	−	−	−	+	−	−	+	13.8953	14.5597	14.4492	13.7064	13.7546	14.3229	14.2224	13.8209
+	−	+	+	−	+	−	+	14.2201	14.3974	15.2757	15.0363	14.1936	14.4295	15.5537	15.2200
+	−	+	+	+	−	+	−	13.5228	13.5828	14.2822	13.8449	14.5640	14.4670	15.2293	15.1099
+	+	−	+	−	−	+	+	14.5335	14.2492	14.6701	15.2799	14.7437	14.1827	14.9695	15.5484
+	+	−	+	+	+	−	−	14.5676	14.0310	13.7099	14.6375	15.8717	15.2239	14.9700	16.0001
+	+	+	−	−	−	−	−	12.9012	12.7071	13.1484	13.8940	14.2537	13.8368	14.1332	15.1681
+	+	+	−	+	+	+	+	13.9532	14.0830	14.1119	13.5963	13.8136	14.0745	14.4313	13.6862

Leaf Spring Experiment

Table 3: Factors and Levels, Leaf Spring Experiment

Control Factor		Level −	Level +
B.	high heat temperature (°F)	1840	1880
C.	heating time (seconds)	23	25
D.	transfer time (seconds)	10	12
E.	hold down time (seconds)	2	3
Noise Factor		**Level −**	**Level +**
Q.	quench oil temperature (°F)	130-150	150-170

Table 4: Cross Array and Height Data, Leaf Spring Experiment

Control Factor *B*	*C*	*D*	*E*	Noise Factor Q^-			Q^+		
−	+	+	−	7.78	7.78	7.81	7.50	7.25	7.12
+	+	+	+	8.15	8.18	7.88	7.88	7.88	7.44
−	−	+	+	7.50	7.56	7.50	7.50	7.56	7.50
+	−	+	−	7.59	7.56	7.75	7.63	7.75	7.56
−	+	−	+	7.94	8.00	7.88	7.32	7.44	7.44
+	+	−	−	7.69	8.09	8.06	7.56	7.69	7.62
−	−	−	−	7.56	7.62	7.44	7.18	7.18	7.25
+	−	−	+	7.56	7.81	7.69	7.81	7.50	7.59

v **Reading**: textbook, 11.1

Strategies for Variation Reduction

- ***Sampling inspection***: passive, sometimes last resort.
- ***Control charting and process monitoring***: can remove special causes. If the process is stable, it can be *followed* by using a *designed experiment*.
- ***Blocking, covariate adjustment***: passive measures but useful in reducing variability, not for removing root causes.
- ***Reducing variation in noise factors***: effective as it may reduce variation in the response but can be expensive. Better approach is to change control factor settings (*cheaper* and *easier* to do) by exploiting control-by-noise interactions, i.e., use robust parameter design!

v **Reading**: textbook, 11.2

Types of Noise Factors

1. Variation in process parameters.
2. Variation in product parameters.
3. Environmental variation.
4. Load Factors.
5. Upstream variation.
6. Downstream or user conditions.
7. Unit-to-unit and spatial variation.
8. Variation over time.
9. Degradation.

- **Traditional design uses 7 and 8.**

v **Reading**: textbook, 11.3

Variation Reduction Through RPD

- Suppose $y = f(x, z)$, x control factors and z noise factors. If x and z interact in their effects on y, then the $var(y)$ can be reduced either by reducing $var(z)$ (i.e. method 4 on p.2-6) or by changing the x values (i.e., RPD).
- An example:

$$
\begin{aligned}
y &= \mu + \alpha x_1 + \beta z + \gamma x_2 z + \varepsilon, \\
&= \mu + \alpha x_1 + (\beta + \gamma x_2) z + \varepsilon.
\end{aligned}
$$

By choosing an appropriate value of x to reduce the coefficient $\beta + \gamma x_2$, the impact of z on y can be reduced. Since β and γ are unknown, this can be achieved by using the control-by-noise interaction plots or other methods to be presented later.

Exploitation of Nonlinearity

- Nonlinearity between y and $\mathbf{x}$ can be exploited for robustness if $\mathbf{x_0}$, nominal values of $\mathbf{x}$, are control factors and deviations of $\mathbf{x}$ around $\mathbf{x_0}$ are viewed as noise factors (called *internal noise*). Expand $y = f(\mathbf{x})$ around $\mathbf{x_0}$,

$$y \approx f(\mathbf{x_0}) + \sum_i \frac{\partial f}{\partial x_i}\bigg|_{x_{i0}} (x_i - x_{i0}). \tag{1}$$

  This leads to

$$\sigma^2 \approx \sum_i \left(\frac{\partial f}{\partial x_i}\bigg|_{x_{i0}} \right)^2 \sigma_i^2,$$

  where $\sigma^2 = var(y)$, $\sigma_i^2 = var(x_i)$, each component x_i has mean x_{i0} and variance σ_i^2.

- From (1), it can be seen that σ^2 can be reduced by choosing x_{i0} with a smaller slope $\frac{\partial f}{\partial x_i}\Big|_{x_{i0}}$. This is demonstrated in Figure 1. Moving the nominal value a to b can reduce $var(y)$ because the slope at b is more flat. This is a **parameter design** step. On the other hand, reducing the variation of x around a can also reduce $var(y)$. This is a **tolerance design** step.

Exploitation of Nonlinearity to Reduce Variation

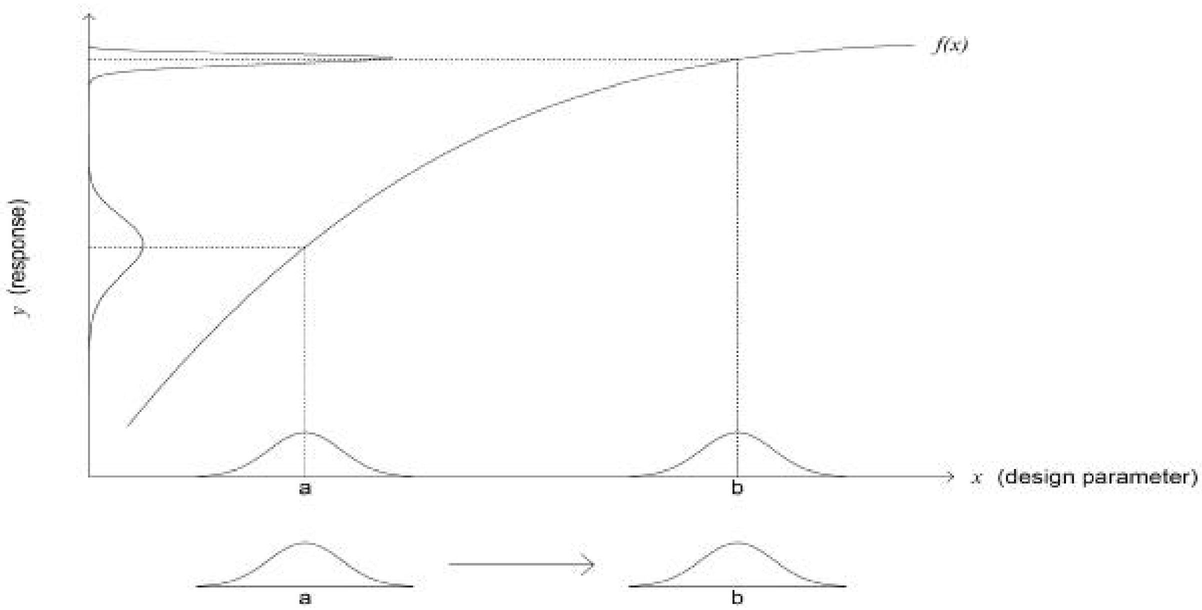

Figure 1: Exploiting the Nonlinearity of $f(x)$ to Reduce Variation

v **Reading**: textbook, 11.4

Cross Array and Location-Dispersion Modeling

- Cross array = control array × noise array,
  control array = array for control factors,
  noise array = array for noise factors.

- Location-dispersion modeling
  - compute $\bar{y}_i, s_i^2$ based on the noise settings for the i^{th} control setting,
  - analyze $\bar{y}_i$ (location), and $\ln s_i^2$ (dispersion), identify significant location and dispersion effects.

Two-step Procedures for Parameter Design Optimization

- **Two-Step Procedure for Nominal-the-Best Problem**

$$\begin{aligned} &(i)\ \textit{select the levels of the dispersion factors to minimize dispersion,} \\ &(ii)\ \textit{select the level of the adjustment factor to bring the location on target.} \end{aligned} \tag{2}$$

- **Two-Step Procedure for Larger-the-Better and Smaller-the-Better Problems**

$$\begin{aligned} &(i)\ \textit{select the levels of the location factors to maximize (or minimize)} \\ &\textit{the location,} \\ &(ii)\ \textit{select the levels of the dispersion factors that are not location} \\ &\textit{factors to minimize dispersion.} \end{aligned} \tag{3}$$

Note that the two steps in (3) are in reverse order from those in (2).
Reason: It is usually harder to increase or decrease the response y in the latter problem, so this step should be the first to perform.

Analysis of Layer Growth Experiment

- From the $\bar{y}_i$ and $\ln s_i^2$ columns of Table 5, compute the factorial effects for location and dispersion respectively. (These numbers are not given in the book.) From the half-normal plots of these effects (Figure 2), D is significant for location and H, A for dispersion.

$$\begin{aligned} \hat{y} &= 14.352 + 0.402 x_D, \\ \hat{z} &= -1.822 + 0.619 x_A - 0.982 x_H. \end{aligned}$$

- Two-step procedure:
  (i) choose A at the "−" level (continuous rotation) and H at the "+" level (nozzle position 6).
  (ii) By solving

$$\hat{y} = 14.352 + 0.402 x_D = 14.5,$$

choose x_D= 0.368.

Layer Growth Experiment: Analysis Results

Table 5: Means, Log Variances and SN Ratios, Layer Growth Experiment

Control Factor											
A	B	C	D	E	F	G	H	$\bar{y}_i$	$\ln s_i^2$	$\ln \bar{y}_i^2$	$\hat{\eta}_i$
−	−	−	+	−	−	−	−	14.79	-1.018	5.389	6.41
−	−	−	+	+	+	+	+	14.86	-3.879	5.397	9.28
−	−	+	−	−	−	+	+	14.00	-4.205	5.278	9.48
−	−	+	−	+	+	−	−	13.91	-1.623	5.265	6.89
−	+	−	−	−	+	−	+	14.15	-5.306	5.299	10.60
−	+	−	−	+	−	+	−	13.80	-1.236	5.250	6.49
−	+	+	+	−	+	+	−	14.73	-0.760	5.380	6.14
−	+	+	+	+	−	−	+	14.89	-1.503	5.401	6.90
+	−	−	−	−	+	+	−	13.93	-0.383	5.268	5.65
+	−	−	−	+	−	−	+	14.09	-2.180	5.291	7.47
+	−	+	+	−	+	−	+	14.79	-1.238	5.388	6.63
+	−	+	+	+	−	+	−	14.33	-0.868	5.324	6.19
+	+	−	+	−	−	+	+	14.77	-1.483	5.386	6.87
+	+	−	+	+	+	−	−	14.88	-0.418	5.400	5.82
+	+	+	−	−	−	−	−	13.76	-0.418	5.243	5.66
+	+	+	−	+	+	+	+	13.97	-2.636	5.274	7.91

Layer Growth Experiment: Plots

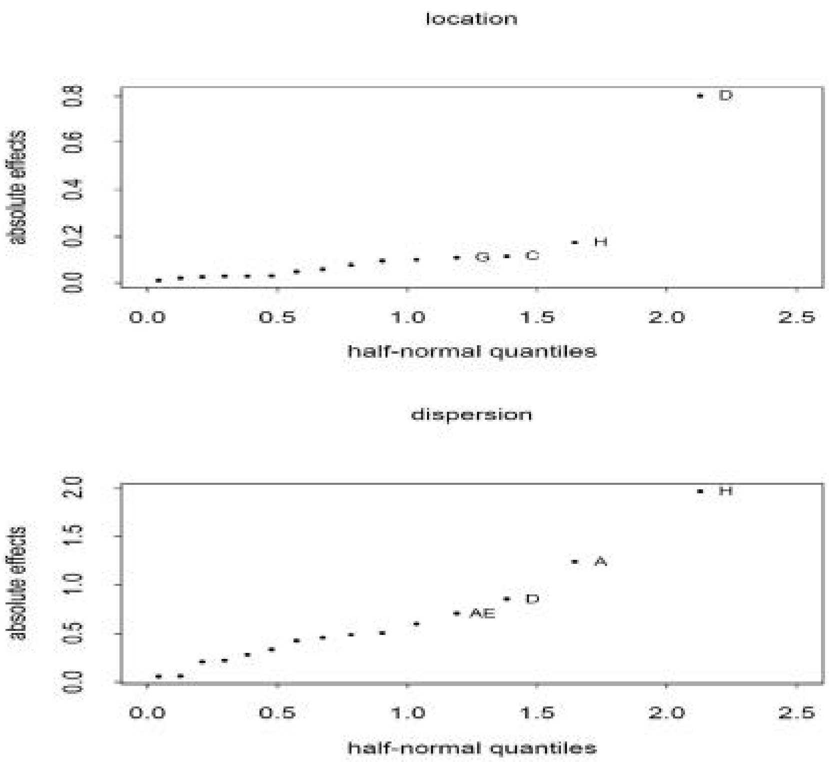

Figure 2: Half-Normal Plots of Location and Dispersion Effects, Layer Growth Experiment

Analysis of Leaf Spring Experiment

- Based on the half-normal plots in Figure 3, B, C and E are significant for location, C is significant for dispersion:

$$\hat{y} = 7.6360 + 0.1106x_B + 0.0881x_C + 0.0519x_E,$$
$$\hat{z} = -3.6886 + 1.0901x_C.$$

- Two-step procedure:
  (i) choose C at $-$.
  (ii) With $x_C = -1$, $\hat{y} = 7.5479 + 0.1106x_B + 0.0519x_E$.

  To achieve $\hat{y} = 8.0$, x_B and x_E must be chosen beyond $+1$, i.e., $x_B = x_E = 2.78$. This is too drastic, and not validated by current data. An alternative is to select $x_B = x_E = x_C = +1$ (not to follow the two-step procedure), then $\hat{y}$=7.89 is closer to 8. (Note that $\hat{y} = 7.71$ with $B_+C_-E_+$.) Reason for the breakdown of the 2-step procedure: its second step cannot achieve the target 8.0.

Leaf Spring Experiment: Analysis Results

Table 6: Means and Log Variances, Leaf Spring Experiment

Control Factor					
B	C	D	E	$\bar{y}_i$	$\ln s_i^2$
−	+	+	−	7.540	-2.4075
+	+	+	+	7.902	-2.6488
−	−	+	+	7.520	-6.9486
+	−	+	−	7.640	-4.8384
−	+	−	+	7.670	-2.3987
+	+	−	−	7.785	-2.9392
−	−	−	−	7.372	-3.2697
+	−	−	+	7.660	-4.0582

Leaf Spring Experiment: Plots

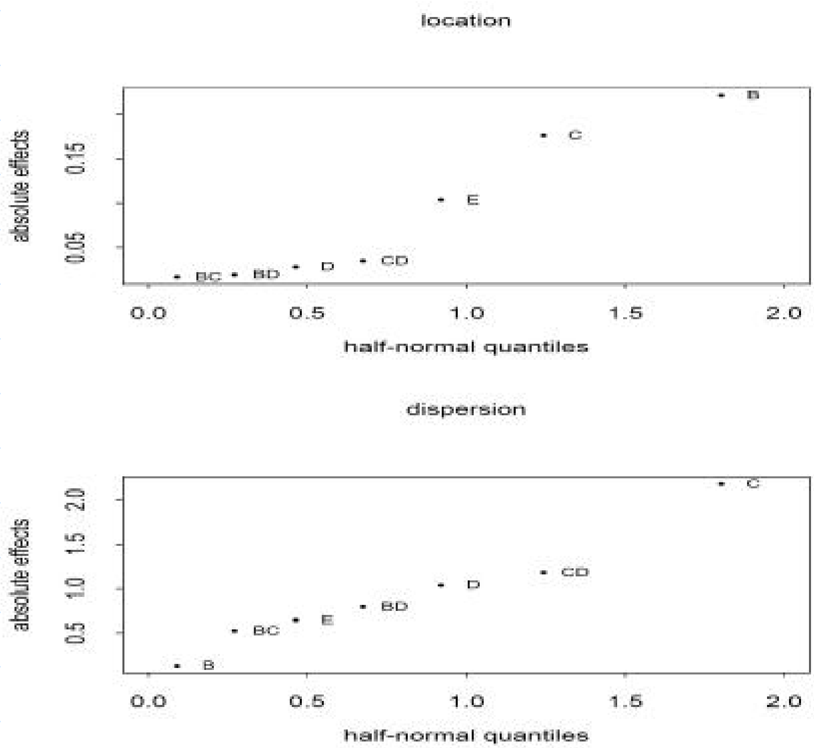

Figure 3: Half-Normal Plots of Location and Dispersion Effects, Leaf Spring Experiment

Response Modeling and Control-by-Noise Interaction Plots

- Response Model: model y_{ij} directly in terms of control, noise effects and control-by-noise interactions.
  - half normal plot of various effects.
  - regression model fitting, obtaining $\hat{y}$.
- Make control-by-noise interaction plots for significant effects in $\hat{y}$, choose **robust** control settings at which y has a flatter relationship with noise.
- Compute $Var(\hat{y})$ with respect to variation in the noise factors. Call $Var(\hat{y})$ the **transmitted variance model**. Use it to identify control factor settings with small transmitted variance.

Half-normal Plot, Layer Growth Experiment

- Define

$$M_l = (M_1 + M_2) - (M_3 + M_4),$$
$$M_q = (M_1 + M_4) - (M_2 + M_3),$$
$$M_c = (M_1 + M_3) - (M_2 + M_4),$$

- From Figure 4, select D, L, HL as the most significant effects.
- How to deal with the next cluster of effects in Figure 4? Use **step-down multiple comparisons**.
- After removing the top three points in Figure 4, make a half-normal plot (Figure 5) on the remaining points. The cluster of next four effects (M_l, H, CM_l, AHM_q) appear to be significant.

Half-normal Plot of Factorial Effects

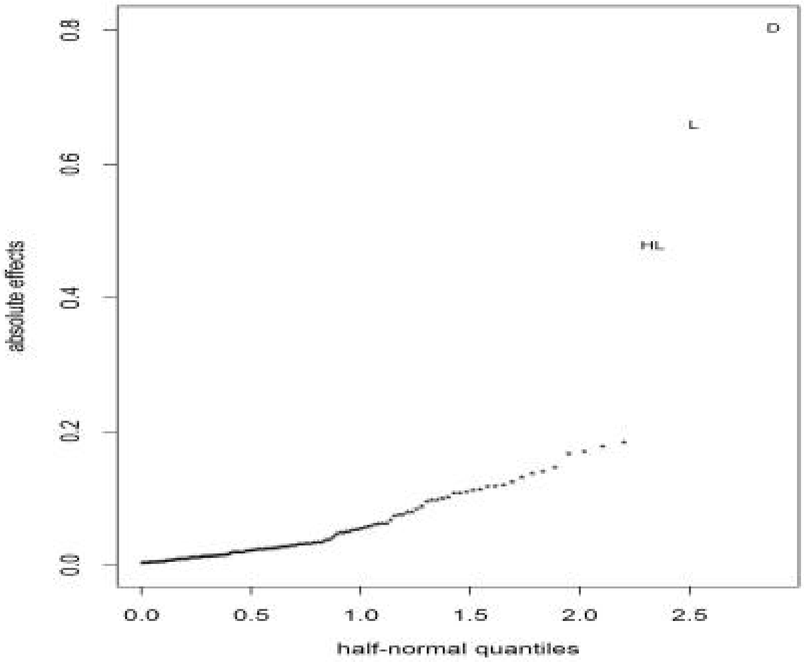

Figure 4: Half-Normal Plot of Response Model Effects, Layer Growth Experiment

Second Half-normal Plot of Factorial Effects

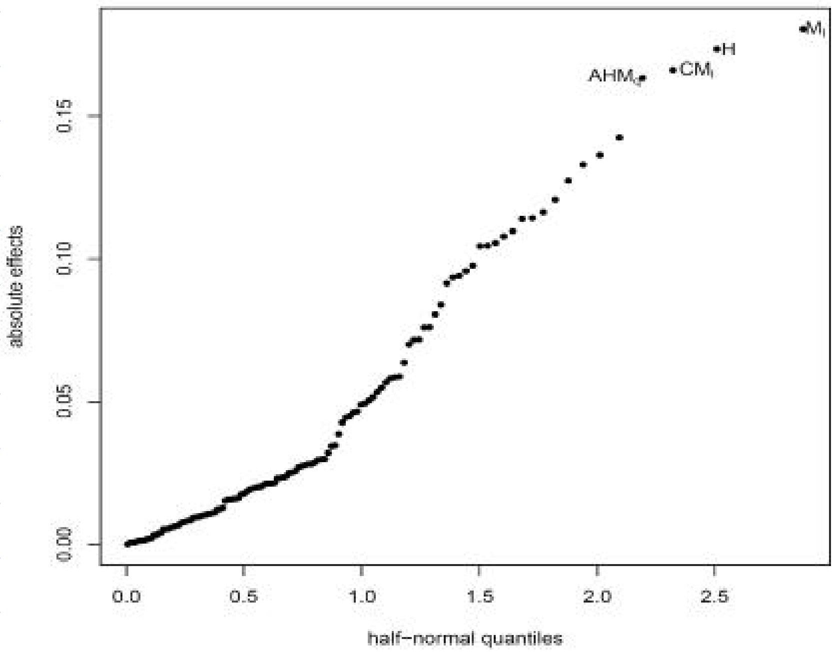

Figure 5: Second Half-Normal Plot of Response Model Effects, Layer Growth Experiment

Control-by-noise Interaction Plots

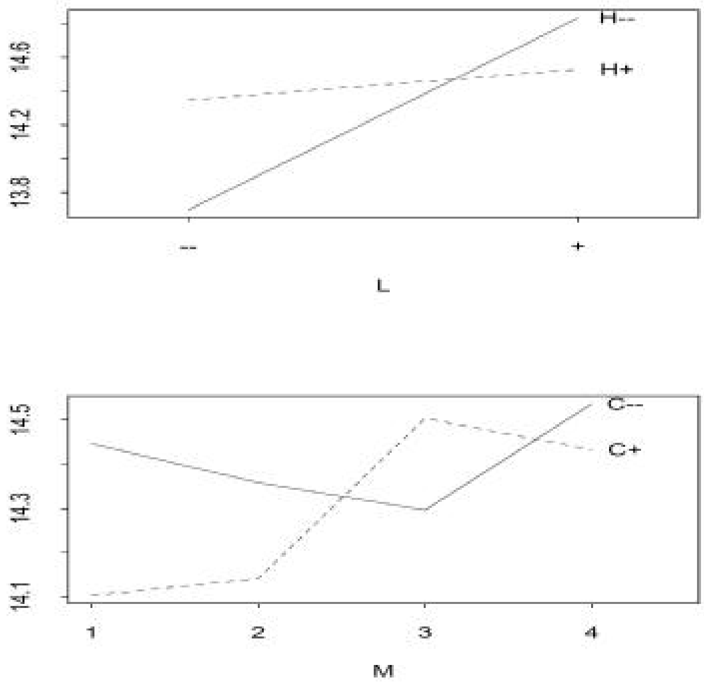

Figure 6: $H \times L$ and $C \times M$ Interaction Plots, Layer Growth Experiment

$A \times H \times M$ Plot

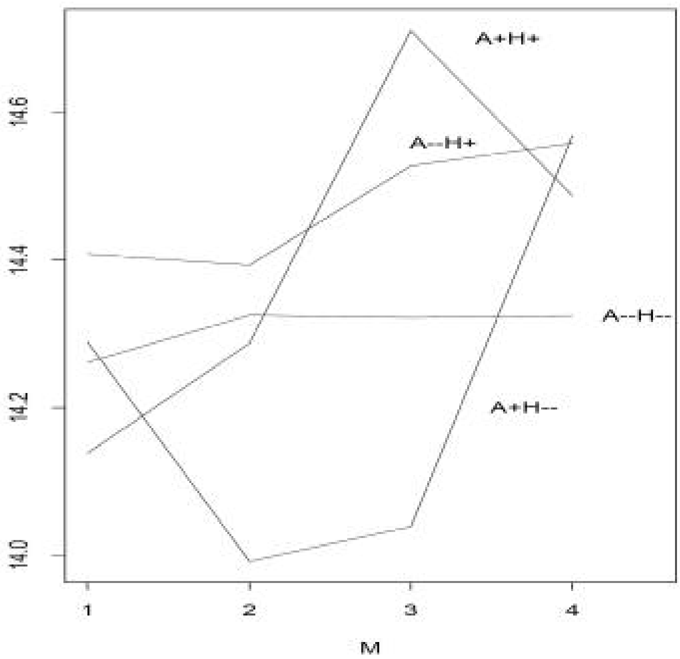

Figure 7: $A \times H \times M$ Interaction Plot, Layer Growth Experiment

Response Modeling, Layer Growth Experiment

- The following model is obtained:

$$\begin{aligned}\hat{y} &= 14.352 + 0.402x_D + 0.087x_H + 0.330x_L - 0.090x_{M_l} \\ &\quad -0.239x_H x_L - 0.083x_C x_{M_l} - 0.082x_A x_H x_{M_q}. \qquad (4)\end{aligned}$$

- Recommendations:
  H: $-$ (position 2) to $+$ (position 6)
  A: $+$ (oscillating) to $-$ (continuous)
  C: $+$ (1210) to $-$ (1220)
  resulting in 37% reduction of thickness standard variation.

Predicted Variance Model

- Assume L, M_l and M_q are random variables, taking -1 and $+1$ with equal probabilities. This leads to

$$\begin{aligned}&x_L^2 = x_{M_l}^2 = x_{M_q}^2 = x_A^2 = x_C^2 = x_H^2 = 1, \\ &E(x_L) = E(x_{M_l}) = E(x_{M_q}) = 0, \qquad (5) \\ &Cov(x_L, x_{M_l}) = Cov(x_L, x_{M_q}) = Cov(x_{M_l}, x_{M_q}) = 0.\end{aligned}$$

- From (4) and (5), we have

$$\begin{aligned}Var(\hat{y}) &= (.330 - .239x_H)^2 Var(x_L) + (-.090 - .083x_C)^2 Var(x_{M_l}) \\ &\quad +(.082x_A x_H)^2 Var(x_{M_q}) \\ &= \text{constant} + (.330 - .239x_H)^2 + (-.090 - .083x_C)^2 \\ &= \text{constant} - 2(.330)(.239)x_H + 2(.090)(.083)x_C \\ &= \text{constant} - .158x_H + .015x_C.\end{aligned}$$

- Choose $H+$ and $C-$. But factor A is not present here. (Why? See explanation on p. 532).

v **Reading**: textbook, 11.5

Estimation Capacity for Cross Arrays

- Example 1. Control array is a 2_{III}^{3-1} design with $\mathbf{I} = \mathbf{ABC}$ and the noise array is a 2_{III}^{3-1} design with $\mathbf{I} = \mathbf{abc}$. The resulting cross array is a 16-run 2_{III}^{6-2} design with $\mathbf{I} = \mathbf{ABC} = \mathbf{abc} = \mathbf{ABCabc}$. Easy to show that all 9 control-by-noise interactions are clear, (but not the 6 main effects). This is indeed a general result stated next.

  Theorem: Suppose a 2^{k-p} design d_C is chosen for the control array, a 2^{m-q} design d_N is chosen for the noise array, and a cross array, denoted by $d_C \otimes d_N$, is constructed from d_C and d_N.

  (i) If $\alpha_1, \ldots, \alpha_A$ are the estimable factorial effects (among the control factors) in d_C and $\beta_1, \ldots, \beta_B$ are the estimable factorial effects (among the noise factors) in d_N, then $\alpha_i, \beta_j, \alpha_i\beta_j$ for $i = 1, \ldots, A,\ j = 1, \ldots, B$ are estimable in $d_C \otimes d_N$.

  (ii) All the *km* control-by-noise interactions (i.e., two-factor interactions between a control factor main effect and a noise factor main effect) are clear in $d_C \otimes d_N$.

Cross Arrays or Single Arrays?

- Three control factors A, B, C two noise factors a, b: $2^3 \times 2^2$ design, allowing all main effects and two-factor interactions to be clearly estimated.

- Use a single array with 16 runs for all five factors: a resolution V 2^{5-1} design with $\mathbf{I} = ABCab$ or $\mathbf{I} = -ABCab$, all main effects and two-factor interactions are clear. (See Table 7)

- Single arrays can have smaller runs, but cross arrays are easier to use and interpret.

32-run Cross Array and 16-run Single Arrays

Table 7: 32-Run Cross Array

			a	+	+	−	−
			b	+	−	+	−
Runs	A	B	C				
1–4	+	+	+	●	○	○	●
5–8	+	+	−	○	●	●	○
9–12	+	−	+	○	●	●	○
13–16	+	−	−	●	○	○	●
17–20	−	+	+	○	●	●	○
21–24	−	+	−	●	○	○	●
25–28	−	−	+	●	○	○	●
29–32	−	−	−	○	●	●	○

$\bullet : \mathbf{I} = ABCab$, $\circ : \mathbf{I} = -ABCab$,

v **Reading**: textbook, 11.6, 11.7

Comparison of Cross Arrays and Single Arrays

- Example 1 (continued) An alternative is to choose a single array 2_{IV}^{6-2} design with $\mathbf{I} = \mathbf{ABCa} = \mathbf{ABbc} = \mathbf{abcC}$. This is not advisable because no 2fi's are clear and only main effects are clear. (Why? We need to have some clear control-by-noise interactions for robust optimization.) A better one is to use a 2_{III}^{6-2} design with $\mathbf{I} = \mathbf{ABCa} = \mathbf{abc} = \mathbf{ABCbc}$. It has 9 clear effects: $A, B, C, Ab, Ac, Bb, Bc, Cb, Cc$ (3 control main effects and 6 control-by-noise interactions).

v **Reading**: textbook, 11.8

Signal-to-Noise Ratio

- Taguchi's SN ratio $\hat{\eta} = \ln \frac{\bar{y}^2}{s^2}$
- Two-step procedure:
  1. Select control factor levels to maximize SN ratio,
  2. Use an adjustment factor to move mean on target.
- **Limitations**
  - maximizing $\bar{y}^2$ not always desired.
  - little justification outside linear circuitry.
  - statistically justifiable only when $Var(y)$ is proportional to $E(y)^2$
- **Recommendation**: Use SN ratio sparingly. Better to use the location-dispersion modeling or the response modeling. The latter strategies can do whatever SN ratio analysis can achieve.

Half-normal Plot for S/N Ratio Analysis

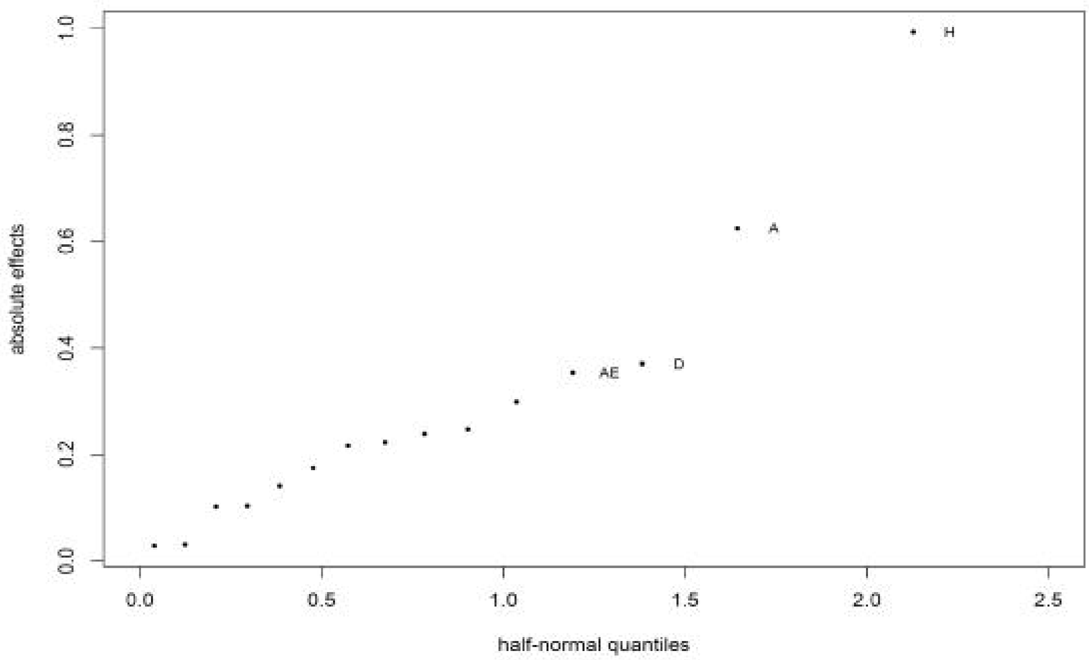

Figure 8: Half-Normal Plots of Effects Based on SN Ratio, Layer Growth Experiment

S/N Ratio Analysis for Layer Growth Experiment

- Based on the $\hat{\eta}_i$ column in Table 5, compute the factorial effects using SN ratio. From Figure 7, the conclusion is similar to location-dispersion analysis. Why? Using

$$\hat{\eta}_i = \ln \bar{y}_i^2 - \ln s_i^2,$$

and from Table 5, the variation among $\ln s_i^2$ is much larger than the variation among $\ln \bar{y}_i^2$; thus maximizing SN ratio is equivalent to minimizing $\ln s_i^2$ in this case.

v **Reading**: textbook, 11.9