(1, 1pt) The answer is one (single-replicated) because the residual sum of squares is zero, i.e., no degrees of freedom left to estimate error variance.

(2, 2pts) Because the RSS is zero, we cannot perform the F-tests in ANOVA. An alternative choice is to use half-normal plot (or Lenth's method) for the *standardized* linear and quadratic effect estimates of these orthogonal components (**note**. not half-normal plot for the effects based on linear-quadratic system).

(3, 1pt) C_l, because it has the largest sum of squares (SS is proportional to $\hat{\beta}^2_{effect}$) among all 1-d.f. effects (i.e., all linear and quadratic effects of orthogonal components).

(4, 2pts) Because of orthogonality, the projected vector of y on the effect C_l is $\hat{\beta}_{C_l}C_l$, and its length square (i.e., $\hat{\beta}^2_{C_l}||C_l||^2$) is the SS of C_l in the ANOVA table. So, the answer is $|\hat{\beta}_{C_l}| = \sqrt{\frac{5516.0}{((-1)^2+0^2+1^2)\times 9}} = 17.506$.

(5, 1pt) Notice that $A\times B$ contains two orthogonal components AB and AB^2, and $C\times D$ contains two orthogonal components CD and CD^2. But, the orthogonal components AB and CD are aliased in the design. Therefore, the dimension of the space spanned by AB, AB^2, CD, and CD^2 is only 6, not 8. Because the three 2-dimensional spaces spanned by $AB(=CD)$, AB^2, and CD^2 respectively are mutually orthogonal, the length square of the projected vector of y on the 6-dimensional space (spanned by AB, AB^2, CD, and CD^2) is the sum of the length squares of the projected vectors of y on each of the three 2-dimensional spaces. So, the ANOVA table is:

source	d.f.	sum of squares	mean square	F
$A\times B$ and $C\times D$	6	339=95.2+215.6+28.2	56.5	NA
residuals	0	0	0	

(6, 3pts) The projected design onto factors A, B, and C is a 3-level full factorial designs. The ANOVA table is:

source	d.f.	sum of squares	mean square	F
A	2	4496.3=SS_A	2248.14	NA
B	2	2768.7=SS_B	1384.35	NA
C	2	5519.8=SS_C	2759.89	NA
$A\times B$	4	310.7=$SS_{AB}+SS_{AB^2}$	77.675	NA
$A\times C$	4	1232.9=$SS_{AC}+SS_{AC^2}$	308.225	NA
$B\times C$	4	669.7=$SS_{BC}+SS_{BC^2}$	167.425	NA
$A\times B\times C$	8	546.3=$SS_D+SS_{AD^2}+SS_{BD^2}+SS_{CD^2}$	68.2875	NA
residuals	0	0	0	

(7, 1pt) The answer is zero because the Model 2 is an identifiable model and it contains 26 $(= 2\times 4 + 3\times \binom{4}{2})$ effects. There are no degrees of freedom left to estimate error variance.

(8, 1pt) No, because the effects in the Model 2 are not mutually orthogonal, which is a property required in the analysis of half-normal plot or Lenth's method.

(9, 1pt) Model (variable) selection is a possible choice.

(10, 2pts) The $\hat{\beta}_{C_l}$ in the Model 2 has the same value as that in the Model 1. It is because C_l is still orthogonal to any other effects in the Model 2. Notice that although the ll, lq, and ql effects are different from the linear and quadratic effects of orthogonal components, the former effects lies in the the space spanned by the latter effects.

(11, 1pt) The control array is a 2_V^{5-1} design with the defining contrast subgroup $I = ABCDE$. The whole cross array is a 2_V^{8-1} design with the defining contrast subgroup $I = ABCDE$.

(12, 2pts) All the control main effects and 2-factor interactions, all the noise main effects and 2-factor interactions, and all the control-by-noise 2-factor interactions are clear. The number of these clear effects is $\left(5 + \binom{5}{2}\right) + \left(3 + \binom{3}{2}\right) + (5 \times 3) = 36$. Out of the 127 $(= 2^{8-1} - 1$) degrees of freedom, 91 $(= 127 - 36)$ are allocated to 3-factor and higher interactions. The design is inefficient if we are only interested in main effects and 2-factor interactions of the 8 factors.

(13, 2pts) The minimum aberration 2^{8-2} has resolution V, which guarantees that all the 8 main effects and $\binom{8}{2} = 28$ two-factor interactions are clear. It has 64 runs, only half of the cross array. But, of course, it is a single array, not a cross array.

(14, 1pt) All the control-by-noise interaction plots, i.e., the interaction plots between one control factor (A, B, C, D, or E) and one noise factor (F, G, or H).

(15, 1pt) Notice that H is a noise factor and none of the control factors have significant interactions with H. When the major sources of variation in y are the effects of noise factors, the ability of control factors to reduce variation is limited.

(16, 1pt) We treat x_H and x_F as uncorrelated random variables each with equal probability on 1 and -1, i.e.,

$$E(x_H) = 0 \text{ and } E(x_F) = 0.$$

So,

$$\begin{aligned} E(\hat{y}|A, B, C, D, E) &= 14.3 - 7.2E(x_H) - 2.6x_B - 1.5x_A - 1.0x_C \\ &\quad +1.3x_C E(x_F) + 0.9x_B E(x_F) - 0.9x_C x_D E(x_F) \\ &= 14.3 - 2.6x_B - 1.5x_A - 1.0x_C. \end{aligned} \quad \text{(i)}$$

(17, 1pt) We should start from the $C \times F$ interaction plot because $x_C x_F$ is the most significant control-by-noise interaction. The $C \times F$ interaction plot suggests to choose $C = -1$. The next one is $C \times D \times F$ interaction plot, which suggests $D = 1$. The last one, $B \times F$ interaction plot, suggests to choose $B = -1$. After substituting $(B, C, D) = (-1, -1, 1)$ into the location model (i), we get

$$E(\hat{y}) = 17.9 - 1.5x_A.$$

We can set $x_A = 0.6$ to make the predicted mean of y reach the target value 17.

(18, 3pts) For dispersion model, Because $Var(x_H) = 1$, $Var(x_F) = 1$, and $Cov(x_H, x_F) = 0$, we have

$$\begin{aligned} Var(\hat{y}|A,B,C,D,E) &= Var[(14.3 - 2.6x_B - 1.5x_A - 1.0x_C) - 7.2x_H \\ &\quad +(1.3x_C + 0.9x_B - 0.9x_C x_D)x_F] \\ &= (7.2)^2 Var(x_H) + (1.3x_C + 0.9x_B - 0.9x_C x_D)^2 Var(x_F) \\ &\quad -2(7.2)(1.3x_C + 0.9x_B - 0.9x_C x_D)Cov(x_F, x_H) \\ &= (7.2)^2 + (1.3x_C + 0.9x_B - 0.9x_C x_D)^2 \\ &= (7.2)^2 + (1.3)^2 x_C^2 + (0.9)^2 x_B^2 + (0.9)^2 x_C^2 x_D^2 \\ &\quad +2(1.3)(0.9)x_B x_C - 2(1.3)(0.9)x_C^2 x_D - 2(0.9)^2 x_B x_C x_D \qquad \text{(ii)} \end{aligned}$$

When x_B, x_C, and x_D are only allowed to be -1 or $+1$, the model (ii) can be simplified to:

$$\begin{aligned} Var(\hat{y}|A,B,C,D,E) &= (7.2)^2 + (1.3)^2 + (0.9)^2 + (0.9)^2 \\ &\quad +2(1.3)(0.9)x_B x_C - 2(1.3)(0.9)x_D - 2(0.9)^2 x_B x_C x_D \\ &= 55.15 + 2.34x_B x_C - 2.34x_D - 1.62x_B x_C x_D \qquad \text{(iii)} \end{aligned}$$

(19, 2pts) To minimize the dispersion model (iii), we can set $(B = +1, C = -1, D = +1)$ or $(B = -1, C = +1, D = +1)$. After substituting the two settings into the location model, we get

$$E(\hat{y}) = 12.7 - 1.5x_A, \qquad \text{(iv)}$$

and

$$E(\hat{y}) = 15.9 - 1.5x_A, \qquad \text{(v)}$$

respectively. To make the model (iv) reach the target value 17, we can set $x_A = -2.87$. Do the same calculation for model (v). We have $x_A = -0.733$. The latter setting for x_A is better because it is located in the experimental region while the former is outside the region. The recommendation is therefore $A = -0.733, B = -1, C = +1$, and $D = +1$.

(20, 1pt) We can find that the $x_B x_C$ term in the dispersion model (iii) suggests the settings of x_B and x_C should have opposite sign. The $B \times C \times F$ interaction plot supports the conclusion because the lines corresponding to $(B, C) = (+1, -1)$ and $(B, C) = (-1, +1)$ are flatter. Notice that although the appearance of the $x_B x_C$ term in model (iii) is due to the significant $B \times F$ and $C \times F$ interactions, the opposite-sign information does not appear in the $B \times F$ and $C \times F$ interaction plots. This explains why the recommended level combinations for problems (17) and (18) are different.