Use of Design Tables

- Minimum aberration (MA) designs are given in the tables in textbook, Appendix 5A.

LNp.6-12 → If two designs are given for same k and p, (k: # of (treatment) factors; p: fraction)

Same if only one design is given.

- the first is an MA design and
- the second is better in having a larger number of clear effects.

Two tables are given on next two slides.

Note. ① MA criterion favors no particular factor (all factors are equally important)
② design not MA but having more clear effects usually favors some particular factors over the other factors.

- In Table 7 (LNp.6-28),
  - the first 2^{9-4} design has MA and 8 clear 2fi's, and
  - the second 2^{9-4} design is (cf.)
    - ∗ the second best according to the MA criterion,
    - ∗ but has 15 clear 2fi's.

Using Rule (iii) in (2) on LNp.6-10, the second design is better because both have resolution IV (Details given on p. 234 of textbook).

- It is not uncommon to find a design with slightly worse aberration but more clear effects. Thus **the number of clear effects** should be used as a *supplementary criterion* to the MA criterion.

Table 6: 16-Run 2^{k-p} FFD $(k-p=4)$

run size 2^4; factors {1,2,3,4} form a 2^4 full factorial in the design matrix.

(k is the number of factors and F&R is the fraction and resolution.)

k	F&R	Design Generators (← check LNp.6-8)	Clear Effects
5	2^{5-1}_{V}	$5 = 1234$	all five main effects, all 10 2fi's
6	2^{6-2}_{IV}	$5 = 123$, $6 = 124$	all six main effects
6*	2^{6-2}_{III}	$5 = 12$, $6 = 134$	3, 4, 6, 23, 24, 26, 35, 45, 56 (9 clear effects)
7	2^{7-3}_{IV}	$5 = 123$, $6 = 124$, $7 = 134$	all seven main effects
8	2^{8-4}_{IV}	$5 = 123$, $6 = 124$, $7 = 134$, $8 = 234$	all eight main effects
9	2^{9-5}_{III}	$5 = 123$, $6 = 124$, $7 = 134$, $8 = 234$, $9 = 1234$	none
10	2^{10-6}_{III}	$5 = 123$, $6 = 124$, $7 = 134$, $8 = 234$, $9 = 1234$, $t_0 = 34$	none
11	2^{11-7}_{III}	$5 = 123$, $6 = 124$, $7 = 134$, $8 = 234$, $9 = 1234$, $t_0 = 34$, $t_1 = 24$	none
12	2^{12-8}_{III}	$5 = 123$, $6 = 124$, $7 = 134$, $8 = 234$, $9 = 1234$, $t_0 = 34$, $t_1 = 24$, $t_2 = 14$	none
13	2^{13-9}_{III}	$5 = 123$, $6 = 124$, $7 = 134$, $8 = 234$, $9 = 1234$, $t_0 = 34$, $t_1 = 24$, $t_2 = 14$, $t_3 = 23$	none
14	2^{14-10}_{III}	$5 = 123$, $6 = 124$, $7 = 134$, $8 = 234$, $9 = 1234$, $t_0 = 34$, $t_1 = 24$, $t_2 = 14$, $t_3 = 23$, $t_4 = 13$	none
15	2^{15-11}_{III}	$5 = 123$, $6 = 124$, $7 = 134$, $8 = 234$, $9 = 1234$, $t_0 = 34$, $t_1 = 24$, $t_2 = 14$, $t_3 = 23$, $t_4 = 13$, $t_5 = 12$	none

Why ?

Table 7: 32 Run 2^{k-p} FFD ($k-p=5$, $6 \le k \le 11$)

factors {1,2,3,4,5} form a 2^5 full factorial in the design matrix.

(k is the number of factors and F&R is the fraction and resolution.)

k	F&R	Design Generators	Clear Effects
6	2^{6-1}_{VI}	$6 = 12345$	all six main effects, all 15 2fi's
7	2^{7-2}_{IV}	$6 = 123$, $7 = 1245$	all seven main effects, 14, 15, 17, 24, 25, 27, 34, 35, 37, 45, 46, 47, 56, 57, 67
8	2^{8-3}_{IV}	$6 = 123$, $7 = 124$, $8 = 1345$	all eight main effects, 15, 18, 25, 28, 35, 38, 45, 48, 56, 57, 58, 68, 78
9	2^{9-4}_{IV}	$6 = 123$, $7 = 124$, $8 = 125$, $9 = 1345$	all nine main effects, 19, 29, 39, 49, 59, 69, 79, 89
9	2^{9-4}_{IV}	$6 = 123$, $7 = 124$, $8 = 134$, $9 = 2345$	all nine main effects, 15, 19, 25, 29, 35, 39, 45, 49, 56, 57, 58, 59, 69, 79, 89
10	2^{10-5}_{IV}	$6 = 123$, $7 = 124$, $8 = 125$, $9 = 1345$, $t_0 = 2345$ (10th factor)	all 10 main effects
10	2^{10-5}_{III}	$6 = 12$, $7 = 134$, $8 = 135$, $9 = 145$, $t_0 = 345$	3, 4, 5, 7, 8, 9, t_0, 23, 24, 25, 27, 28, 29, $2t_0$, 36, 46, 56, 67, 68, 69, $6t_0$
11	2^{11-6}_{IV}	$6 = 123$, $7 = 124$, $8 = 134$, $9 = 125$, $t_0 = 135$, $t_1 = 145$ (11th factor)	all 11 main effects
11	2^{11-6}_{III}	$6 = 12$, $7 = 13$, $8 = 234$, $9 = 235$, $t_0 = 245$, $t_1 = 1345$	4, 5, 8, 9, t_0, t_1, 14, 15, 18, 19, $1t_0$, $1t_1$

Choice of Fractions and Avoidance of Specific Level Combinations

2^p parallel flats

- A 2^{k-p} design has 2^p choices. ← isomorphic designs; p independent defining relations: $I = W_1 = W_2 = \cdots = W_p$ (check LNp.6)
- In general, use randomization to choose one of them.

  For example, the 2^{7-3} design has 8 choices

  $4 = \pm 12$, $5 = \pm 13$, $6 = \pm 23$. (2 × 2 × 2)

  Randomly choose the signs.

  different "+" & "−" signs for $W_1, \ldots, W_p$ can generate different choices of the 2^{k-p} design (check LNp.6-6). But, they are isomorphic designs.

- If specific level combinations, e.g.,

  $(+, +, +)$ for high pressure, high temperature, high concentration, (might cause explosion)

  are deemed undesirable or even disastrous, they can be avoided by choosing a fraction that does not contain them. Example on p.237 of textbook.

❖ **Reading**: textbook, 5.5

Blocking in FF Designs

Recall. Arrange full factorial in blocks (LNp. 5-36 ~ 40)

16-run FFD → six treatment factors

- Example: Arrange the 2^{6-2} design in four ($=2^2$) blocks with

DCSG for treatment factors → $I = 1235 = 1246 = 3456$. ($5 = 123$, $6 = 124$)

one block factor with 4 levels, block size = 4

factors {1,2,3,4} form a full factorial

block1 block2 block3 block4 (4 4 4 4) → 2^{6-2}

- Suppose we choose

2 pseudo block factors → $b_1 = 134$, $b_2 = 234$, $b_1 b_2 = 12$. (a block ME)

$I = 134b_1$, $I = 234b_2$, $I = 12b_1b_2$

DCSG for block factors → $I = 134b_1 = 234b_2 = 12b_1b_2$

- Then

$$\begin{aligned} b_1 &= 134 = 245 = 236 = 156, \\ b_2 &= 234 = 145 = 136 = 256, \\ b_1b_2 &= 12 = 35 = 46 = 123456; \end{aligned}$$

confounding between treatment and block effects (caused by mixed-type words)

aliasing between treatment effects (caused by pure-type words)

$$\begin{aligned} 13 &= 25 = 2346 = 1456, \\ 14 &= 26 = 2345 = 1356, \\ 15 &= 23 = 2456 = 1346, \\ 16 &= 24 = 2356 = 1345, \\ 34 &= 56 = 1245 = 1236, \\ 36 &= 45 = 1256 = 1234. \end{aligned}$$

$1 = 235 = 246 = 13456$
$2 = 135 = 146 = 23456$
$3 = 125 = 12346 = 456$
$4 = 12345 = 126 = 356$
$5 = 123 = 12456 = 346$
$6 = 12356 = 124 = 345$

The $4 \times 3 = 12$ factorial effects are confounded with block effects and cannot be used for estimation. Among the remaining 12 degrees of freedom, six are main effects and the rest are given above.

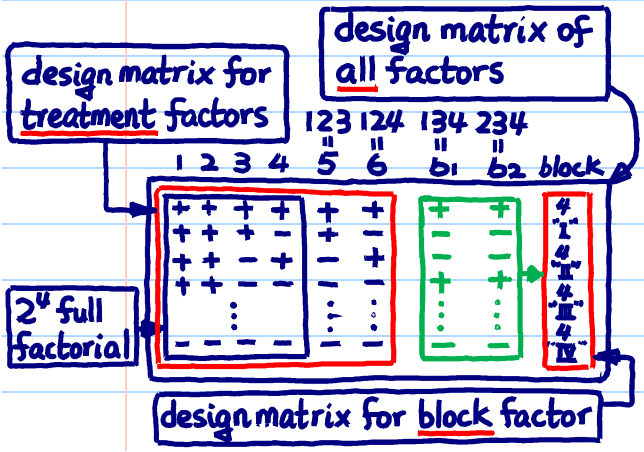
Use of Design Tables for Blocking

- Among the 15 degrees of freedom for the blocked design on LNp.6-30, 3 are allocated for block effects and 6 are for clear main effects (see Table 8 in LNp.6-32). The remaining 6 degrees of freedom are six pairs of aliased two-factor interactions.
- For the 2^{6-2} design with $5 = 12$, $6 = 134$, if we use the block generators $b_1 = 13$, $b_2 = 14$, there are a total of 9 clear effects (see Table 8 in LNp.6-32):

worse aberration than the 2^{6-2} design in LNp.6-30

$3, 4, 6, 23, 24, 26, 35, 45, 56$. ← (exercise)

  - Thus, the total number of clear effects for this blocked design is 3 more than the total number of clear effects for the blocked design on LNp.6-30.
  - However, only the main effects 3, 4, 6 are clear.



pure-type words

2^{8-4}: $I = 1235 = 1246 = 3456$

mixed-type words

$= 134b_1 = 245b_1 = 236b_1 = 156b_1$
$= 234b_2 = 145b_2 = 136b_2 = 256b_2$
$= 12b_1b_2 = 35b_1b_2 = 46b_1b_2 = 123456b_1b_2$

- alias sets:

$1 = 235 = 246 = 13456$
$= 346_1 = 12456_1 = \cdots = 23456b_1b_2$

$134 = 245 = 236 = 156 = b_1$
$= 12356_1 = 1246_1 = \cdots = 256b_1b_2$

Table 8: Sixteen-Run 2^{k-p} Fractional Factorial Designs in 2^q Blocks

of treatment factors; fraction; run size; # of pseudo block factors; # of blocks or # of levels of the block factor; k-p=4

k	p	q	Design Generators	Block Generators	Clear Effects
5	1	1	$5 = 1234$	$b_1 = 12$	all five main effects, all 2fi's except 12
5	1	2	$5 = 1234$	$b_1 = 12$, $b_2 = 13$	all five main effects, 14, 15, 24, 25, 34, 35, 45
5	1	3	$5 = 123$	$b_1 = 14$, $b_2 = 24$, $b_3 = 34$	all five main effects
6	2	1	$5 = 123, 6 = 124$	$b_1 = 134$	all six main effects
6	2	1	$5 = 12, 6 = 134$	$b_1 = 13$	3, 4, 6, 23, 24, 26, 35, 45, 56
6	2	2	$5 = 123, 6 = 124$	$b_1 = 134$, $b_2 = 234$	all six main effects
6	2	2	$5 = 12, 6 = 134$	$b_1 = 13$, $b_2 = 14$	3, 4, 6, 23, 24, 26, 35, 45, 56
6	2	3	$5 = 123, 6 = 124$	$b_1 = 13$, $b_2 = 23$, $b_3 = 14$	all six main effects

2 blocks, block size=8; 4 blocks, block size=4; 8 blocks, block size=2

cf; MA; larger # of clear treatment effects

Q: How to generate MA criterion to blocked FFDs?

W_t: WLP of pure-type words

W_b: WLP of mixed-type words

combine into a WLP in a reasonable way, and then sequentially minimize

Table 8: Sixteen-Run 2^{k-p} Fractional Factorial Designs in 2^q Blocks (Cont.)

k-p=4

k	p	q	Design Generators	Block Generators	Clear Effects
7	3	1	$5 = 123, 6 = 124, 7 = 134$	$b_1 = 234$	all seven main effects
7	3	2	$5 = 123, 6 = 124, 7 = 134$	$b_1 = 12$, $b_2 = 13$	all seven main effects
7	3	3	$5 = 123, 6 = 124, 7 = 134$	$b_1 = 12$, $b_2 = 13$, $b_3 = 14$	all seven main effects
8	4	1	$5 = 123, 6 = 124, 7 = 134, 8 = 234$	$b_1 = 12$	all eight main effects
8	4	2	$5 = 123, 6 = 124, 7 = 134, 8 = 234$	$b_1 = 12$, $b_2 = 13$	all eight main effects
8	4	3	$5 = 123, 6 = 124, 7 = 134, 8 = 234$	$b_1 = 12$, $b_2 = 13$, $b_3 = 14$	all eight main effects
9	5	1	$5 = 12, 6 = 13, 7 = 14, 8 = 234, 9 = 1234$	$b_1 = 23$	none
9	5	2	$5 = 12, 6 = 13, 7 = 14, 8 = 234, 9 = 1234$	$b_1 = 23$, $b_2 = 24$	none

Ok for screening designs.

- More FF designs in blocks are given in Appendix 5B of textbook.

❖ **Reading**: textbook, 5.6