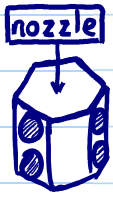
Epitaxial Layer Growth Experiment

factorial design (因子設計) ← multi-way layout
often used in exp'ts with a larger # of factors ← cf. in industrial era

previous exp'ts & designs in CH3 (comparative exp'ts) $Y = X\beta + \varepsilon$

- An AT&T experiment based on 2^4 design; four treatment factors each at two levels. There are 6 replicates for each of the 16 ($=2^4$) level combinations; data given on LNp.5-2.



★ response: thickness
★ treatment factors: A, B, C, D
All 2 levels — −1, +1

Q: Why is fewer # of levels often used in factorial exp'ts?

"qualitative or quantitative" is not an important issue in 2-level case [why? both can use (0,1) or (−1,1) codings]

qualitative or quantitative ?

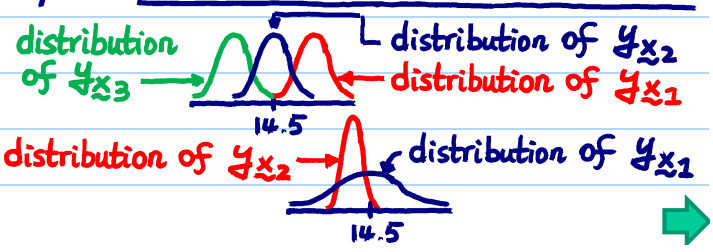
Table 1: Factors and Levels, Adapted Epitaxial Layer Growth Experiment

Treatment Factor		− Level	+
A.	susceptor-rotation method	continuous	oscillating
B.	nozzle position	2	6
C.	deposition temperature (°C)	1210	1220
D.	deposition time	low	high

★ Exp'tal unit: a group of 6 wafers
homogeneous — EU1 ··· EU16 (A, B, C, D)
⇒ 16 whole plots, 96 subplots
⇒ all 4 factors are WP factors

cf. the objectives in the comparative exp'ts

Objective : Reduce variation of y (=layer thickness) around its target 14.5 μm by changing factor level combinations.



Data from Epitaxial Layer Growth Experiment

$E(y_A) = \beta_0 + \beta_1 x_A$
All d.f. = 10 = 2 + 8
for parameters β | for $\mathrm{Var}(y) = \sigma^2$

2^4 full factorial design

Table 2: Design Matrix and Thickness Data, Adapted Epitaxial Layer Growth Experiment

unreplicated | replicated

Run	A	AB	B	C	D	Thickness						$\bar{y}$	s^2	$\ln s^2$
1	−	+	−	−	+	14.506	14.153	14.134	14.339	14.953	15.455	14.59	0.270	−1.309
2	−	+	−	−	−	12.886	12.963	13.669	13.869	14.145	14.007	13.59	0.291	−1.234
3	−	+	−	+	+	13.926	14.052	14.392	14.428	13.568	15.074	14.24	0.268	−1.317
4	−	+	−	+	−	13.758	13.992	14.808	13.554	14.283	13.904	14.05	0.197	−1.625
5	−	−	+	−	+	14.629	13.940	14.466	14.538	15.281	15.046	14.65	0.221	−1.510
6	−	−	+	−	−	14.059	13.989	13.666	14.706	13.863	13.357	13.94	0.205	−1.585
7	−	−	+	+	+	13.800	13.896	13.887	14.902	14.461	14.454	14.40	0.222	−1.505
8	−	−	+	+	−	13.707	13.623	14.210	14.042	14.881	14.378	14.14	0.215	−1.537
9	+	−	−	−	+	15.050	14.361	13.916	14.431	14.968	15.294	14.67	0.269	−1.313
10	+	−	−	−	−	14.249	13.900	13.065	13.143	13.708	14.255	13.72	0.272	−1.302
11	+	−	−	+	+	13.327	13.457	14.368	14.405	13.932	13.552	13.84	0.220	−1.514
12	+	−	−	+	−	13.605	13.190	13.695	14.259	14.428	14.223	13.90	0.229	−1.474
13	+	+	+	−	+	14.274	13.904	14.317	14.754	15.188	14.923	14.56	0.227	−1.483
14	+	+	+	−	−	13.775	14.586	14.379	13.775	13.382	13.382	13.88	0.253	−1.374
15	+	+	+	+	+	13.723	13.914	14.913	14.808	14.469	13.973	14.30	0.250	−1.386
16	+	+	+	+	−	14.031	14.467	14.675	14.252	13.658	13.578	14.11	0.192	−1.650

AC, ···
ABC, ···
ABCD

check LNp 4-65

The # of different level combinations in the exp't ⇒ can be used to study 16 parameters (1 intercept, 15 effects)

model matrix

contain what information?
$E(y_x)$ and $\mathrm{Var}(y_x)$ ← pure error information

★ conceptual model

$$z \sim \beta_0 + A + B + C + D + AB + AC + AD + BC + BD + CD + ABC + ABD + ACD + BCD + ABCD + \varepsilon$$

for Thickness | for $\bar{y}$ / s^2 / $\ln(s^2)$

① response: Thickness
# of all obs. = 16 × 6 = 96
all d.f.
16 → for effects
80 → for $\mathrm{Var}(z)$

② response: $\bar{y}$ / s^2 / $\ln s^2$
# of all obs. = 16
all d.f.
16 → for effects
0 → for $\mathrm{Var}(z)$

❖ **Reading**: textbook, 4.1

Designs: A General discussion

2^k: k = # of factors; 2 = # of levels; 2^k = # of distinct level combinations

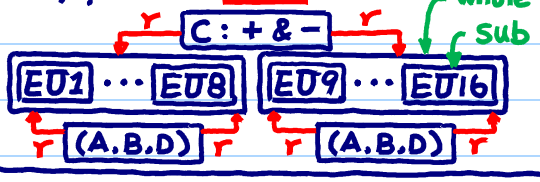
- Full factorial design (全因子設計)
  - $2 \times 2 \times \ldots \times 2 = 2^k$ design.

Actually, 2^k full factorial design
⇒ an OA of strength k (orthogonality of design matrix)
⇒ All effects in model matrix are geometrically orthogonal (under sum codings)

cf. Design matrix → + model + codings

- Planning matrix vs model matrix → $Z = X\beta + \varepsilon$ (see Tables 4.3, 4.5, textbook, p.158 & 161).
- Run order and restricted randomization (see Table 4.4, textbook, p.160).
  - randomization

∵ factor C is a hard-to-change factor
⇒ each of its 2 levels 1210, 1220°C only performs once



- *Balance*: each factor level appears the same number of times in the design matrix.
- *Orthogonality* : for any pair of factors, each possible level combination appears the same number of times in the design matrix.

defined on DM

OA of strength 1 ⇒ In model matrix, ME ⊥ 1

OA of strength 2 (sum codings) ⇒ In model matrix, ME ⊥ ME

- Replicated vs unreplicated experiment.
  - Thickness
  - $\bar{y}/s^2/\ln s^2$

Replicates can bring in information about $\mathrm{Var}(Z_x) \leftrightarrow x$

❖ **Reading**: textbook, 4.2