

Incorrect Model and Analysis

EU's are treated as in completely randomized design (homogeneous EU's)
Two-way layout model (treatment factors \sim LNp.45-46

A and B with n replicates):

different from the notations i, j, k in LNp.4-49~55

$$y_{ijk} = \eta + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk},$$

where $i = 1, \dots, I$; $j = 1, \dots, J$; $k = 1, \dots, n$.

(In the case, $I = 2$, $J = 4$, $n = 3$.)

★ conceptual model:

$$y \sim \underbrace{\beta_0}_{(I-1) \text{ parameters}} + \underbrace{A}_{(I-1) \text{ parameters}} + \underbrace{B}_{(J-1) \text{ parameters}} + \underbrace{A \times B}_{(I-1)(J-1) \text{ parameters}} + \underbrace{\varepsilon}_{(I-1)(J-1) \text{ parameters}}$$

$$\text{cov}(\varepsilon) = \sigma^2 I$$

df. in $\sigma^2 = IJn - IJ$

- ANOVA (table on LNp.4-57) shows that only factor A is significant; neither B nor $A \times B$ is significant.
- The model is wrong: A and B use different randomization schemes. The error component should be separated into two parts—the whole plot error and the subplot error. To test the significance of various effects, we need to compare their respective mean squares with two different error components.

Suppose the experiment is really from CRD. Some possible choices for $\text{Var}(\varepsilon_{ijk})$:

(a) $24 \varepsilon^R, 24 \varepsilon^W, 24 \varepsilon^S \Rightarrow \text{cov}(\underline{y}) = (\sigma_R^2 + \sigma_W^2 + \sigma_S^2) I$

(b) $1 \varepsilon^R, 24 \varepsilon^W, 24 \varepsilon^S \Rightarrow \text{cov}(\underline{y}) = (\sigma_W^2 + \sigma_S^2) I$

(c) $1 \varepsilon^R, 1 \varepsilon^W, 24 \varepsilon^S \Rightarrow \text{cov}(\underline{y}) = \sigma_S^2 I$

Incorrect ANOVA Table

Table 25: Incorrect ANOVA Table, Wood Experiment

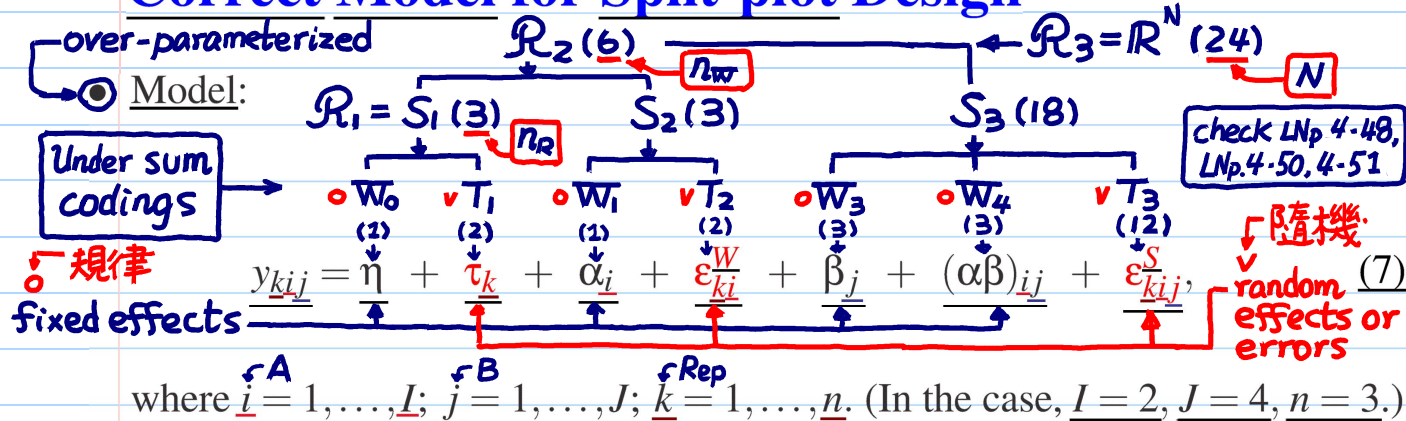
Source	Degrees of Freedom	Sum of Squares	Mean Squares	F	p -value
A	1	782.04	782.04	13.49	0.002 ✓
B	3	266.00	88.67	1.53	0.245
$A \times B$	3	62.79	20.93	0.36	0.783
Residual	16	927.88	57.99		
Total	23	2038.72			

cf. Table 27 in LNp.64

$$\hat{\sigma}^2$$

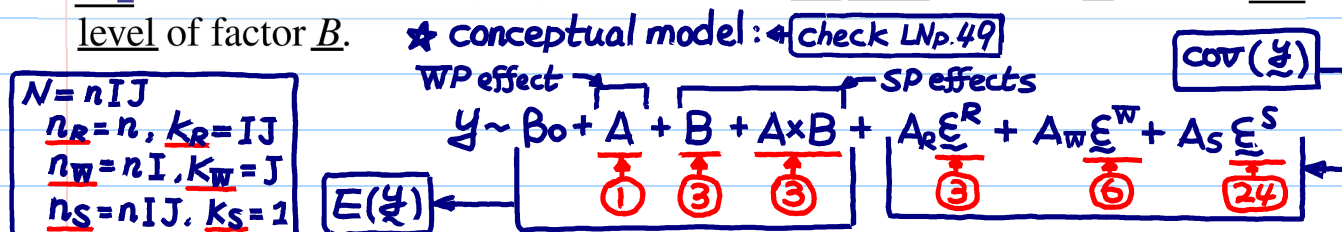
- Only A is significant.

Correct Model for Split-plot Design

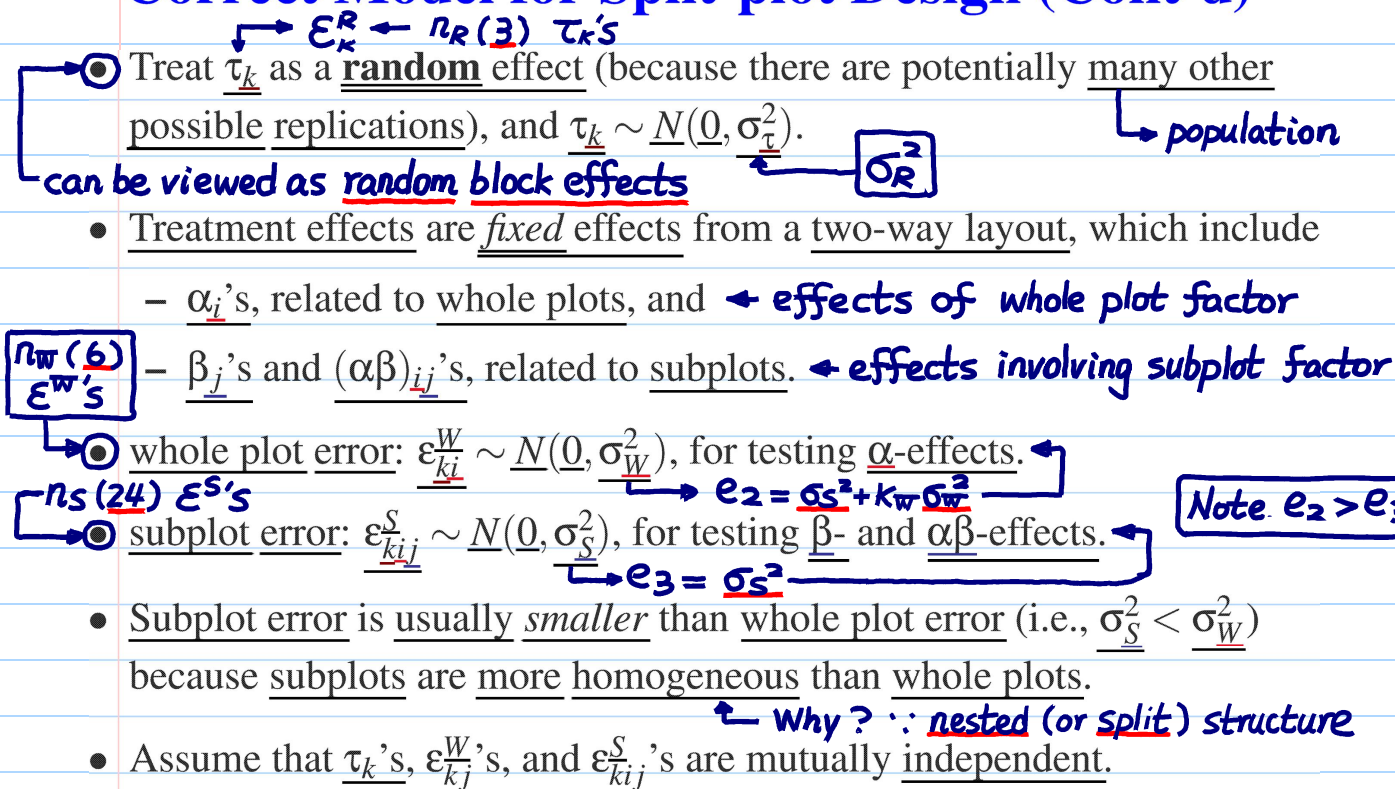


Terms related to the whole plot		Terms related to the subplot	
ϵ_k^R	effect of \underline{k} th replicate	β_j	\underline{j} th main effect of \underline{B}
α_i	\underline{i} th main effect for \underline{A}	$(\alpha\beta)_{ij}$	$(\underline{i}, \underline{j})$ th interaction $\underline{A} \times \underline{B}$
ϵ_{ki}^W	$(\underline{k}, \underline{i})$ th whole plot error	ϵ_{kij}^S	$(\underline{k}, \underline{i}, \underline{j})$ th subplot error

- y_{kij} : observation for the \underline{k} th replicate of the \underline{i} th level of factor \underline{A} and the \underline{j} th level of factor \underline{B} .



Correct Model for Split-plot Design (Cont'd)



ANOVA Decomposition

$$W_i = \text{Span}\{X_i\}$$

Note. All these SS's are independent.

- Use the zero-sum constraints: \rightarrow sum codings

\uparrow By (L6) in LNp.55

$$\alpha_{I-} = -(\alpha_1 + \dots + \alpha_{I-1}) \xrightarrow{\text{e.g.}} \sum_{i=1}^I \alpha_i = \sum_{j=1}^J \beta_j = 0, \quad \text{e.g. } \beta_J = -(\beta_1 + \dots + \beta_{J-1}) \rightarrow$$

$$\alpha\beta_{iJ} = -(\alpha\beta_{i1} + \dots + \alpha\beta_{i,J-1}) \xrightarrow{\text{e.g.}} \sum_{j=1}^J (\alpha\beta)_{ij} = 0, \quad \text{for } i = 1, \dots, I, \quad \left. \begin{array}{l} W_2 = \text{Span}\{X_2\} \\ W_3 = \text{Span}\{X_3\} \end{array} \right\}$$

$$\alpha\beta_{I,j} = -(\alpha\beta_{1,j} + \dots + \alpha\beta_{I-1,j}) \xrightarrow{\text{e.g.}} \sum_{i=1}^I (\alpha\beta)_{ij} = 0, \quad \text{for } j = 1, \dots, J,$$

treated as a 3-level factor

to break up the total sum of squares as a three-way layout with factors A, B, and Rep: $SS_{\underline{T}} = \|P_{\underline{T}} \underline{y}\|^2$ A: 2 levels, B: 4 levels, Rep: 3 levels

By Δ in LNp.4-53

$$SST = SS_{\text{Rep}}^{(2)} + SS_A^{(1)} + SS_B^{(3)} + SS_{\text{Rep} \times A}^{(2)} + SS_{A \times B}^{(3)} + SS_{\text{Rep} \times B}^{(6)} + SS_{\text{Rep} \times A \times B}^{(6)}$$

\uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow
 $R^N \ominus W_0$ T_1 W_1 W_2 T_2 W_3 T_3

- Define the sum of squares for the whole plot error SS_{whole} and the sum of squares for the subplot error SS_{sub} as:

W_i 's, T_j 's are the spaces generated by these effects | e.g. $\text{Rep} \times A \subset \mathcal{R}_2$ and $\text{Rep} \times A \perp W_0 \perp \text{Rep} \perp A$ (LNp.4-48)

$$(k_w \sigma_w^2 + \sigma_s^2) \xleftarrow{\text{est.}} SS_{\text{whole}} = SS_{\text{Rep} \times A}, \quad SS_{\text{sub}} = SS_{\text{Rep} \times B} + SS_{\text{Rep} \times A \times B}$$

- ANOVA decomposition for the split-plot model:

By (L5) in LNp.55

$$SST = SS_{\text{Rep}} + SS_A + SS_{\text{whole}} + SS_B + SS_{A \times B} + SS_{\text{sub}}$$

$\xrightarrow{f \sim e_1 \chi_{\dim(T_1)}^2}$ $\xrightarrow{f \sim e_2 \chi_{\dim(T_2)}^2}$ $\xrightarrow{f \sim e_3 \chi_{\dim(T_3)}^2}$
 S_2 S_3