Robust Parameter Design

- Statistical/engineering method for product/process improvement (Taguchi).
- Two types of factors in a system (product/process):
  - control factors: once chosen, values remain fixed.
  - noise factors: hard-to-control during normal process or usage.
- **Robust Parameter design** (**RPD** or **PD**): choose control factor settings to make response less sensitive (i.e., more robust) to noise variation; exploiting control-by-noise interactions.

A Robust Design Perspective of Layer-growth and Leaf Spring Experiments

- The original AT&T layer growth experiment had
  - 8 control factors,
  - 2 noise factors (location and facet).

  Goal was to achieve *uniform* thickness around 14.5 μm over the noise factors. See Tables 1 and 2 (LNp.10-3∼4).

- The original leaf spring experiment had
  - 4 control factors,
  - 1 noise factor (quench oil temperature). The quench oil temperature is not controllable; with efforts it can be set in two ranges of values 130-150, 150-170.

  Goal is to achieve *uniform* free height around 8 inches over the range of quench oil temperature. See Tables 3 and 4 (LNp.10-5).

- Must understand the role of *noise factors* in achieveing *robustness*.

Layer Growth Experiment: Factors and Levels

Table 1: Factors and Levels, Layer Growth Experiment

Control Factor		Level –	Level +
A.	susceptor-rotation method	continuous	oscillating
B.	code of wafers	668G4	678D4
C.	deposition temperature(°C)	1210	1220
D.	deposition time	short	long
E.	arsenic flow rate(%)	55	59
F.	hydrochloric acid etch temperature(°C)	1180	1215
G.	hydrochloric acid flow rate(%)	10	14
H.	nozzle position	2	6
Noise Factor		**Level –**	**Level +**
L.	location	bottom	top
M.	facet	1 2	3 4

Layer Growth Experiment: Thickness Data

Table 2: Cross Array and Thickness Data, Layer Growth Experiment

Control Factor *A B C D E F G H*	L-Bottom *M*-1	L-Bottom *M*-2	L-Bottom *M*-3	L-Bottom *M*-4	L-Top *M*-1	L-Top *M*-2	L-Top *M*-3	L-Top *M*-4
− − − + − − − −	14.2908	14.1924	14.2714	14.1876	15.3182	15.4279	15.2657	15.4056
− − − + + + + +	14.8030	14.7193	14.6960	14.7635	14.9306	14.8954	14.9210	15.1349
− − + − − − + +	13.8793	13.9213	13.8532	14.0849	14.0121	13.9386	14.2118	14.0789
− − + − + + − −	13.4054	13.4788	13.5878	13.5167	14.2444	14.2573	14.3951	14.3724
− + − − − + − +	14.1736	14.0306	14.1398	14.0796	14.1492	14.1654	14.1487	14.2765
− + − − + − + −	13.2539	13.3338	13.1920	13.4430	14.2204	14.3028	14.2689	14.4104
− + + + − + + −	14.0623	14.0888	14.1766	14.0528	15.2969	15.5209	15.4200	15.2077
− + + + + − − +	14.3068	14.4055	14.6780	14.5811	15.0100	15.0618	15.5724	15.4668
+ − − − − + + −	13.7259	13.2934	12.6502	13.2666	14.9039	14.7952	14.1886	14.6254
+ − − − + − − +	13.8953	14.5597	14.4492	13.7064	13.7546	14.3229	14.2224	13.8209
+ − + + − + − +	14.2201	14.3974	15.2757	15.0363	14.1936	14.4295	15.5537	15.2200
+ − + + + − + −	13.5228	13.5828	14.2822	13.8449	14.5640	14.4670	15.2293	15.1099
+ + − + − − + +	14.5335	14.2492	14.6701	15.2799	14.7437	14.1827	14.9695	15.5484
+ + − + + + − −	14.5676	14.0310	13.7099	14.6375	15.8717	15.2239	14.9700	16.0001
+ + + − − − − −	12.9012	12.7071	13.1484	13.8940	14.2537	13.8368	14.1332	15.1681
+ + + − + + + +	13.9532	14.0830	14.1119	13.5963	13.8136	14.0745	14.4313	13.6862

Leaf Spring Experiment

Table 3: Factors and Levels, Leaf Spring Experiment

Control Factor	Level −	Level +
B. high heat temperature (°F)	1840	1880
C. heating time (seconds)	23	25
D. transfer time (seconds)	10	12
E. hold down time (seconds)	2	3
Noise Factor	**Level −**	**Level +**
Q. quench oil temperature (°F)	130-150	150-170

Table 4: Cross Array and Height Data, Leaf Spring Experiment

Control Factor B C D E	Noise Factor Q^-			Noise Factor Q^+		
− + + −	7.78	7.78	7.81	7.50	7.25	7.12
+ + + +	8.15	8.18	7.88	7.88	7.88	7.44
− − + +	7.50	7.56	7.50	7.50	7.56	7.50
+ − + −	7.59	7.56	7.75	7.63	7.75	7.56
− + − +	7.94	8.00	7.88	7.32	7.44	7.44
+ + − −	7.69	8.09	8.06	7.56	7.69	7.62
− − − −	7.56	7.62	7.44	7.18	7.18	7.25
+ − − +	7.56	7.81	7.69	7.81	7.50	7.59

❖ **Reading**: textbook, 11.1

Strategies for Variation Reduction

1. ***Sampling inspection***: passive, sometimes last resort.
2. ***Control charting and process monitoring***: can remove special causes. If the process is stable, it can be *followed* by using a *designed experiment*.
3. ***Blocking, covariate adjustment***: passive measures but useful in reducing variability, not for removing root causes.
4. ***Reducing variation in noise factors***: effective as it may reduce variation in the response but can be expensive. Better approach is to change control factor settings (*cheaper* and *easier* to do) by exploiting control-by-noise interactions, i.e., use robust parameter design!

❖ **Reading**: textbook, 11.2

Types of Noise Factors

1. Variation in process parameters.
2. Variation in product parameters.
3. Environmental variation.
4. Load Factors.
5. Upstream variation.
6. Downstream or user conditions.
7. Unit-to-unit and spatial variation.
8. Variation over time.
9. Degradation.

- **Traditional design uses 7 and 8.**

❖ **Reading**: textbook, 11.3

Variation Reduction Through RPD

- Suppose $y = f(\mathbf{x}, \mathbf{z})$, $\mathbf{x}$ control factors and $\mathbf{z}$ noise factors. If $\mathbf{x}$ and $\mathbf{z}$ interact in their effects on y, then the $var_{\mathbf{z}}(y)$ can be reduced either by reducing $var(\mathbf{z})$ (i.e., method 4 in LNp.10-6) or by changing the $\mathbf{x}$ values (i.e., RPD).
- An example:

$$\begin{aligned} y &= \mu + \alpha x_1 + \beta z + \gamma x_2 z + \varepsilon, \\ &= \mu + \alpha x_1 + (\beta + \gamma x_2) z + \varepsilon. \end{aligned}$$

By choosing an appropriate value of x_2 to reduce the coefficient $\beta + \gamma x_2$, the impact of z on y can be reduced. Since β and γ are unknown, this can be achieved by using the control-by-noise interaction plots or other methods to be presented later.

Exploitation of Nonlinearity

- Nonlinearity between y and $\mathbf{x}$ can be exploited for robustness if $\mathbf{x}_0$, nominal values of $\mathbf{x}$, are control-factor settings and deviations of $\mathbf{x}$ around $\mathbf{x}_0$ (i.e., $\mathbf{x}-\mathbf{x}_0$) are viewed as noise factors (called *internal noise*). Expand $y = f(\mathbf{x})$ around $\mathbf{x}_0$,

$$y \approx f(\mathbf{x}_0) + \sum_i \left(\frac{\partial f}{\partial x_i} \bigg|_{x_{i0}} \right) (x_i - x_{i0}).$$

- This leads to

$$\sigma^2 \approx \sum_i \left(\frac{\partial f}{\partial x_i} \bigg|_{x_{i0}} \right)^2 \sigma_i^2, \qquad (1)$$

where $\sigma^2 = var(y)$, $\sigma_i^2 = var(x_i)$, each component x_i has mean x_{i0} and variance σ_i^2.

- From (1), it can be seen that σ^2 can be reduced by choosing x_{i0} with a smaller slope $\frac{\partial f}{\partial x_i}\big|_{x_{i0}}$. This is demonstrated in Figure 1. Moving the nominal value a to b can reduce $var(y)$ because the slope at b is more flat. This is a **parameter design** step.
- On the other hand, reducing the variation of x around a can also reduce $var(y)$. This is a **tolerance design** step.

Exploitation of Nonlinearity to Reduce Variation

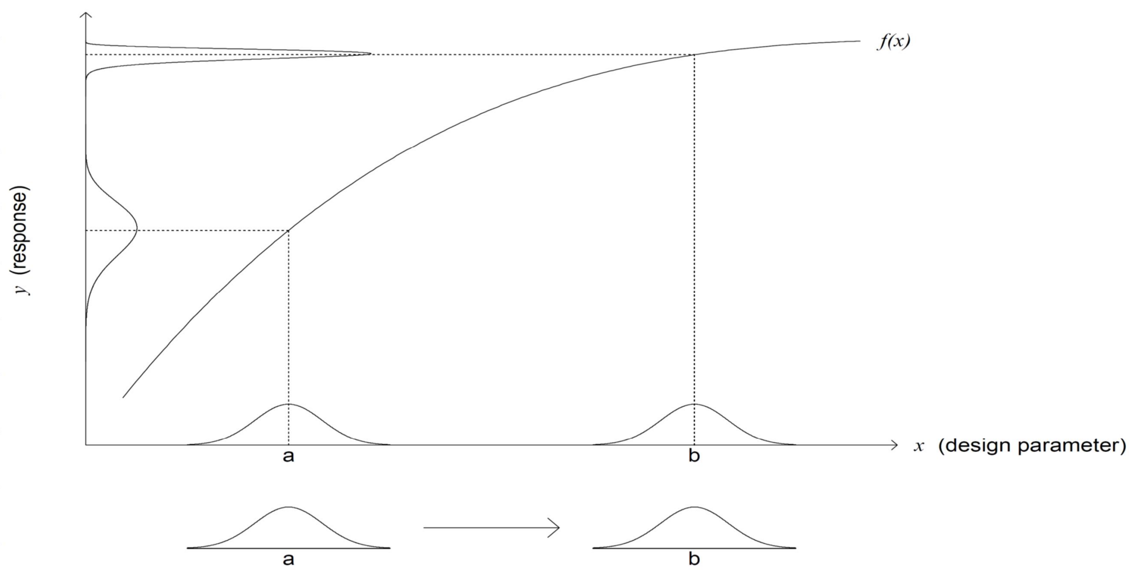


Figure 1: Exploiting the Nonlinearity of $f(x)$ to Reduce Variation

❖ **Reading**: textbook, 11.4

Cross Array and Location-Dispersion Modeling

- Cross array = control array $\times$ noise array,
  - control array (or inner array)
    = array (design matrix) for control factors,
  - noise array (or outer array)
    = array (design matrix) for noise factors.
- Location-dispersion modeling
  - compute $\bar{y}_i$, s_i^2 based on the noise settings for the i^{th} control setting,
  - analyze $\bar{y}_i$ (location), and $\ln s_i^2$ (dispersion), identify significant location and dispersion effects.

Two-step Procedures for RPD Optimization

- **Two-Step Procedure for Nominal-the-Best Problem**

  (i) select the levels of the dispersion factors to minimize dispersion,
  (ii) select the level of the adjustment factor (if exists) to bring the location on target. (2)

- **Two-Step Procedure for Larger-the-Better and Smaller-the-Better Problems**

  (i) select the levels of the location factors to maximize (or minimize) the location,
  (ii) select the levels of the dispersion factors that are not location factors to minimize dispersion. (3)

- Note that the two steps in (3) are in reverse order from those in (2).
  Reason: It is usually harder to increase or decrease the response y in the latter problem, so this step should be the first to perform.

Analysis of Layer Growth Experiment

- From the $\bar{y}_i$ and $\ln s_i^2$ columns of Table 5 (LNp.10-14), compute the factorial effect estimates for location and dispersion respectively. (These numbers are not given in the textbook.) From the half-normal plots of these effects (Figure 2, LNp.10-15), D is significant for location and H, A for dispersion.

$$\hat{y} = 14.352 + 0.402 x_D,$$
$$\ln \hat{s}^2 = -1.822 + 0.619 x_A - 0.982 x_H.$$

- Two-step procedure:
  - (i) Choose A at the "−" level (continuous rotation) and H at the "+" level (nozzle position = 6).
  - (ii) By solving
  $$\hat{y} = 14.352 + 0.402 x_D = 14.5,$$
  choose $-1 < x_D = 0.368 < 1$.

Layer Growth Experiment: Analysis Results

Table 5: Means, Log Variances and SN Ratios, Layer Growth Experiment

Control Factor											
A	B	C	D	E	F	G	H	$\bar{y}_i$	$\ln s_i^2$	$\ln \bar{y}_i^2$	$\hat{\eta}_i$
−	−	−	+	−	−	−	−	14.79	−1.018	5.389	6.41
−	−	−	+	+	+	+	+	14.86	−3.879	5.397	9.28
−	−	+	−	−	−	+	+	14.00	−4.205	5.278	9.48
−	−	+	−	+	+	−	−	13.91	−1.623	5.265	6.89
−	+	−	−	−	+	−	+	14.15	−5.306	5.299	10.60
−	+	−	−	+	−	+	−	13.80	−1.236	5.250	6.49
−	+	+	+	−	+	+	−	14.73	−0.760	5.380	6.14
−	+	+	+	+	−	−	+	14.89	−1.503	5.401	6.90
+	−	−	−	−	+	+	−	13.93	−0.383	5.268	5.65
+	−	−	−	+	−	−	+	14.09	−2.180	5.291	7.47
+	−	+	+	−	+	−	+	14.79	−1.238	5.388	6.63
+	−	+	+	+	−	+	−	14.33	−0.868	5.324	6.19
+	+	−	+	−	−	+	+	14.77	−1.483	5.386	6.87
+	+	−	+	+	+	−	−	14.88	−0.418	5.400	5.82
+	+	+	−	−	−	−	−	13.76	−0.418	5.243	5.66
+	+	+	−	+	+	+	+	13.97	−2.636	5.274	7.91

Layer Growth Experiment: Plots

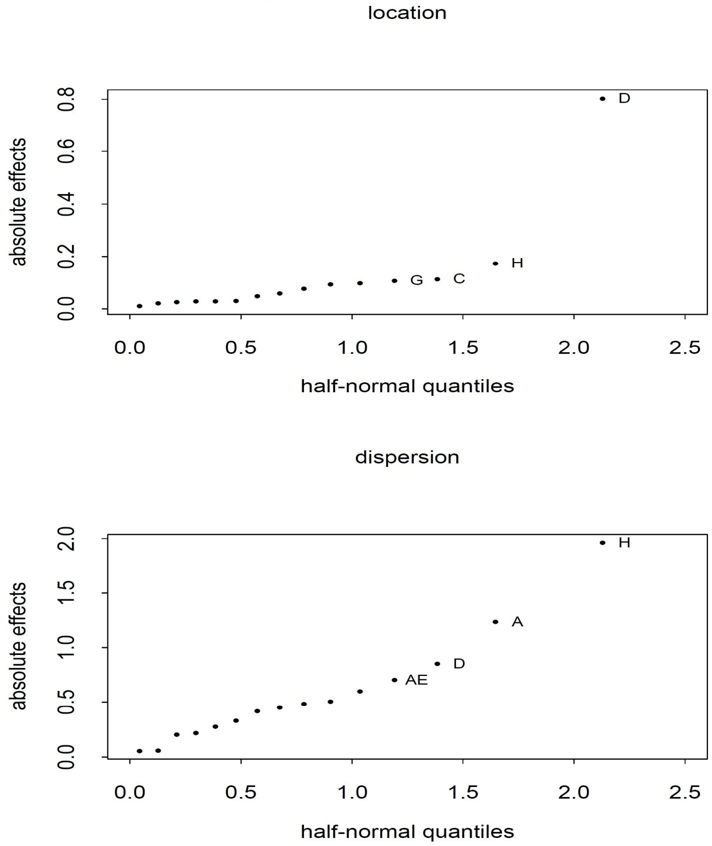


Figure 2: Half-Normal Plots of Location and Dispersion Effects, Layer Growth Experiment

Analysis of Leaf Spring Experiment

- From the $\bar{y}_i$ and $\ln s_i^2$ columns of Table 6 (LNp.10-17), compute the factorial effect estimates for location and dispersion respectively. Based on the half-normal plots in Figure 3 (LNp.10-18), B, C and E are significant for location, C is significant for dispersion:

$$\hat{y} = 7.6360 + 0.1106 x_B + 0.0881 x_C + 0.0519 x_E,$$
$$\ln \hat{s}^2 = -3.6886 + 1.0901 x_C.$$

- Two-step procedure:

  (i) Choose C at $-$.

  (ii) With $x_C = -1$, $\hat{y} = 7.5479 + 0.1106 x_B + 0.0519 x_E$.

  * To achieve $\hat{y} = 8.0$, x_B and x_E must be chosen beyond $+1$ (e.g., $x_B = x_E = 2.78$). This is too drastic, and not validated by current data.
  * An alternative is to select $x_B = x_E = x_C = +1$ (not to follow the two-step procedure), then $\hat{y}$=7.89 is closer to 8. (Note that $\hat{y} = 7.71$ with $B_+ C_- E_+$.)
  * Reason for the breakdown of the 2-step procedure: its second step cannot achieve the target 8.0.

Leaf Spring Experiment: Analysis Results

Table 6: Means and Log Variances, Leaf Spring Experiment

Control Factor					
B	C	D	E	$\bar{y}_i$	$\ln s_i^2$
$-$	$+$	$+$	$-$	7.540	-2.4075
$+$	$+$	$+$	$+$	7.902	-2.6488
$-$	$-$	$+$	$+$	7.520	-6.9486
$+$	$-$	$+$	$-$	7.640	-4.8384
$-$	$+$	$-$	$+$	7.670	-2.3987
$+$	$+$	$-$	$-$	7.785	-2.9392
$-$	$-$	$-$	$-$	7.372	-3.2697
$+$	$-$	$-$	$+$	7.660	-4.0582

Leaf Spring Experiment: Plots

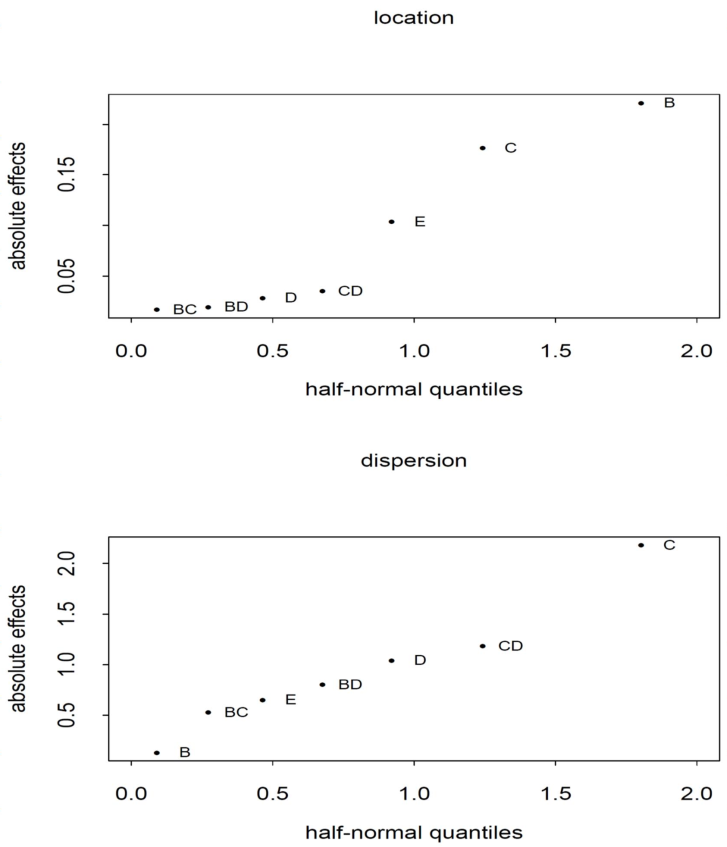


Figure 3: Half-Normal Plots of Location and Dispersion Effects, Leaf Spring Experiment

Response Modeling and Control-by-Noise Interaction Plots

- Response Model: model y_{ij} directly in terms of control and noise main effects and control-by-noise interactions.
  - half normal plot of various effects.
  - regression model fitting, obtaining $\hat{y}$.
- Make control-by-noise interaction plots for significant effects in $\hat{y}$, choose **robust** control settings at which y has a flatter relationship with noise factors.
- Compute $Var_N(\hat{y}_{\mathbf{x}})$ with respect to variation in the noise factors. Call $Var_N(\hat{y}_x)$ the **transmitted variance model**. Use it to identify control factor settings with small transmitted variance.

Half-normal Plot, Layer Growth Experiment

- Define

$$M_l = (M_1 + M_2) - (M_3 + M_4),$$

$$M_q = (M_1 + M_4) - (M_2 + M_3),$$

$$M_c = (M_1 + M_3) - (M_2 + M_4),$$

- From Figure 4 (LNp.10-21), select the effects D, L, HL as the most significant effects.
- How to deal with the next cluster of effects in Figure 4? Use **step-down multiple comparisons**.
- After removing the top three points in Figure 4, make a half-normal plot (Figure 5, LNp.10-22) on the remaining points. The cluster of next four effects (M_l, H, CM_l, AHM_q) appear to be significant.

Half-normal Plot of Factorial Effects

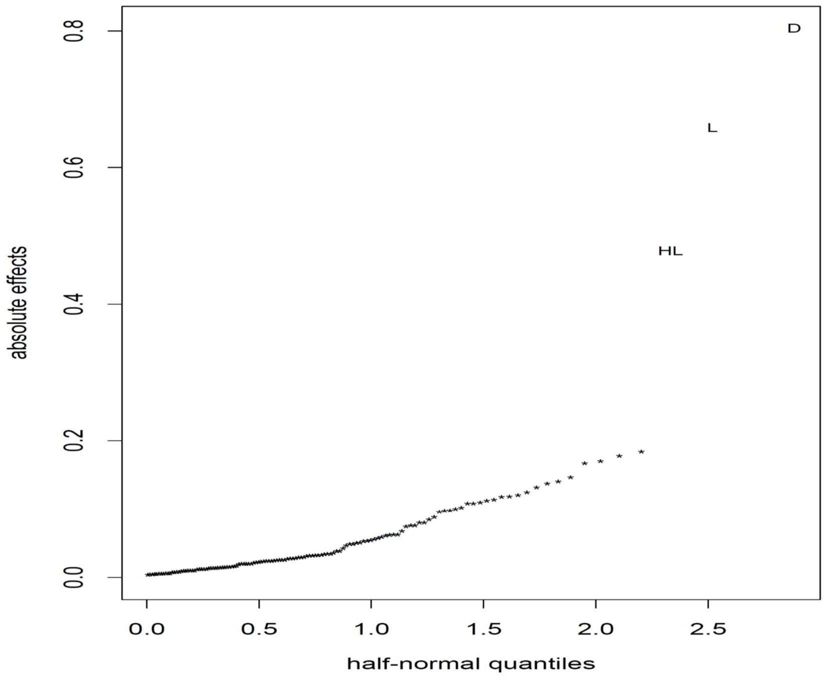


Figure 4: Half-Normal Plot of Response Model Effects, Layer Growth Experiment

Second Half-normal Plot of Factorial Effects

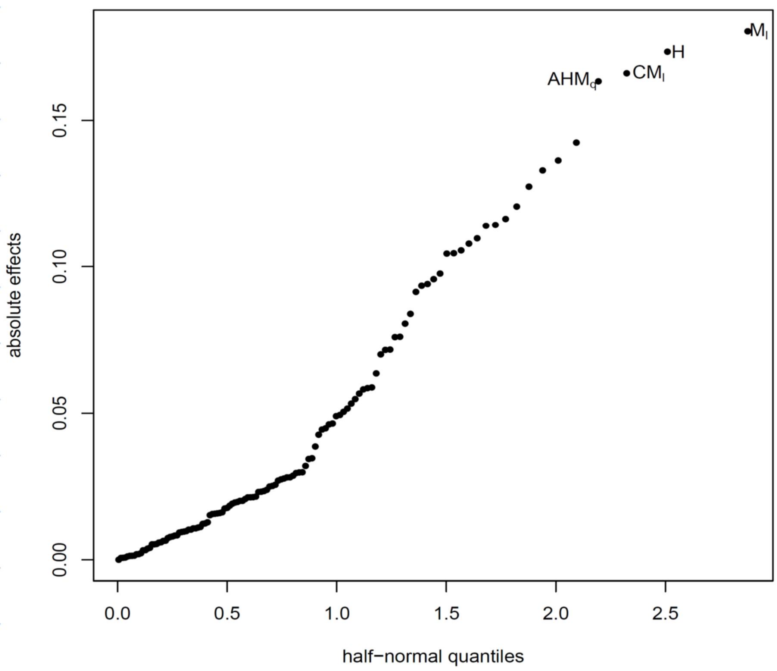
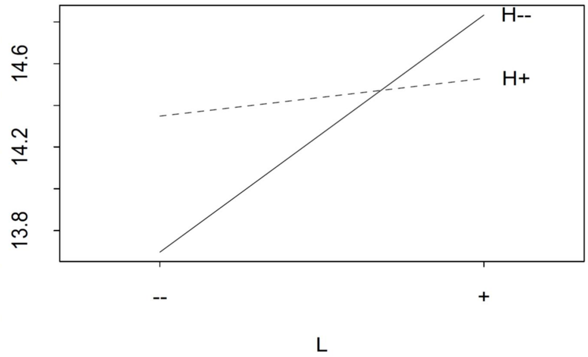


Figure 5: Second Half-Normal Plot of Response Model Effects, Layer Growth Experiment

Control-by-noise Interaction Plots



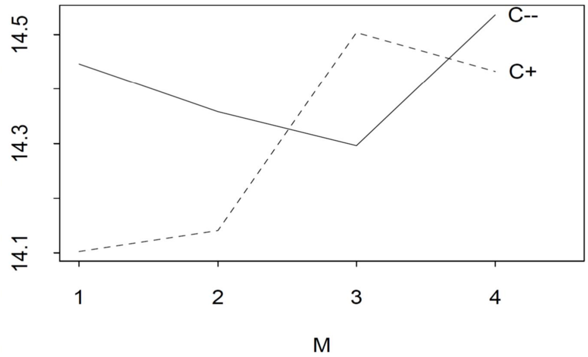


Figure 6: $H \times L$ and $C \times M$ Interaction Plots, Layer Growth Experiment

Three-factor Interaction Plot: $A \times H \times M$

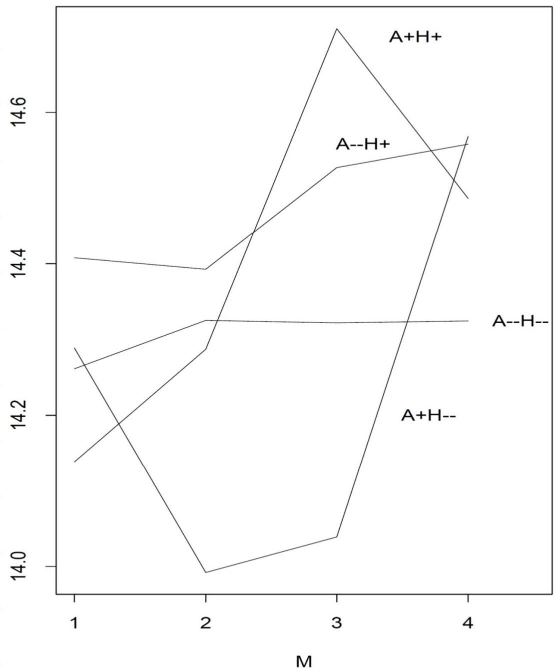


Figure 7: $A \times H \times M$ Interaction Plot, Layer Growth Experiment

Response Modeling, Layer Growth Experiment

- The following model is obtained:

$$\begin{aligned}\hat{y} &= 14.352 + 0.402x_D + 0.087x_H + 0.330x_L - 0.090x_{M_l} \\ &\quad -0.239x_H x_L - 0.083x_C x_{M_l} - 0.082x_A x_H x_{M_q}.\end{aligned} \tag{4}$$

- Recommendations:

H:	$-$ (position 2)	to	$+$ (position 6)
A:	$+$ (oscillating)	to	$-$ (continuous)
C:	$+$ (1210)	to	$-$ (1220)

resulting in 37% reduction of thickness standard variation.

Transmitted Variance Model

- Assume L, M_l and M_q are random variables, taking -1 and $+1$ with equal probabilities. This leads to

$$\begin{aligned}&x_L^2 = x_{M_l}^2 = x_{M_q}^2 = x_A^2 = x_C^2 = x_H^2 = 1, \\ &E(x_L) = E(x_{M_l}) = E(x_{M_q}) = 0, \\ &Cov(x_L, x_{M_l}) = Cov(x_L, x_{M_q}) = Cov(x_{M_l}, x_{M_q}) = 0, \\ &Var(x_L) = Var(x_{M_l}) = Var(x_{M_q}) = 1.\end{aligned} \tag{5}$$

- From (4) and (5), we have

$$\begin{aligned}Var_N(\hat{y}_{\mathbf{x}}) &= (.330 - .239x_H)^2 Var(x_L) + (-.090 - .083x_C)^2 Var(x_{M_l}) \\ &\quad + (.082x_A x_H)^2 Var(x_{M_q}) \\ &= \text{constant} + (.330 - .239x_H)^2 + (-.090 - .083x_C)^2 \\ &= \text{constant} - 2(.330)(.239)x_H + 2(.090)(.083)x_C \\ &= \text{constant} - .158x_H + .015x_C.\end{aligned}$$

- Choose $H+$ and $C-$. But factor A is not present here. (Why? See explanation on textbook, p.532).

❖ **Reading**: textbook, 11.5

Estimation Capacity for Cross Arrays

- Example.
  - Control array is a 4-run 2^{3-1}_{III} design with

$$\mathbf{I} = ABC.$$

  - Noise array is a 4-run 2^{3-1}_{III} design with

$$\mathbf{I} = abc.$$

  - The resulting cross array is a 16-run 2^{6-2}_{III} design with

$$\mathbf{I} = ABC = abc = ABCabc.$$

  - Easy to show that all 9 control-by-noise interactions are clear, (but not the 6 main effects).
- This is indeed a general result stated in next slide.

Estimation Capacity for Cross Arrays (Cont.)

- **Theorem**. Suppose
  - a 2^{k-p} design d_C is chosen for the control array,
  - a 2^{m-q} design d_N is chosen for the noise array, and
  - a cross array, denoted by $d_C \otimes d_N$, is constructed from d_C and d_N.

  (i) If
  - $\{\alpha_1, \ldots, \alpha_A\}$ are the estimable factorial effects (among the control factors) in d_C and
  - $\{\beta_1, \ldots, \beta_B\}$ are the estimable factorial effects (among the noise factors) in d_N,

  then $\{\alpha_i, \beta_j, \alpha_i\beta_j\}$ for $i = 1, \ldots, A,\ j = 1, \ldots, B$ are estimable in $d_C \otimes d_N$.

  (ii) All the km control-by-noise two-factor interactions (i.e., two-factor interactions between a control factor main effect and a noise factor main effect) are clear in $d_C \otimes d_N$.

Cross Arrays or Single Arrays?

- Three control factors A, B, C and two noise factors a, b:
  Cross array requires $2^3 \otimes 2^2$ full factorial design (32 runs)
  for allowing all main effects and two-factor interactions
  to be clearly estimated.

- Use a single array with 16 runs for all five factors:
  In the resolution V 2^{5-1} design with

  $$\mathbf{I} = ABCab \quad \text{or} \quad \mathbf{I} = -ABCab,$$

  all main effects and two-factor interactions are clear.
  (See Table 7, LNp.10-30)

- Single arrays can have smaller runs, but
  cross arrays are easier to use and interpret.

32-run Cross Array and 16-run Single Arrays

Table 7: 32-Run Cross Array

			a	+	+	−	−
			b	+	−	+	−
Runs	A	B	C				
1–4	+	+	+	•	○	○	•
5–8	+	+	−	○	•	•	○
9–12	+	−	+	○	•	•	○
13–16	+	−	−	•	○	○	•
17–20	−	+	+	○	•	•	○
21–24	−	+	−	•	○	○	•
25–28	−	−	+	•	○	○	•
29–32	−	−	−	○	•	•	○

• : $\mathbf{I} = ABCab$; ○ : $\mathbf{I} = -ABCab$

❖ **Reading**: textbook, 11.6, 11.7

Comparison of Cross Arrays and Single Arrays

- Example 1 (continued)
  - An alternative is to choose a single array 2_{IV}^{6-2} design with $\mathbf{I} = \mathbf{ABCa} = \mathbf{ABbc} = \mathbf{abcC}$. This is not advisable because no 2fi's are clear and only main effects are clear. (Why? We need to have some clear control-by-noise interactions for robust optimization.)
  - A better one is to use a 2_{III}^{6-2} design with $\mathbf{I} = \mathbf{ABCa} = \mathbf{abc} = \mathbf{ABCbc}$. It has 9 clear effects: $A, B, C, Ab, Ac, Bb, Bc, Cb, Cc$ (3 control main effects and 6 control-by-noise interactions).

❖ **Reading**: textbook, 11.8

Signal-to-Noise Ratio

- Taguchi's SN ratio $\hat{\eta}_{\mathbf{x}} = \ln \frac{\bar{y}_{\mathbf{x}}^2}{s_{\mathbf{x}}^2}$
- Two-step procedure:
  1. Select control factor levels to maximize SN ratio
  2. Use an adjustment factor to move mean on target.
- **Limitations**
  - maximizing $\bar{y}_{\mathbf{x}}^2$ not always desired.
  - little justification outside linear circuitry.
  - statistically justifiable only when $Var_N(y_{\mathbf{x}})$ is proportional to $[E_N(y_{\mathbf{x}})]^2$
- **Recommendation**: Use SN ratio sparingly. Better to use the location-dispersion modeling or the response modeling. The latter strategies can do whatever SN ratio analysis can achieve.

S/N Ratio Analysis for Layer Growth Experiment

- Based on the $\hat{\eta}_i$ column in Table 5 (LNp.10-14), compute the factorial effects using SN ratio. A half-normal plot of the effects for $\hat{\eta}_i$ is given in Figure 8 (LNp.10-34). From Figure 8, the conclusion is similar to location-dispersion analysis. Why? Using

$$\hat{\eta}_i = \ln \bar{y}_i^2 - \ln s_i^2,$$

and from Table 5, the variation among $\ln s_i^2$ is much larger than the variation among $\ln \bar{y}_i^2$; thus maximizing SN ratio is equivalent to minimizing $\ln s_i^2$ in this case.

Half-normal Plot for S/N Ratio Analysis

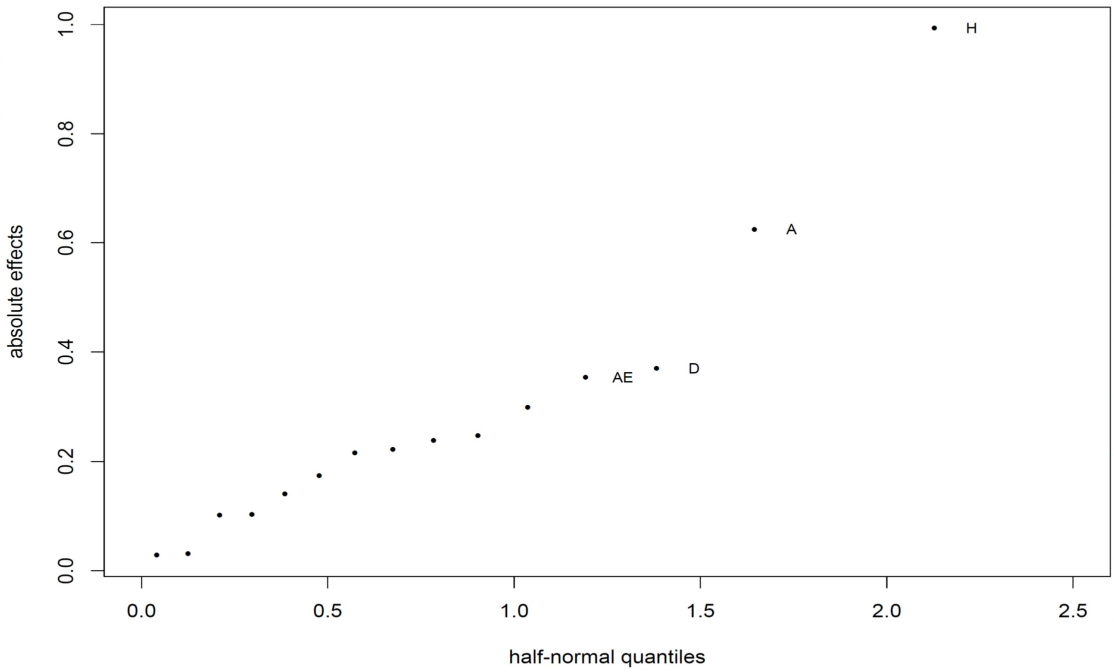


Figure 8: Half-Normal Plots of Effects Based on SN Ratio, Layer Growth Experiment

❖ **Reading**: textbook, 11.9