Illustration with Adapted Epi-Layer Growth Experiment

1. In Table 4 (LNp.5-11),
   - median$|\hat{\theta}_i| = 0.078$, ← median of 15 $|\hat{\theta}_i|$'s
   - $s_0 = 1.5 \times 0.078 = 0.117$.
   - trimming constant $2.5 s_0 = 2.5 \times 0.117 = 0.292$, which eliminates 0.490 (D) and 0.345 (CD). → 2 out of 15 effects
   - $\text{median}_{\{|\hat{\theta}_i| < 2.5 s_0\}} |\hat{\theta}_i| = 0.058$ ← median of 13 smaller $|\hat{\theta}_i|$'s
   - $PSE = 1.5 \times 0.058 = 0.087 \xleftrightarrow{\text{cf.}}$ s.e.$(\hat{\theta}_i)$

   closer to zero

   The corresponding $|t_{PSE}|$ values appear in Table 6 (LNp.5-28).
2. Choose $\alpha = 0.01$.
   - $\text{IER}_{0.01} = 3.63$ for $I = 15$. By comparing with the $|t_{PSE}|$ values, D and CD are significant at 0.01 level.
   - $\text{EER}_{0.01} = 6.45$ (for $I = 15$). No effect is detected as significant.

   larger

- Analysis of the $|t_{PSE}|$ values for $\ln s^2$ (Table 6, LNp.5-28) detects no significant effect (details on textbook, p.182), thus confirming the half-normal plot analysis in Figure 4.10 of section 4.8 (textbook, p.179).

$|t_{PSE}|$ Values for Adapted Epi-Layer Growth Experiment

Table 6: $|t_{PSE}|$ Values, Adapted Epitaxial Layer Growth Experiment

Effect	$\bar{y}$	$\ln s^2$
A	0.90	0.25
B	1.99	1.87
C	0.90	1.78
D	✓ $\boxed{5.63}$	0.89
AB	0.09	0.71
AC	1.07	0.41
AD	0.57	0.46
BC	0.67	1.27
BD	0.34	0.16
CD	✓ $\boxed{3.97}$	1.35
ABC	1.13	0.51
ABD	0.29	0.67
ACD	0.34	0.00
BCD	1.26	0.05
$ABCD$	0.23	1.63

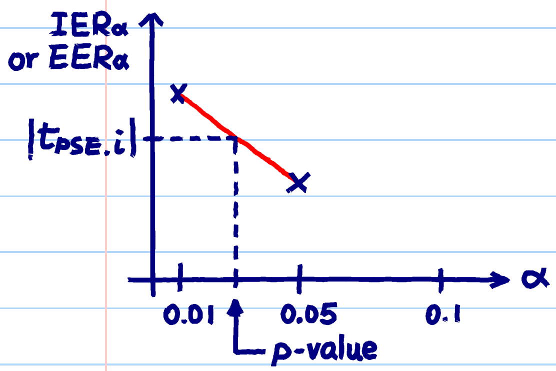
unreplicated

$y_x \sim N(\mu_x, \sigma_x^2)$

$\boxed{E(y_x)}$

$\Rightarrow \begin{cases} \hat{\mu}_x \approx \hat{\beta}_0 + \hat{\beta}_D x_D + \hat{\beta}_{CD} x_{CD} \\ \hat{\sigma}_x^2 \approx \text{a constant} \end{cases}$

$\boxed{Var(y_x)}$



- p-values of effects can be obtained from packages or by interpolating the critical values in the tables in appendix H (textbook, p.701). (See textbook, p.182 for illustration).

❖ **Reading**: textbook, 4.9

Nominal-the-Best Problem ← Recall objective in LNp.5-1

- There is a nominal or target value t (=14.5 in the case) based on engineering design requirements.
- Define a quantitative loss due to deviation of $y_{\mathbf{x}}$ from t.

Quadratic loss: $L(y_{\mathbf{x}}, t) = \cancel{c} \cdot (y_{\mathbf{x}} - t)^2$. $\boxed{-E(y_{\mathbf{x}})+E(y_{\mathbf{x}})}$

$\boxed{\underset{\mathbf{x}}{\text{argmin}}}$ → $E(L(y_{\mathbf{x}}, t)) = \cancel{c} \cdot Var(y_{\mathbf{x}}) + \cancel{c} \cdot [E(y_{\mathbf{x}}) - t]^2$.

↑ variance ↑ bias²

$\boxed{\mu_{\mathbf{x}} = E(y_{\mathbf{x}}) \approx X_1 \hat{\beta}_1}$

$\boxed{\ln \sigma_{\mathbf{x}}^2 = \ln(Var(y_{\mathbf{x}})) \approx X_2 \hat{\beta}_2}$

- **Two-step procedure for nominal-the-best problem**:

  (i) Select levels of some factors to minimize $Var(y_{\mathbf{x}})$.

  (ii) Select the level of a factor not in (i) to move $E(y_{\mathbf{x}})$ closer to t.

  - A factor in step (ii) is an *adjustment* factor if it has a significant effect on $E(y_{\mathbf{x}})$ but not on $Var(y_{\mathbf{x}})$. ← $\boxed{\text{a factor whose effects appears in } \hat{\beta}_1 \text{, but not } \hat{\beta}_2}$
  - Procedure is effective only if an adjustment factor can be found. This is often done on engineering ground. → If an adjustment factor does not exist, some trade-off between (i) and (ii) is required.
  - Examples of adjustment factors : deposition time in surface film deposition process, mold size in tile fabrication, location and spacing of markings on the dial of a weighing scale.

❖ **Reading**: textbook, 4.10

Why Take $\ln s^2$? → as a response of linear model

- It maps s^2 over $(0,\infty)$ to $\ln s^2$ over $(-\infty,\infty)$. Regression and ANOVA assume the responses are nearly normal, i.e. over $(-\infty,\infty)$.
- Better for variance prediction.
  - Suppose $z_{\mathbf{x}} = \ln s_{\mathbf{x}}^2$.
  - $\hat{z}_{\mathbf{x}}$ = predicted value of $\ln \sigma_{\mathbf{x}}^2$
  - $e^{\hat{z}_{\mathbf{x}}}$ = predicted value of $\sigma_{\mathbf{x}}^2$, always nonnegative.

response $\xleftarrow{\text{approximate}}$ linear structure

$s^2 \in (0,\infty)$

$\ln(s^2) \in (-\infty,\infty)$ ← $X\beta + \varepsilon \in (-\infty,\infty)$

- Most physical laws have a multiplicative component. Log converts *multiplicity* into *additivity*.
- Variance stabilizing property: next slide. → produce "constant" error variance in the model: $\ln s_{\mathbf{x}}^2 = X\beta + \varepsilon$

→ error part of y:

$\varepsilon = \varepsilon_1 \times \cdots \times \varepsilon_k$

Assume independence and zero means:

$Var(\varepsilon) = Var(\varepsilon_1) \times \cdots \times Var(\varepsilon_k)$ ← c.f.

→ $\ln(Var(\varepsilon)) = \sum_{i=1}^{k} \ln(Var(\varepsilon_i))$

$\boxed{\text{may get a better approximation by linear structure } X\beta \text{ and follow normal.}}$

$\ln s^2$ as a Variance Stabilizing Transformation

- Assume $y_{\mathbf{x},j} \overset{i.i.d.}{\sim} N(\mu_{\mathbf{x}}, \sigma^2_{\mathbf{x}})$, $j = 1, \ldots, n_{\mathbf{x}}$. Then,

$$(n_{\mathbf{x}}-1)s^2_{\mathbf{x}} = \sum_{j=1}^{n_{\mathbf{x}}} (y_{\mathbf{x},j} - \bar{y}_{\mathbf{x}})^2 \sim \sigma^2_{\mathbf{x}} \chi^2_{n_{\mathbf{x}}-1},$$

and

$E(\ln S^2_{\mathbf{x}}) = ?$ $Var(\ln S^2_{\mathbf{x}}) = ?$

$$\underbrace{\ln s^2_{\mathbf{x}}}_{\text{a random variable}} = \ln \sigma^2_{\mathbf{x}} + \ln \underbrace{\left(\overbrace{\chi^2_{n_{\mathbf{x}}-1}}^{\text{a random variable} \sim \chi^2_{n_{\mathbf{x}}-1}}/(n_{\mathbf{x}}-1)\right)}_{W_{\mathbf{x}}}. \quad (3)$$

- $W_{\mathbf{x}}$: a random variable, h: a smooth function, by δ-method,

$$E(h(W_{\mathbf{x}})) \approx h(E(W_{\mathbf{x}})) \quad \text{and} \quad Var(h(W_{\mathbf{x}})) \approx [h'(E(W_{\mathbf{x}}))]^2 Var(W_{\mathbf{x}}) \quad (4)$$

- Suppose $W_{\mathbf{x}} \sim \chi^2_{\nu_{\mathbf{x}}}/\nu_{\mathbf{x}}$. Then, $E(W_{\mathbf{x}}) = 1$ and $Var(W_{\mathbf{x}}) = 2/\nu_{\mathbf{x}}$.
- Take $h = \ln$. Applying (4) to $W_{\mathbf{x}}$ $(\sim \chi^2_{\nu_{\mathbf{x}}}/\nu_{\mathbf{x}})$ leads to

$$E(\ln(W_{\mathbf{x}})) \approx \ln(E(W_{\mathbf{x}})) = \ln(1) = 0, \rightarrow E(\ln S^2_{\mathbf{x}}) \approx \ln \sigma^2_{\mathbf{x}}$$
$$Var(\ln(W_{\mathbf{x}})) \approx [h'(1)]^2(2/\nu_{\mathbf{x}}) = 2/\nu_{\mathbf{x}}. \rightarrow Var(\ln S^2_{\mathbf{x}}) \approx 2/\nu_{\mathbf{x}}$$

In LM, $Z_{\mathbf{x}} \sim N(\mu_{\mathbf{x}}, \sigma^2)$ cf

- In (3), $\nu_{\mathbf{x}} = n_{\mathbf{x}} - 1$, we have $\ln s^2_{\mathbf{x}} \sim N(\ln \sigma^2_{\mathbf{x}}, 2(n_{\mathbf{x}}-1)^{-1})$. The variance of $\ln s^2_{\mathbf{x}}$, i.e., $2(n_{\mathbf{x}}-1)^{-1}$, is nearly constant for $n_{\mathbf{x}} - 1 \geq 9$.

$\sigma^2_{\mathbf{x}}$ only appears in the mean structure of $\ln(S^2_{\mathbf{x}})$

a constant if we have same # of replicates for each run

❖ **Reading**: textbook, 4.11 (exercise) Compare the result with $E(S^2_{\mathbf{x}})$ and $Var(S^2_{\mathbf{x}})$

Epi-layer Growth Experiment Revisited

- Original data from Shoemaker, Tsui and Wu (1991)

Table 2 (LNp 5-2) cf Table 7: Design Matrix and Thickness Data, Original Epitaxial Layer Growth Experiment

2^4 design

original response (6 replicates) unreplicated

A	B	C	D	Thickness (y)						$\bar{y}$	s^2	$\ln s^2$
−	−	−	+	14.812	14.774	14.772	14.794	14.860	14.914	14.821	0.003	−5.771
−	−	−	−	13.768	13.778	13.870	13.896	13.932	13.914	13.860	0.005	−5.311
−	−	+	+	14.722	14.736	14.774	14.778	14.682	14.850	14.757	0.003	−5.704
−	−	+	−	13.860	13.876	13.932	13.846	13.896	13.870	13.880	0.001	−6.984
−	+	−	+	14.886	14.810	14.868	14.876	14.958	14.932	14.888	0.003	−5.917
−	+	−	−	14.182	14.172	14.126	14.274	14.154	14.082	14.165	0.004	−5.485
−	+	+	+	14.758	14.784	15.054	15.058	14.938	14.936	14.921	0.016	−4.107
−	+	+	−	13.996	13.988	14.044	14.028	14.108	14.060	14.037	0.002	−6.237
+	−	−	+	15.272	14.656	14.258	14.718	15.198	15.490	14.932	0.215	−1.538
+	−	−	−	14.324	14.092	13.536	13.588	13.964	14.328	13.972	0.121	−2.116
+	−	+	+	13.918	14.044	14.926	14.962	14.504	14.136	14.415	0.206	−1.579
+	−	+	−	13.614	13.202	13.704	14.264	14.432	14.228	13.907	0.226	−1.487
+	+	−	+	14.648	14.350	14.682	15.034	15.384	15.170	14.878	0.147	−1.916
+	+	−	−	13.970	14.448	14.326	13.970	13.738	13.738	14.032	0.088	−2.430
+	+	+	+	14.184	14.402	15.544	15.424	15.036	14.470	14.843	0.327	−1.118
+	+	+	−	13.866	14.130	14.256	14.000	13.640	13.592	13.914	0.070	−2.653

② Use $Z_{\mathbf{x}} = \ln S^2_{\mathbf{x}}$ to build a variance model

✱ Conceptual model:

$Z_{\mathbf{x}} \sim \beta_0 + \beta_A x_A + \cdots$
$+ \beta_{AB} x_A x_B + \cdots$
$+ \beta_{ABC} x_A x_B x_C + \cdots$
$+ \beta_{ABCD} x_A x_B x_C x_D$
$+ \underline{\varepsilon}$ ← constant variance

① Use $Z_{\mathbf{x}} = \bar{y}_{\mathbf{x}}$ to build a mean model

(Q: Why not use $Z_{\mathbf{x}} = y_{\mathbf{x}}$ to build a mean model?
Ans. $Var(y_{\mathbf{x}})$ not regarded constant over $\mathbf{x}$)

(Q: Why $\bar{y}_{\mathbf{x}}$ better than $y_{\mathbf{x}}$?
Ans. $0 < Var(y_{\mathbf{x}}) = \sigma^2_{\mathbf{x}}$
$0 < Var(\bar{y}_{\mathbf{x}}) = \sigma^2_{\mathbf{x}}/6$)

Epi-layer Growth Experiment: Effect Estimates

Table 8: Factorial Effects, Original Epitaxial Layer Growth Experiment

$2\hat{\beta}_i = \hat{\theta}_i$

$(X^TX)^{-1}X^TZ$

$16I$

Effect	$\bar{y}$	$\ln s^2$
A	−0.055	3.834
B	0.142	0.078
C	−0.109	0.077
D	0.836	0.632
AB	−0.032	−0.428
AC	−0.074	0.214
AD	−0.025	0.002
BC	0.047	0.331
BD	0.010	0.305
CD	−0.037	0.582
ABC	0.060	−0.335
ABD	0.067	0.086
ACD	−0.056	−0.494
BCD	0.098	0.314
$ABCD$	0.036	0.109

Understand the meaning of these effect estimates, e.g.,

$ME(A) = -0.055 = \bar{Z}(A+) - \bar{Z}(A-)$

$INT(A,B) = -0.032 = ME(A|B+) - ME(A|B-)$

⋮

Epi-layer Growth Experiment: Half-Normal Plots

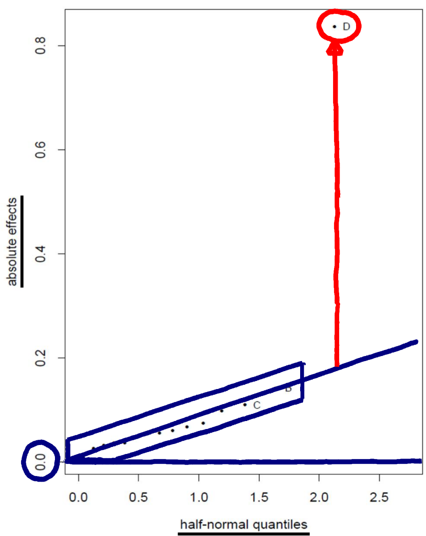


Figure 7 : Location effects ($\bar{y}_{\mathbf{x}}$)

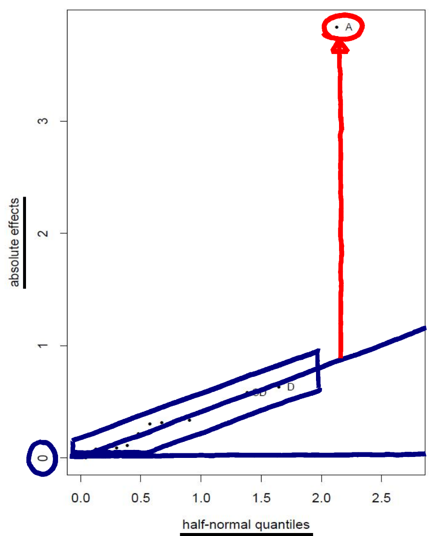


Figure 8 : Dispersion effects ($\ln s^2_{\mathbf{x}}$)

(exercise) Perform Lenth's method to identify significant effects

Epi-layer Growth Experiment: Analysis and Optimization

positive sign ⇒ increasing effect

- From the two plots, ME D is significant for $z = \bar{y}$ and ME A is significant for $z = \ln s^2$. Fitted models :

ME(D)/2

location model → $\hat{y}_{\mathbf{x}} = \hat{\beta}_0 + \hat{\beta}_D x_D = 14.389 + 0.418 x_D$, → used to reach the target value, i.e., $E(y_x) \approx 14.5$

dispersion model → $\ln \hat{s}^2_{\mathbf{x}} = \hat{\gamma}_0 + \hat{\gamma}_A x_A = -3.772 + 1.917 x_A$. → used to minimize variance

ME(A)/2

- Factor D is an adjustment factor.
- All the effects related to factors B and C do not appear in the fitted models.

B,C could be important variables with small exp'tal range

useful information?

- Two-step procedure: — qualitative factor

(i) Choose A at $-$ level (continuous rotation). ← Why not set A=-2,-3,...

— quantitative factor

(ii) Choose $x_D = 0.266$ to satisfy $14.5 = 14.389 + 0.418 x_D$. (If D = 30 and 40 sec for $x_D = -1$ and $+1$, $x_D = 0.266$ corresponds to $35 + 0.266(5) = 36.33$ sec.)

cf.

- Predicted variance (Q: What if D is a qualitative factor?)

$$\hat{\sigma}^2 = \exp[-3.772 + 1.917(-1)] = (0.058)^2 \approx 0.0034$$

check LNp.5-32

optimal setting

x_A	x_B	x_C	x_D
-1	?	?	0.266

This is too optimistic! Predicted values should be validated with a *confirmation experiment*.

(Note. $s^2_{\mathbf{x}}$ estimates subplot error variance)

Q: Do we have information about the whole-plot error variance?

Ans. Check Fig 7 in LNp.5-34

❖ **Reading**: textbook, 4.12 — check LNp.5-3

2^k Designs in 2^q Blocks

one block factor, 2^q levels

block size 2^{k-q} (∵ total 2^k EUs)

k treatment factors, each 2 levels

full factorial ⇒ 2^k treatments (⇒ need 2^k EUs)

- Example: Arranging a 2^3 design in 2 blocks (of size 4). Use the 123 column in Table 9 (LNp.5-37) to define the blocking scheme:

$\boxed{\frac{2^3}{2}}$

block size 2^{k-q} $< 2^k$ treatments ⇒ incomplete blocking

block I if 123 = $-$, and block II if 123 = $+$.

✱ conceptual model:

$z \sim B_0 +$ block effects (2^q-1 parameters) + all treatment effects (2^k-1 parameters) $+ \varepsilon$

⇒ no interaction btwn block & treatment factors

- The block effect estimate $\bar{y}(II) - \bar{y}(I)$ is identical to the estimate of the 123 interaction $\bar{y}(123 = +) - \bar{y}(123 = -)$. The block effect b and the interaction 123 are called **confounded**. Notationally,

aliased (Ch5) cf.

$$b = 123.$$

① In design matrix, column b = (column 1)×(column 2)×(column 3)

② In model matrix, block effect column = 123 interaction column

③ $B_b + B_{123}$ is jointly estimated

sacrifying

- By giving up the ability to estimate 123, this blocking scheme increases the precision in the estimates of main effects and 2fi's by arranging 8 runs in two *homogeneous* blocks.

∵ the block effect ① can reduce $\hat{\sigma}^2$ ② is orthogonal to other treatment effects (their estimates not biased by block effect)

a good choice?

- Why sacrificing 123?

ans: Effect hierarchy principle. → 123 is the least important effect among all the 8 factorial effects.

Arrangement of 2^3 Design in 2 Blocks

block 1: EU11, EU13, EU12, EU14 — (1, 2, 3)
block 2: EU21, EU23, EU22, EU24 — (1, 2, 3)

4 ← how? → 4

8 treatments
(−, −, −)
(−, −, +)
⋮
(+, +, +)

conceptual model: (Note. not interested in β_b)

$$Z = \beta_0 + \beta_b x_b + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_{12} x_{12} + \beta_{13} x_{13} + \beta_{23} x_{23} + \beta_{123} x_{123} + \varepsilon$$

block effect

$$= \beta_0 + (\beta_b + \beta_{123}) x_b + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_{12} x_{12} + \beta_{13} x_{13} + \beta_{23} x_{23} + \varepsilon$$

Table 9: Arranging a 2^3 Design in Two Blocks of Size Four
(The 3 factors are denoted by **1**, **2**, and **3**)

intercept

123 = Block ⇒ $x_b = x_{123}$

Run	I	1	2	3	12	13	23	123	123	Block
1	+	−	−	−	+	+	+	−	−	I
2	+	−	−	+	+	−	−	+	+	II
3	+	−	+	−	−	+	−	+	+	II
4	+	−	+	+	−	−	+	−	−	I
5	+	+	−	−	−	−	+	+	+	II
6	+	+	−	+	−	+	−	−	−	I
7	+	+	+	−	+	−	−	−	−	I
8	+	+	+	+	+	+	+	+	+	II

4 "+"
4 "−"

design matrix of block factor

# of possible choices: $\binom{8}{4} = \frac{8!}{4!4!}$

Why might not be good choices? Ans. It would be better to have orthogonality.

# of possible columns: 7 ⇒ guarantee orthogonality

In the model matrix, the 8 d.f. are used to estimate
β_0 (intercept),
$\beta_1, \beta_2, \beta_3$,
$\beta_{12}, \beta_{13}, \beta_{23}$,
$\beta_b + \beta_{123}$ ← a joint effect

model matrix (all treatment effects)

design matrix of treatment factors

combine → whole design matrix

A 2^3 Design in 4 Blocks

8 treatments

one block factor, 4 levels ($= 2^2$), block size 2

block 1: EU11, EU12 — (1, 2, 3)
block 2: EU21, EU22 — (1, 2, 3)
block 3: EU31, EU32 — (1, 2, 3)
block 4: EU41, EU42 — (1, 2, 3)

2 2 (−, −, −) (−, −, +) ⋮ (+, +, +) 2 2

- Similarly we can use $b_1 = \mathbf{12}$ and $b_2 = \mathbf{13}$ to define two independent blocking variables. (each with 2 levels) The 4 blocks I, II, III and IV are defined by $b_1 = \pm$ and $b_2 = \pm$:

pseudo block factors

b_2 \ b_1	−	+
−	I	III
+	II	IV

block	b_1 (β_{b_1})	b_2 (β_{b_2})	b_1b_2 ($\beta_{b_1b_2}$)
I	−	−	+
II	−	+	−
III	+	−	−
IV	+	+	+

a block factor with 4 levels needs 3 df. for its block effects.

This is not an interaction effect. It represents a block main effect.

A group structure in Algebra, with 2 generators b_1, b_2

Q: Why not sacrify 123?

$\{I, b_1, b_2, b_1b_2\}$

identity element → Intercept

I	b_1	b_2	b_1b_2
123	1 ×		23
	2 ×		13
	3 ×		12
	12		3 ×
	13		2 ×
	23		1 ×

- A 2^3 design in 4 blocks is given in Table 10 (LNp.5-39). Confounding relationships:
$b_1 = \mathbf{12}$, $b_2 = \mathbf{13}$, $b_1 b_2 = \mathbf{12} \times \mathbf{13} = \mathbf{23}$. $= 1^2 23$
Thus **12**, **13** and **23** are confounded with block effects and thus sacrificed.

Note. In model matrix, for every columns x_i's, we have $x_i^2 = I$

Arranging a 2^3 Design in 4 Blocks

(exercise)
- coceptual model = ?
- model matrix with block & treatment effects = ?
- 8 df. are used to estimate what ?

Table 10: Arranging a 2^3 Design in Four Blocks of Size Two

$B_{b_2} + B_{13}$ ← b_2, $B_{b_1} + B_{12}$ ← b_1, $B_{b_1 b_2} + B_{23}$ ← $b_1 b_2$

$\binom{7}{2}$ choices → b_1, b_2

# of possible choices: $\frac{8}{2\,2\,2\,2}$

					b_1	b_2	b_1b_2		
Run	I	1	2	3	12	13	23	123	block
1	+	−	−	−	+	+	+	−	IV
2	+	−	−	+	+	−	−	+	III
3	+	−	+	−	−	+	−	+	II
4	+	−	+	+	−	−	+	−	I
5	+	+	−	−	−	−	+	+	I
6	+	+	−	+	−	+	−	−	II
7	+	+	+	−	+	−	−	−	III
8	+	+	+	+	+	+	+	+	IV

2 "I", 2 "II", 2 "III", 2 "IV"

model matrix (treatment effects)

whole design matrix

True model: $Z = X_1\beta_1 + X_2\beta_2 + \varepsilon$, $X_2\beta_2 = H_1X_2\beta_2 + (I-H_1)X_2\beta_2$

Fitted model: $Z = X_1\beta_1 + \varepsilon$ (X_2: block effects)

(a) If $X_1 = \begin{array}{c} I\;1\;2\;3 \\ \left[\,|\;|\;|\;|\,\right] \end{array}$ ⇒ $H_1X_2\beta_2 = 0$; But, $\hat\sigma^2$ is larger than it should be.

(b) If $X_1 = \begin{array}{c} I\;1\;2\;3\;12\;13\;23\;123 \\ \left[\,|\;|\;|\;|\;|\;|\;|\;|\,\right] \end{array}$ ⇒ $(I-H_1)X_2\beta_2 = 0$; But, $\hat\beta_{12}, \hat\beta_{13}, \hat\beta_{23}$ are biased

- $\{I, 12, 13, 23\}$ forms the **block defining contrast subgroup** for the 2^3 design in 4 blocks. For a more complicated example (2^5 design in 8 blocks), see textbook, p.196.

3 pseudo block factors b_1, b_2, b_3
⇒ 7 block effects: $b_1, b_2, b_3, b_1b_2, b_1b_3, b_2b_3, b_1b_2b_3$

Minimum Aberration Blocking Scheme

criterion

Only consider block schemes of the form: $B = \{I, b_1, b_2, b_1b_2\}$ ← a group

chosen from the columns in the model matrix of treatment effects (Q: What is the best choice?)

# of treatment factors

- For any blocking scheme B and $1 \le i \le k$, let

also called a word of length i

$$g_i(B) = \text{number of } i\text{-factor effects that are confounded with block effects.}$$

of i-factor (treatment) effects that are sacrified.

  – Must require $g_1(B) = 0$ (because no main effect should be confounded with block effects).

- For any two blocking schemes B_1 and B_2, let

$$r = \text{smallest } i \text{ such that } g_i(B_1) \neq g_i(B_2).$$

wordlength pattern (WLP) of block scheme

  – If $g_r(B_1) < g_r(B_2)$, scheme B_1 is said to have *less aberration* than scheme B_2.
  – A blocking scheme B has *minimum aberration* (MA) if no other blocking schemes have less aberration than B.

$(g_1(B), g_2(B), \cdots, g_k(B))$ sequentially minimize

MA criterion

- The minimum aberration criterion is justified by the effect hierarchy principle.
- Minimum aberration blocking schemes are given in Table 4A.1 (textbook, p.207).
- Theory is developed under the assumption of no *block* × *treatment* interactions.

❖ **Reading**: textbook, 4.15