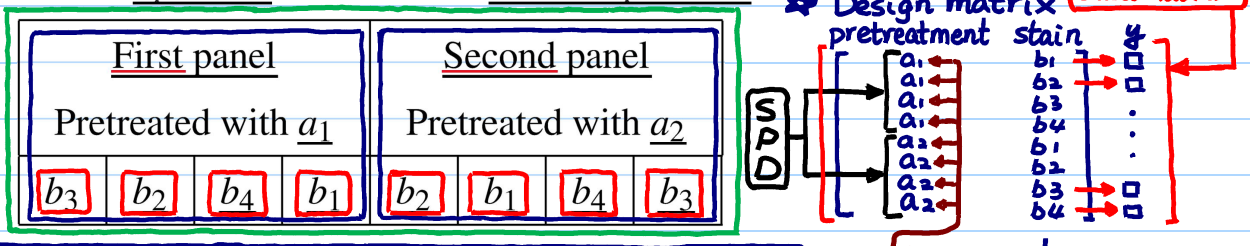
Split-plot Design

Design matrix and data matrix look the same as those in 2-way layout

- Alternative Design: split-plot design in Table 24.

Table 24: Split-Plot Version of the Wood Experiment

First panel				Second panel			
Pretreated with a_1				Pretreated with a_2			
b_3	b_2	b_4	b_1	b_2	b_1	b_4	b_3



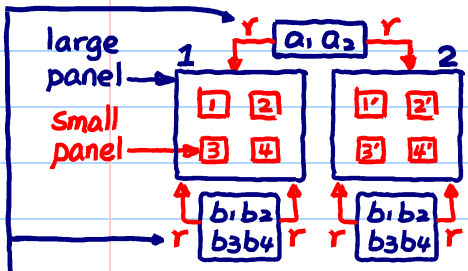
Q: How many times pretreatments (a_1 or a_2) are applied?
Ans: 2 times

CRD

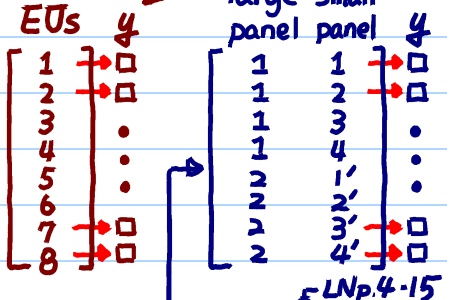
規律 XB
structures in y
隨機 ε

- Justification: Easier to apply pretreatment to *large* wood panels.

★ Exp'tal units (Q: what are the exp'tal units):



- EUs for A and B are different
- EUs for A: large panels
- EUs for B: small panels
- large panels are split into small panels



"small panel" is nested in "large panel"

LNp.4-15

Split-plot Design (Cont'd)

Why is it called "plot"?

- Split-plot design (and the name) has its origin in agriculture.
- Some factors need to be applied to large plots, called *whole plots*. In the example, the two big wood panels to which pretreatement a_1 and a_2 are applied are whole plots.
- Split each whole plot into smaller plots, called *subplots*. In the example, the four small wood panels within the large panels are subplots.

Exp'tal unit ↔ ε (check LNp.4-45)

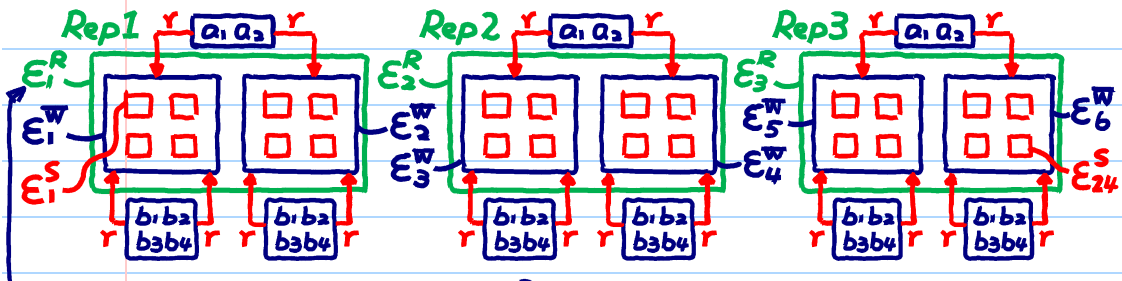
- Wood Experiment:
  - A: whole-plot factor
  - B: subplot factor

$\varepsilon^R_1, \varepsilon^R_2, \varepsilon^R_3$ ← 3 replications (treated as 3 blocks with random effects),

$\varepsilon^W_1, \cdots, \varepsilon^W_6$ ← 6 whole plots (two large panels for a_1 and a_2 per replication),

$\varepsilon^S_1, \cdots, \varepsilon^S_{24}$ ← 24 subplots (four small panels for b_1, b_2, b_3, and b_4 per large panel).

similar to the 1-way random effects model discussed in LNp 3-31~38



★ Conceptual model:

$y \sim \beta_0 + A + B + A\times B + \varepsilon^R + \varepsilon^W + \varepsilon^S$

$E(y)$

$\mathrm{cov}(y)$

★ in LNp4-45 ← cf.

Data from the Wood Experiment

3 parameters about error variance in Σ: $\sigma_R^2, \sigma_W^2, \sigma_S^2$

Whole plot	Sub-plot	Pretreatment (A)	Stain type (B)	Replication (Rep)	Resistance (Y)
4	1 ε_1^S	a_2 -1	b_2 0 1 0	1	53.5
4	2 ε_1^S	a_2 :	b_4 -1 -1 -1	1	32.5
4	3	a_2 :	b_1 1 0 0	1	46.6
4	4	a_2 -1	b_3 0 0 1	1	35.4
1	5	a_1 1	b_3	1	40.8
1	6	a_1 :	b_1	1	43.0
1	7	a_1 :	b_2	1	51.8
1	8	a_1 1	b_4	1	45.5
5		a_2 -1	b_4 •	2	44.6
5		a_2 :	b_1	2	52.2
5		a_2 :	b_3	2	45.9
5		a_2 -1	b_2 •	2	48.3
2		a_1 1	b_2	2	60.9
2		a_1 :	b_4	2	55.3
2		a_1 :	b_3	2	51.1
2		a_1 1	b_1 •	2	57.4
6		a_2 -1	b_1	3	32.1
6		a_2 :	b_4	3	30.1
6		a_2 :	b_2	3	34.4
6		a_2 -1	b_3	3	32.2
3		a_1 1	b_1 1 0 0	3	52.8
3		a_1 :	b_3 0 0 1	3	51.7
3		a_1 :	b_4 -1 -1 -1	3	55.3
3	23 ε_{23}^S	a_1 :	b_2 0 1 0	3	59.2
3	24 ε_{24}^S	a_1 1		3	

$\mathcal{R}_2$: σ_W^2; $\mathcal{R}_3$: σ_S^2; W_A, $W_{A\times B}$, W_B; $\mathcal{R}_1$: σ_R^2

ε_1^R, ε_2^R, ε_3^R; ε_1^W, ε_2^W, ε_3^W, ε_4^W, ε_5^W, ε_6^W

Sum codings

$\mathrm{cov}(\underset{\sim}{y}) = \sigma_R^2 \equiv \Sigma_R + \sigma_W^2 \equiv \Sigma_W + \sigma_S^2 \equiv \Sigma_S \equiv \Sigma \quad (= \Sigma_R + \Sigma_W + \Sigma_S)$

$\underset{\sim}{Y} \sim N(X\underset{\sim}{\beta}, \Sigma)$ cf. REM in LNp 3-32

not $\sigma^2 I$

assume all ε's are independent

$\mathrm{cov}(y_1, y_2) = \mathrm{cov}(\varepsilon_1^R + \varepsilon_4^W + \varepsilon_2^S, \varepsilon_1^R + \varepsilon_4^W + \varepsilon_3^S) = \mathrm{Var}(\varepsilon_1^R + \varepsilon_4^W) = \sigma_R^2 + \sigma_W^2$

$\mathrm{cov}(y_1, y_2) = \mathrm{cov}(\varepsilon_1^R + \varepsilon_4^W + \varepsilon_4^S, \varepsilon_1^R + \varepsilon_1^W + \varepsilon_8^S) = \mathrm{Var}(\varepsilon_1^R) = \sigma_R^2$

$\mathrm{Var}(y) = \sigma_R^2 + \sigma_W^2 + \sigma_S^2$

$\mathrm{cov}(y_1, y_2) = \mathrm{cov}(\varepsilon_2^R + \varepsilon_2^W + \varepsilon_{16}^S, \varepsilon_3^R + \varepsilon_6^W + \varepsilon_{17}^S) = 0$

$W_A \subset \mathcal{R}_2 \cap \mathcal{R}_1^\perp = S_2$

$W_B \subset \mathcal{R}_3 \cap \mathcal{R}_2^\perp = S_3$

$W_{A\times B} \subset \mathcal{R}_3 \cap \mathcal{R}_2^\perp = S_3$