Comparison of Cross Arrays and Single Arrays

special case

cross array: 2^{6-2}_{III}

D1 → $I = ABC = abc = ABCabc$

	eligible	clear
D1	A, B, C, a, b, c	Aa, Ab, Ac, Ba, Bb, Bc, Ca, Cb, Cc
D2	All 15 2fi's (aliased)	A, B, C, a, b, c
D3	a, b, c, Aa, Ba, Ca, AB, AC, BC	A, B, C, Ab, Bb, Cb, Ac, Bc, Cc

(aliased)

- Example 1 (continued, LNp.27)
  - An alternative is to choose a single array 2^{6-2}_{IV} design with D2 → $\mathbf{I} = \mathbf{ABCa} = \mathbf{ABbc} = \mathbf{abcC}$. This is not advisable because no 2fi's are clear and only main effects are clear. (Why? We need to have some clear control-by-noise interactions for robust optimization.)

    minimum aberration design

  - A better one is to use a 2^{6-2}_{III} design with $\mathbf{I} = \mathbf{ABCa} = \mathbf{abc} = \mathbf{ABCbc}$. ← D3 It has 9 clear effects: $A, B, C, Ab, Ac, Bb, Bc, Cb, Cc$ (3 control main effects and 6 control-by-noise interactions).

✱ inadequency of the conventional resolution and minimum aberration creteria for RPD ← They do not distinguish the 2 types of factors.

✱ possible modification of effect hierarchy.

$C == N == C \times N \gg C \times C == C \times C \times N \gg N \times N \cdots$

Recall similar situation in blocked FFD, where we have block factors and treatment factors.

❖ **Reading**: textbook, 11.8

Signal-to-Noise Ratio

$\overset{cf.}{\longleftrightarrow}$ coefficient of variation $\sigma_{\mathbf{x}}/\mu_{\mathbf{x}}$

↑ information theory, comunication system ↑ dimensionless

$y_{\mathbf{x}1}, \ldots, y_{\mathbf{x}r}$ iid $N(\mu_{\mathbf{x}}, \sigma^2_{\mathbf{x}})$

parameter version: $\eta_{\mathbf{x}} = \ln(\mu^2_{\mathbf{x}}/\sigma^2_{\mathbf{x}})$

- Taguchi's SN ratio $\hat{\eta}_{\mathbf{x}} = \ln \dfrac{\bar{y}^2_{\mathbf{x}}}{s^2_{\mathbf{x}}} = \ln(\bar{y}^2_{\mathbf{x}}) - \ln(s^2_{\mathbf{x}})$

  2 extreme cases:
  - If $\bar{y}^2_{\mathbf{x}}$ dominates $\hat{\eta}_{\mathbf{x}}$ ①
  - If $s^2_{\mathbf{x}}$ dominates $\hat{\eta}_{\mathbf{x}}$ ②

  t^2 for testing $H_0: \mu_{\mathbf{x}} = 0$

- Two-step procedure:

  For ①, maximize $\bar{y}^2_{\mathbf{x}}$

  For ②, minimize $s^2_{\mathbf{x}}$

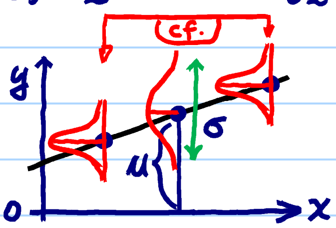
  $\hat{\eta}_{\mathbf{x}} = X_1 \hat{\beta}_1$

  1. Select control factor levels to maximize SN ratio

  $\hat{\mu}_{\mathbf{x}} = X_2 \hat{\beta}_2$ cf. minimize $\ln(s^2_{\mathbf{x}})$

  2. Use an adjustment factor to move mean on target.

  2 steps in LNp.10-12



- **Limitations**

  For ①, why get a setting with large $|\bar{y}_{\mathbf{x}}|$ in step 1, then adjust $\bar{y}_{\mathbf{x}}$ to target in step 2?

  ⊖ maximizing $\bar{y}^2_{\mathbf{x}}$ not always desired.

  - little justification outside linear circuitry.
  - statistically justifiable only when $Var_N(y_{\mathbf{x}})$ is proportional to $[E_N(y_{\mathbf{x}})]^2$

    or $y_{\mathbf{x}} \sim$ log-normal (LNp.10-33)

model: $y_{\mathbf{x}} = \mu(x_1, x_2) \times \varepsilon(x_1)$, where ε r.v. $E[\varepsilon(x_1)] = 1$, $Var[\varepsilon(x_1)] = \delta^2_{x_1}$

Then, $E(y_{\mathbf{x}}) = \mu(x_1, x_2)$

$Var(y_{\mathbf{x}}) = [\mu(x_1, x_2)]^2 \delta^2_{x_1}$

$\Rightarrow \eta_{\mathbf{x}} = \ln(\mu^2/\mu^2\delta) = -\ln(\delta^2_{x_1})$

- **Recommendation**: Use SN ratio sparingly. Better to use the location-dispersion modeling or the response modeling. The latter strategies can do whatever SN ratio analysis can achieve.

especially when assume $y_{\mathbf{x}} \sim normal(\mu_{\mathbf{x}}, \sigma^2_{\mathbf{x}})$

S/N Ratio Analysis for Layer Growth Experiment

- Based on the $\hat{\eta}_i$ column in Table 5 (LNp.10-14), compute the factorial effects using SN ratio. A half-normal plot of the effects for $\hat{\eta}_i$ is given in Figure 8 (LNp.10-34). From Figure 8, the conclusion is similar to location-dispersion analysis. Why? Using

From the fitted model of $\hat{\eta}_i$'s → H+A − (cf. LNp.10-13)

$$\hat{\eta}_i = \ln \bar{y}_i^2 - \ln s_i^2,$$

case 2 in LNp.10-32

and from Table 5, the variation among $\ln s_i^2$ is much larger than the variation among $\ln \bar{y}_i^2$; thus maximizing SN ratio is equivalent to minimizing $\ln s_i^2$ in this case.

If $y_x \sim$ log-normal(μ_x, σ_x^2), then $Z_x = \ln(y_x) \sim N(\mu_x, \sigma_x^2)$ → $E(Z_x) = \mu_x$, $Var(Z_x) = \sigma_x^2$

$$E(y_x^n) = \exp(n\mu_x + n^2\sigma_x^2/2)$$

$$Var(y_x) = E(y_x^2) - [E(y_x)]^2 = \exp(2\mu_x + 2\sigma_x^2) - [\underbrace{\exp(\mu_x + \sigma_x^2/2)}_{E(y_x)}]^2$$

$$= \exp(2\mu_x + \sigma_x^2)[\exp(\sigma_x^2) - 1] = [E(y_x)]^2[\exp(\sigma_x^2) - 1]$$

$$\Rightarrow \eta_x = \frac{[E(y_x)]^2}{Var(y_x)} = [\exp(\sigma_x^2 - 1)]^{-1} \uparrow \text{ iff } \downarrow \sigma_x^2 = \{Var[\ln(y_x)]\}$$

a Box-Cox transformation

maximizing η_x is equivalent to minimizing $\ln[Var(\ln y_x)] = \ln[Var(Z_x)]$

Half-normal Plot for S/N Ratio Analysis

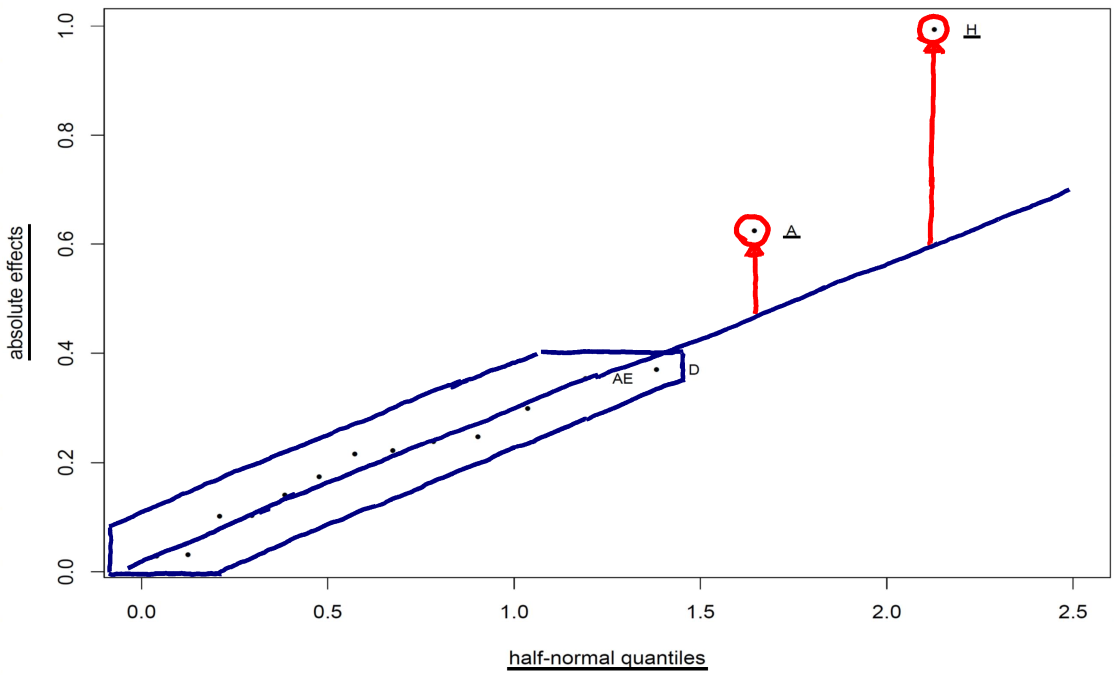


Figure 8: Half-Normal Plots of Effects Based on SN Ratio, Layer Growth Experiment

❖ **Reading**: textbook, 11.9