Analysis of Bolt Experiment

anova($y \sim B_0$ + media + plating + media × plating)

Table 10: ANOVA Table, Bolt Experiment

Source	Degrees of Freedom	Sum of Squares	Mean Squares	F
W_α media	1	821.400	821.400	22.46
W_β plating	2	2290.633	1145.317	31.31
W_ω media × plating	2	665.100	332.550	9.09
$\Omega^\perp$ residual	54	1975.200	$\hat{\sigma}^2 \leftarrow$ 36.578	

H_0, H_A models = ?

Q : What if we use anova($y \sim B_0$ + plating + media + plating × media) ?

- **Conclusions :** Both factors and their interactions are significant. Multiple comparisons of C&W, HT and P&O can be performed by using Tukey method with $k = 3$ and 54 error df's.
  ↳ compare $\bar{\mu}_{\cdot j}$'s $\xleftarrow{e}$ $\bar{y}_{\cdot j \cdot}$'s
  answer "how different" problem
- Another method is considered in the following pages.
  t-tests in LM

Note. Interactions are significant (check LNp.16)

Two Qualitative Factors: a Regression Modeling Approach

to answer "how different" problem

over-parameterized

	0	0		0
	ω_{11}	ω_{12}	$\cdots$	ω_{1J}
0 =	ω_{21}	ω_{22}	$\cdots$	ω_{2J}
	$\vdots$	$\vdots$	$\ddots$	$\vdots$
0 =	ω_{I1}	ω_{I2}	$\cdots$	ω_{IJ}

I+J−1 constraints

- Motivation: need to find a model that allows the comparison and estimation between levels of the qualitative factors. The parameters α_i and β_j in model (5) (LNp.4-16) are not estimable without putting constraints. → must have a model that is not over-parameterized

can estimate & test individual effects α_i's, β_j's, ω_{ij}'s

- For qualitative factors, use the **baseline constraint** for the bolt experiment:

$$\alpha_1 = \beta_1 = 0 \text{ and } w_{1j} = w_{i1} = 0, \quad i = 1,2, \; j = 1,2,3.$$

cf. → treatment codings (LNp. 3-8) — (0, 1) codings → $\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ ← baseline level

→ Then, we can do estimation & t-tests for individual effects

- It can be shown that (media, plating) = (1, 1)

$$\underbrace{\eta + \alpha_1 + \beta_1 + \omega_{11}}_{\alpha_1=\beta_1=\omega_{11}=0} = \mu_{11} = E(y_{11}) = \eta, \quad \mu_{12} = E(y_{12}) = \eta + \beta_2, \quad \mu_{13} = E(y_{13}) = \eta + \beta_3,$$

β_2 → $\mu_{12} - \mu_{11}$

$$\mu_{21} = E(y_{21}) = \eta + \alpha_2,$$

α_2 → $\mu_{21} - \mu_{11}$

$$\mu_{22} = E(y_{22}) = \eta + \alpha_2 + \beta_2 + \omega_{22}, \quad \mu_{23} = E(y_{23}) = \eta + \alpha_2 + \beta_3 + \omega_{23}.$$

→ ✱ sum codings in LNp.4-16~17 ⇒ (−1, 1) codings → for ANOVA (∵ ⊥)

Regression Model (continued)

- In the regression model $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \varepsilon$, (matrix form)

model + codings → model matrix

model (5) in LNp.16

Design Matrix

media	plating	y
1(B)	1(C)	□ ⋯ □
1(B)	2(H)	□ ⋯ □
1(B)	3(P)	•
2(M)	1(C)	•
2(M)	2(H)	•
2(M)	3(P)	□ ⋯ □

E(y) → μ_{11}, E(y) → μ_{12}, E(y) → μ_{23}

$$\underline{\boldsymbol{\mu}} = \begin{pmatrix} \mu_{11} \\ \mu_{12} \\ \mu_{13} \\ \mu_{21} \\ \mu_{22} \\ \mu_{23} \end{pmatrix} \qquad \underline{\mathbf{X}} = \begin{array}{c} \begin{array}{cccccccccccc} \eta & \alpha_1 & \alpha_2 & \beta_1 & \beta_2 & \beta_3 & \omega_{21} & \omega_{22} & \omega_{23} & \omega_{11} & \omega_{12} & \omega_{13} \end{array} \\ \begin{pmatrix} 1 & \cancel{1} & 0 & \cancel{1} & 0 & 0 & \cancel{0} & 0 & 0 & \cancel{1} & \cancel{0} & \cancel{0} \\ 1 & \cancel{1} & 0 & \cancel{0} & 1 & 0 & \cancel{0} & 0 & 0 & \cancel{0} & \cancel{1} & \cancel{0} \\ 1 & \cancel{1} & 0 & \cancel{0} & 0 & 1 & \cancel{0} & 0 & 0 & \cancel{0} & \cancel{0} & \cancel{1} \\ 1 & \cancel{0} & 1 & \cancel{1} & 0 & 0 & \cancel{1} & 0 & 0 & \cancel{0} & \cancel{0} & \cancel{0} \\ 1 & \cancel{0} & 1 & \cancel{0} & 1 & 0 & \cancel{0} & 1 & 0 & \cancel{0} & \cancel{0} & \cancel{0} \\ 1 & \cancel{0} & 1 & \cancel{0} & 0 & 1 & \cancel{0} & 0 & 1 & \cancel{0} & \cancel{0} & \cancel{0} \end{pmatrix} \end{array}$$

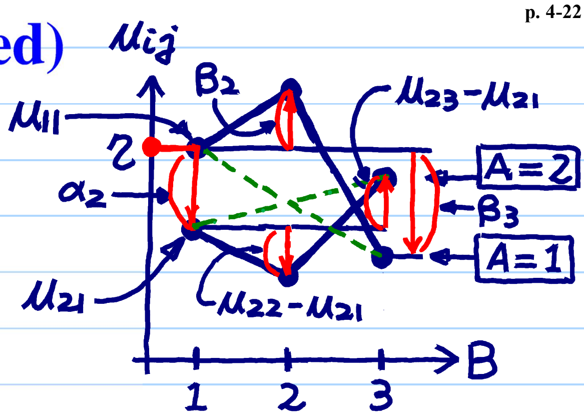
(0,1) coding (0,1) coding $x_{ij} = x_i \times x_j$

$$\underline{\boldsymbol{\beta}} = \begin{pmatrix} \eta \\ \alpha_2 \\ \beta_2 \\ \beta_3 \\ \omega_{22} \\ \omega_{23} \end{pmatrix}$$

- η ← intercept
- α_2 ← 1 dummy variable for ME of media ← I-1
- β_2, β_3 ← 2 dummy variables for ME of plating ← J-1
- ω_{22}, ω_{23} ← 2 dummy variables for their interaction ← (I-1)(J-1)

$\underline{\mu} = X\underline{\beta}$
$\Rightarrow \underline{\beta} = X^{-1}\underline{\mu}$

Regression Model (continued)



- Interpretation of parameters

Sum coding

$\eta = \dfrac{\mu_{11}+\mu_{12}+ \cdots +\mu_{23}}{6}$ ⟷ cf.

$\alpha = \dfrac{\mu_{11}+\mu_{12}+\mu_{13}}{3} - \dfrac{\mu_{21}+\mu_{22}+\mu_{23}}{3}$ ⟷ cf.

$\underline{\beta} = X^{-1}\underline{\mu}$ (exercise)

$$\begin{aligned} \underline{\eta} &= \underline{\mu_{11}}, \\ \underline{\alpha_2} &= \underline{\mu_{21}} - \boxed{\underline{\mu_{11}}}, \\ \underline{\beta_2} &= \underline{\mu_{12}} - \boxed{\underline{\mu_{11}}}, \\ \underline{\beta_3} &= \underline{\mu_{13}} - \boxed{\underline{\mu_{11}}}, \\ \underline{\omega_{22}} &= (\underline{\mu_{22}-\mu_{21}}) - (\underline{\mu_{12}-\mu_{11}}), \\ \underline{\omega_{23}} &= (\underline{\mu_{23}-\mu_{21}}) - (\underline{\mu_{13}-\mu_{11}}). \end{aligned}$$

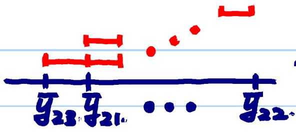
baseline level, (media, plating) = (1, 1)

← β_2 ← β_3

Exercise 1. Express η, α_i's, β_j's, ω_{ij}'s as functions of μ_{ij}'s under sum codings.

Exercise 2. Under sum codings, use the graph to interpret the meanings of η, α_i's, β_j's, ω_{ij}'s.

Regression Analysis Results



multiple comparison result

cf.

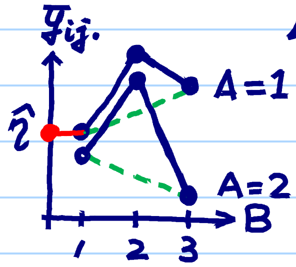


Table 11: Tests, Bolt Experiment

Why different?

Why not significant?

contradict to ANOVA result in LNp.4-19?

What H_0 H_A models?

Effect	Estimate	Standard Error	t	p-value
* η	17.4000	1.9125	9.10	0.000
α_2	−0.5000	2.7047	−0.18	0.854
* β_2	17.3000	2.7047	6.40	0.000
* β_3	13.1000	2.7047	4.84	0.000
ω_{22}	−4.8000	3.8251	−1.25	0.215
* ω_{23}	−15.9000	3.8251	−4.16	0.000

Q: How to test $H_0: \mu_{12} = \mu_{13}$ ($\beta_2 = \beta_3$)

Note. We may get different conclusion if using different coding

EX1. Use sum codings to perform the regression analysis, and compare the results with Table 11 & with ANOVA in LNp.4-19.

EX2. Interpret the meanings of $\hat{\eta}$, α_i's, β_j's under main-effect-only model.

$\underset{\sim}{\mu} = X\underset{\sim}{\beta} \Rightarrow X^T\underset{\sim}{\mu} = (X^TX)\underset{\sim}{\beta}$
$\Rightarrow \underset{\sim}{\beta} = (X^TX)^{-1}X^T\underset{\sim}{\mu}$

- Significant effects: $\hat{\beta}_2$ (C&W and HT are different when media=bolt), $\hat{\beta}_3$ (C&W and P&O are different when media=bolt), $\hat{\omega}_{23}$ (difference between C&W and P&O varies from bolt to mandrel)

  B=1 B=2 A=1 B=1 B=3 B=1 B=1→3 A=1 A=2

- $\hat{\alpha}_2$ not significant: no difference between bolt and mandrel when plating=C&W.

Adjusted *p*-values

Why doing it? It follows the same concept as for multiple testing problem in the previous multiple comparison methods.

- The p-values in Table 11 (LNp.4-23) are for each individual effect.
- Since five effects (excluding η) are considered *simultaneously*, we should, strictly speaking, adjust the p-values when making a *joint* statement about the five effects.

  multiple comparison

In Bonferroni method for k tests, reject if
$p\text{-value} < \alpha \times \frac{1}{k}$
$\Leftrightarrow k \times p\text{-value} < \alpha$

- In the spirit of the Bonferroni method, again justified by the Bonferroni's inequality in Eqn. (2.15) of the textbook (LNp.3-11), we multiply the individual p-value by the number of tests to obtain **adjusted p-value**.

Note. Tukey method need independence assumption, which might not hold on $\hat{\eta}$, $\hat{\alpha}_i$'s, $\hat{\beta}_j$'s, $\hat{\omega}_{ij}$'s.
$\text{cov}(\hat{\underset{\sim}{\beta}}) = \sigma^2(X^TX)^{-1}$

- For $\hat{\omega}_{23}$, the adjusted p-value is $5 \times 0.0001 = 0.0005$, still very significant. The adjusted p-values, for $\hat{\beta}_2$ and $\hat{\beta}_3$ are smaller.

may increase if we use more tests to answer "how different" problem

Box-Whisker Plot of Residuals: Bolt Experiment

Exercise. Do the box plots for $y_{ij\ell}$'s, and compare their difference.

larger variation in the 10 obs. (media, plating) = (1, 3)

∵ saturated model $\sum_{\ell=1}^{n} \hat{\varepsilon}_{ij\ell} = 0$ for any (i, j)

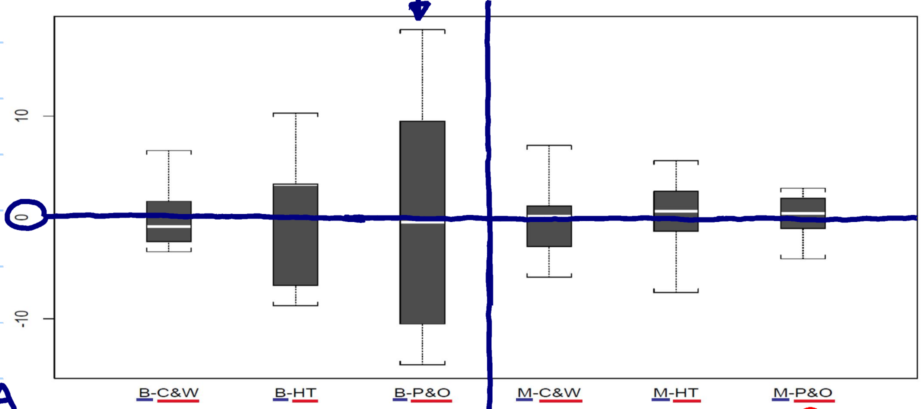


Figure 1: Box-Whisker Plots of Residuals, Bolt Experiment

- The plot suggests that
  - the constant variance assumption in model (5) (LNp.4-16) does not hold,
  - the variance of y for bolt is larger than that for mandrel. (A=1) (A=2)

These are aspects that cannot be discovered by regression analysis alone.

Multiple-Way Layout

more than 2 treatment factors, but no block factor

conceptual model: ← before exp't

$y \sim \beta_0 + \underbrace{A + B + C}_{\text{ME}} + \underbrace{A{*}B + A{*}C + B{*}C}_{\text{2fi's}} + \underbrace{A{*}B{*}C}_{\text{3fi's}} + \varepsilon$ (df: $I-1$, $J-1$, $K-1$, $(I-1)(J-1)(K-1)$)

need n (≥ 1) replicates for ANOVA

Table 12: ANOVA Table for Three-Way Layout

Source	df	Sum of Squares
A	$I-1$	$\sum_{i=1}^{I} nJK(\hat{\alpha}_i)^2$
B	$J-1$	$\sum_{j=1}^{J} nIK(\hat{\beta}_j)^2$
C	$K-1$	$\sum_{k=1}^{K} nIJ(\hat{\delta}_k)^2$
$A \times B$	$(I-1)(J-1)$	$\sum_{i=1}^{I}\sum_{j=1}^{J} nK(\widehat{(\alpha\beta)}_{ij})^2$
$A \times C$	$(I-1)(K-1)$	$\sum_{i=1}^{I}\sum_{k=1}^{K} nJ(\widehat{\alpha\delta})_{ik})^2$
$B \times C$	$(J-1)(K-1)$	$\sum_{j=1}^{J}\sum_{k=1}^{K} nI(\widehat{\beta\delta})_{jk})^2$
$A \times B \times C$	$(I-1)(J-1)(K-1)$	$\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} n(\hat{\gamma}_{ijk})^2$
residual	$IJK(n-1)$	$\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{l=1}^{n}(y_{ijkl} - \bar{y}_{ijk\cdot})^2$
total	$IJKn-1$	$\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{l=1}^{n}(y_{ijkl} - \bar{y}_{\cdots\cdot})^2$

Under sum codings

$\bar{y}_{i\cdots} - \bar{y}_{\cdots\cdot}$

∵ balanced design

$W_0, W_A, W_B, W_C, W_{A\times B}, W_{A\times C}, W_{B\times C}, W_{A\times B\times C}, \Omega^{\perp}$ mutually orthogonal

$\bar{y}_{ij\cdot\cdot} - \hat{\alpha}_i - \hat{\beta}_j - \hat{\eta} = \bar{y}_{ij\cdot\cdot} - \bar{y}_{i\cdots} - \bar{y}_{\cdot j\cdot\cdot} + \bar{y}_{\cdots\cdot}$

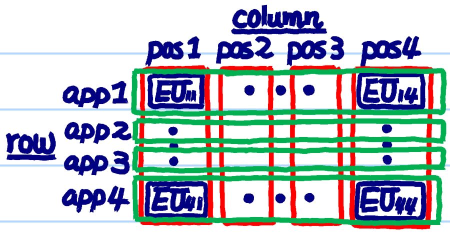
$\sum_i \gamma_{ijk} = \sum_j \gamma_{ijk} = \sum_k \gamma_{ijk} = 0$

- $\hat{\alpha}_i$, $\hat{\beta}_j$, $\widehat{(\alpha\beta)}_{ij}$, $\hat{\gamma}_{ijk}$, etc given in Eqn. (3.35) of the textbook.
- Estimation, F-tests, residual analysis are similar to those for two-way layout.

❖ **Reading**: textbook, 3.3, 3.5

Latin Square Design : Wear Experiment

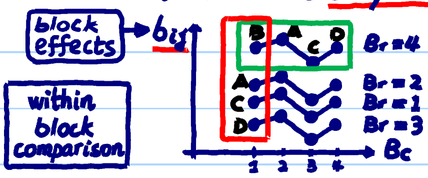
★ Exp'tal units: a position at an application (16 EUs: hetrogeneous)



- **Wear Experiment :** Testing the abrasion resistance of rubber-covered fabric,
  - ★response y: loss in weight over a period of time
  - ★one treatment factor : material type A, B, C, D, → qualitative, 4 levels (check LNp 4-15)
  - ★two blocking factors (crossing): row-column structure
    - (1) four positions (1, 2, 3, 4) on the tester, → qualitative, 4 levels, block size = 4
    - (2) four applications (1, 2, 3, 4; four different times for setting up the tester) → qualitative, 4 levels, block size = 4

do not work for block factors with nesting structure.

Assume block factors have only ME

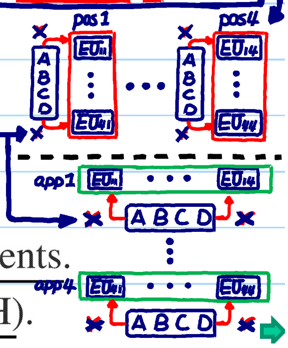


df. of the residuals in this RBD $= (k-1)^2 \geq k-1$ (if $k \geq 2$) ⇒ may accommodate one more k-level factor

- **Latin square (LS) design of order k :** Each of the k Latin letters (i.e., treatments) appears once in each of k rows and once in each of k columns. ← block factor

RBD → $(k!)^k$ cf. $(k!)^2$ ← level permutation of the three factors

more restriction on randomization than RBD



- It is an extension of RBD to accommodate *two* blocking factors. ← 1 block factor, 1 treatment factor
- Randomization applied to assignments to rows, columns, treatments.

(Collection of Latin Square Tables given in Appendix 3A of WH).

Wear Experiment : Design and Data

Compare τ_A vs. τ_B (within block comparison)
$\bar{y}_A - \bar{y}_B$
column, row — block effects are removed in residuals
It's achieved because each treatment levels appear equally often in rows and columns

Table 13: Latin Square Design (columns correspond to positions, rows correspond to applications and Latin letters correspond to materials), Wear Experiment

Application (row)	Position (column) 1	2	3	4
1	C	D	B	A
2	A	B	D	C
3	D	C	A	B
4	B	A	C	D

★ block factor — Design Matrix — treatment factor — randomization ⇔ level permutation

Data Matrix

y	Pos	App	Mat
235	1	1	C
	1	2	A
	1	3	D
	1	4	B
⋮	2	1	D
	2	2	B
	2	3	C
	2	4	A
⋮	3	1	B
	3	2	D
	3	3	A
	3	4	C
⋮	4	1	A
	4	2	C
	4	3	B
225	4	4	D

full factorial design — cf. — fractional factorial design

any 2 columns in design matrix are said to be orthogonal, if all level combinations of the 2 columns appear equally often

How to construct? → orthogonal array (OA) of strength 2 (textbook, p323)

Table 14: Weight Loss Data, Wear Experiment

Application	Position 1	2	3	4
1	235	236	218	268
2	251	241	227	229
3	234	273	274	226
4	195	270	230	225

# of different level combinations of the three factors $= 4 \times 4 \times 4 = 64$. But, only 16 of them appear in the design matrix.

say, (1, 1, A)? ← (Q: Can we perform all 64 level combinations?)

Model for Latin Square Design

(k-1) parameters

Conceptual model: $y \sim B_0$ + row + column + treatment + ε ← main-effect-only model ⇒ no interaction (Why?)

before exp't

- Model: sum codings → W_r, W_c, W_t ⇒ $W_0 \perp W_r \perp W_c \perp W_t \perp \Omega^{\perp}$ Why?

over-parameterized →
$$y_{ijl} = \eta + \alpha_i + \beta_j + \tau_l + \varepsilon_{ijl},$$
(k) (k) (k)

∵ ① DM is an OA of strength 2, ② main-effect-only model ③ sum codings

where $i, j, l = 1, \ldots, k$, and

Note. l is a function of (i,j) → l = Latin letter in the (i, j) cell of the LS,

α_i = ith row effect,

β_j = jth column effect,

Note. distinction of orthogonality in design & model matrices

τ_l = lth treatment (i.e., Latin letter) effect,

ε_{ijl} are independent $N(0, \sigma^2)$.

Note. If there exist interactions btwn the 2 block factors (or in the nesting case) ← LNp.15
⇒ conceptual model: $y \sim B_0$ + row + column + row:column + treatment + ε ← $(k-1)^2$ parameters
⇒ can treat the 2 block factors as 1 block factor with k^2 levels
⇒ But, in LS, no d.f. left for treatment
⇒ need to have multiple EUs in each of k^2 blocks
⇒ Then, can use {RBD or BIBD

- There are only k^2 values in the triplet (i, j, l) dictated by the particular LS; ← Note. not k^3

this set is denoted by S.

Sum codings

$$y_{ijl} = \hat{\eta} + \hat{\alpha}_i + \hat{\beta}_j + \hat{\tau}_l + r_{ijl}$$
$$\underset{\sim}{y} = \underbrace{\bar{y}_{\cdots}}_{P_{W_0}(\underset{\sim}{y})} + \underbrace{(\bar{y}_{i\cdot\cdot} - \bar{y}_{\cdots})}_{P_{W_r}(\underset{\sim}{y})} + \underbrace{(\bar{y}_{\cdot j\cdot} - \bar{y}_{\cdots})}_{P_{W_c}(\underset{\sim}{y})} + \underbrace{(\bar{y}_{\cdot\cdot l} - \bar{y}_{\cdots})}_{P_{W_t}(\underset{\sim}{y})}$$
$$+ (y_{ijl} - \bar{y}_{i\cdot\cdot} - \bar{y}_{\cdot j\cdot} - \bar{y}_{\cdot\cdot l} + 2\bar{y}_{\cdots}),$$

∵ orthogonality

- ANOVA decomposition: similar formula (see (3.40) of WH)

ANOVA for Latin Square Design

anova($y \sim B_0$ + row + column + treatment) ← In sequential ANOVA, block factors should always be put before treatment factor

Table 15: ANOVA Table for Latin Square Design

	Source	Degrees of Freedom	Sum of Squares	MS =?	E(MS) =?
W_r	row	$k-1$	$k\sum_{i=1}^{k}(\bar{y}_{i\cdot\cdot} - \bar{y}_{\cdots})^2$		
W_c	column	$k-1$	$k\sum_{j=1}^{k}(\bar{y}_{\cdot j\cdot} - \bar{y}_{\cdots})^2$		
W_t	treatment	$k-1$	$k\sum_{l=1}^{k}(\bar{y}_{\cdot\cdot l} - \bar{y}_{\cdots})^2$		
$\Omega^{\perp}$	residual	$(k-1)(k-2)$	$\sum_{(i,j,l)\in S}(y_{ijl} - \bar{y}_{i\cdot\cdot} - \bar{y}_{\cdot j\cdot} - \bar{y}_{\cdot\cdot l} + 2\bar{y}_{\cdots})^2$	$\hat{\sigma}^2$	
$\mathbb{R}^N \ominus W_0$	total	$k^2 - 1$	$\sum_{(i,j,l)\in S}(y_{ijl} - \bar{y}_{\cdots})^2$		

$\Omega \ominus W_0$ = $W_r \oplus W_c \oplus W_t$

Sum of Squares: $\|P_{W_i}(\underset{\sim}{y})\|^2$

use (P4) in LNp.2-36

Ω_t: $y \sim$ row + column + treatment + ε
ω_t: $y \sim$ row + column + ε

B_0

$k^2 - 1 - 3(k-1) = (k-1)(k+1-3)$
$= (k-1)^2 - (k-1)$ ← d.f. of residuals in RBD

$\hat{\sigma}^2$ estimated from an RSS not containing block variation

Table 16: ANOVA Table, Wear Experiment

Source	Degrees of Freedom	Sum of Squares	Mean Squares	F
application	3	986.5	328.833	5.37
position	3	1468.5	489.500	7.99
material	3	4621.5	1540.500	25.15
residual	6	367.5	61.250	

4-1

largest SS

$\hat{\sigma}^2$ → 61.250

F-Test and Multiple Comparisons

F-test & multiple comparison for LSD use the same principles as in RBD, except that 2 block factors in the former while 1 in the latter

$\Omega_t: y \sim row + column + treatment$
$\omega_t: y \sim row + column$

- $H_0: \tau_1 = \cdots = \tau_k$, can be tested by using the F-statistic

$$F = \frac{\overbrace{SS_t}^{RSS_{\omega_t}-RSS_{\Omega_t}}/\overbrace{(k-1)}^{df_{\omega_t}-df_{\Omega_t}}}{\underbrace{SS_r}_{RSS_{\Omega_t}}/\underbrace{(k-1)(k-2)}_{df_{\Omega_t}}},$$

Note. ① $\bar{y}_{\cdot\cdot j} - \bar{y}_{\cdot\cdot i}$ is not biased by blocks ($\because$ orthogonality)
② $\hat{\sigma}^2 = RSS_{\Omega_t}/df_{\Omega_t}$ does not contain block variation.

The F-test rejects H_0 at level α if $F > F_{k-1,(k-1)(k-2),\alpha}$.

$Y = X_1\beta_1 + X_2\beta_2 + \varepsilon$

$\cancel{H_1X_2\beta_2}$ ① $\quad \cancel{(I-H_1)X_2\beta_2}$ ②

- If H_0 is rejected, multiple comparisons of the τ_j should be performed. t-statistics for making multiple comparisons :

$H_0^{ij}: \tau_i = \tau_j \; (\tau_j - \tau_i = 0)$
– for specific (i,j) → use t-dist. for null
– for all (i,j)'s

$$t_{ij} = \frac{\overbrace{\bar{y}_{\cdot\cdot j} - \bar{y}_{\cdot\cdot i}}^{\hat{\tau}_j - \hat{\tau}_i} - 0}{\hat{\sigma}\sqrt{1/k + 1/k}},$$

$Var(\bar{y}_{\cdot\cdot j} - \bar{y}_{\cdot\cdot i}) = \sigma^2/k + \sigma^2/k$

where $\hat{\sigma}^2$ is the mean square error in the ANOVA table. $\hat{\sigma}^2 = \frac{SS_r}{(k-1)(k-2)}$

- At level α, the Tukey multiple comparison method identifies "treatments i and j as different" if

$$|t_{ij}| > \frac{1}{\sqrt{2}} q_{k,(k-1)(k-2),\alpha}.$$

Analysis Results

- The p-values for application (row) and position (column) are 0.039 (=$Prob(F_{3,6} > 5.37)$) and 0.016 (=$Prob(F_{3,6} > 7.99)$), respectively. This indicates that blocking is important.

- The treatment factor (material) has the most significance as indicated by a p-value of 0.0008 (= $Prob(F_{3,6} > 25.15)$).

- With k=4 and $(k-1)(k-2)$=6, the critical value for the Tukey multiple comparison method is

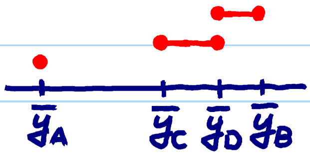
$$\frac{1}{\sqrt{2}} q_{4,6,0.05} = \frac{4.90}{\sqrt{2}} = 3.46$$

at the 0.05 level.

- By comparing the multiple comparisons t-statistics given in Table 17 (LNp.4-33) with 3.46, material A and B, A and C, A and D and B and C are identified as different at 0.05 level.

Multiple Comparisons Tables

Table 17: Multiple Comparison t-statistics, Wear Experiment



A vs. B	A vs. C	A vs. D	B vs. C	B vs. D	C vs. D
−8.27	−4.34	−6.37	3.93	1.90	−2.03

anova($y \sim B_0$ + treatment) ← treated as one-way layout

Table 18: ANOVA Table (Ignoring Blocking), Wear Experiment ↔ cf. Table 16 (LNp. 30)

Source	Degrees of Freedom	Sum of Squares	Mean Squares	F
material	3	4621.5	1540.500	6.55
residual	12	2822.5	235.21	

same ← cf.

235.21 ← $\hat{\sigma}^2$ ← estimated from an RSS containing block variation

6 ← cf. (12); (2822.5) = 986.5 + 1468.5 + 367.5; cf. (235.21) → 61.25

- Effectiveness of blocking:
  - With blocking, $Pr(F_{3,6} > 25.15) = 0.0008$.
  - Without blocking, $Pr(F_{3,12} > 6.55) = 0.007$.

  become less significant when block factors are significant

Therefore blocking can make a difference in decision making if treatment effects are smaller.

Q. What if some block factor is found insignificant.

❖ **Reading**: textbook, 3.6