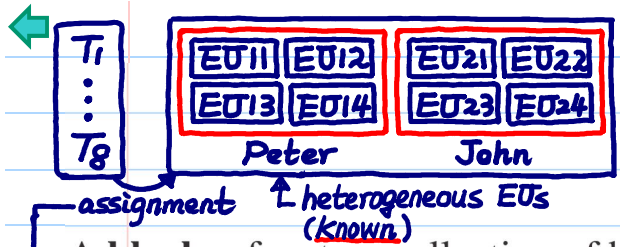
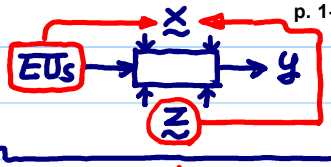


for known $\underset{\sim}{z}$

Blocking



Fitted model: $Y = X\beta + Z\gamma + \varepsilon$ cf. $Y = X\beta + \varepsilon'$ (LNp.1-19)

① $Var(\varepsilon') \geq Var(\varepsilon)$
② $\hat{\beta}$ not biased by $Z\gamma$

A **block** refers to a collection of homogeneous units. Effective blocking : larger between-block variations than within-block variations.

(Examples: hours, batches, lots, street blocks, pairs of twins.)

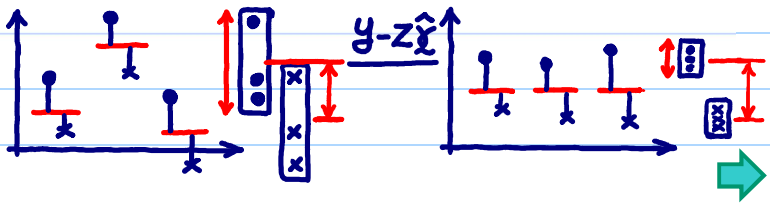
→ can try to achieve $X \perp Z$ by design ⇒ orthogonality

- Run and compare treatments within the same blocks. (Use randomization within blocks.) It can eliminate block-block variation and reduce variability of treatment effects estimates.

∵ $Z\gamma$ is put in the systematic part of the fitted model.

- **Block what you can and randomize what you cannot.**

- Discuss **typing experiment** to demonstrate possible elaboration of the blocking idea. See LNp.1-24



Blocking

homogeneous EUs or same z value

a known z

Q: If operator effect is identified as significant before exp't, what can we do?

	A	B	C	operator	
1	low	low	low	Peter	EU11
2	low	low	high	John	EU21
3	low	high	low	John	EU22
4	low	high	high	Peter	EU12
5	high	low	low	John	EU23
6	high	low	high	Peter	EU13
7	high	high	low	Peter	EU14
8	high	high	high	John	EU24

Known $\underset{\sim}{z}$ or systematic structure in EUs

randomization result in LNp.1-21

cf. Orthogonality

block factor : factors that are controllable and may influence the response but in which we are ***not directly interested***. (cf. treatment factors) $\underset{\sim}{x}$

& usually no interaction with $\underset{\sim}{x}$

Examples of blocking factors:

lot-to-lot, brand-to-brand, operator-to-operator, day-to-day, ...

block what you can & randomize what you cannot

Illustration: Typing Experiment

- To compare two keyboards A and B in terms of typing efficiency. Six manuscripts 1-6 are given to the same typist.
- Several designs (i.e., orders of test sequence) are considered:

1. ★extra variable (factor or unknown): tying order
   2 levels: I, II, → block factor.

   1. $\underset{\text{I}}{A},\underset{\text{II}}{B}$, 2. $\underset{\text{I}}{A},\underset{\text{II}}{B}$, 3. $\underset{\text{I}}{A},\underset{\text{II}}{B}$, 4. $\underset{\text{I}}{A},\underset{\text{II}}{B}$, 5. $\underset{\text{I}}{A},\underset{\text{II}}{B}$, 6. $\underset{\text{I}}{A},\underset{\text{II}}{B}$.

   (A always followed by B, why bad ?)

2. Randomizing the order leads to a new sequence like this

   1 $\underset{\text{I}}{A},\underset{}{B}$, 2. $\underset{\text{II}}{B},\underset{}{A}$, 3 $\underset{\text{I}}{A},\underset{}{B}$, 4. $\underset{\text{II}}{B},\underset{}{A}$, 5 $\underset{\text{I}}{A},\underset{}{B}$, 6 $\underset{\text{I}}{A},\underset{}{B}$.

   (an improvement, but there are four with A,B and two with B,A. Why is this not desirable? Impact of *learning effect*.)

3. *Balanced randomization*: To mitigate the learning effect, randomly choose three with A,B and three with B,A. (Produce one such plan on your own).

4. Other improved plans?

- factors
  - ★keyboard: 2 levels: A, B; treatment factor
  - ★manuscript: 6 levels: 1~6; block factor
- EUs: a manuscript type once; 6 x 2 = 12 EUs

treated as unknown

treated as block factor

orthogonality

only one typist. What if several typists included? representative of typist population

1. (known: manu., order; unknown: KB)

KB	manu.	order
A	1	I
B	1	II
A	2	I
B	2	II
A	3	I
B	3	II
A	4	I
B	4	II
A	5	I
B	5	II
A	6	I
B	6	II

confounded

2. (known: manu., order; unknown: KB)

KB	manu.	order
A	1	I
B	1	II
B	2	I
A	2	II
A	3	I
B	3	II
B	4	I
A	4	II
A	5	I
B	5	II
A	6	I
B	6	II

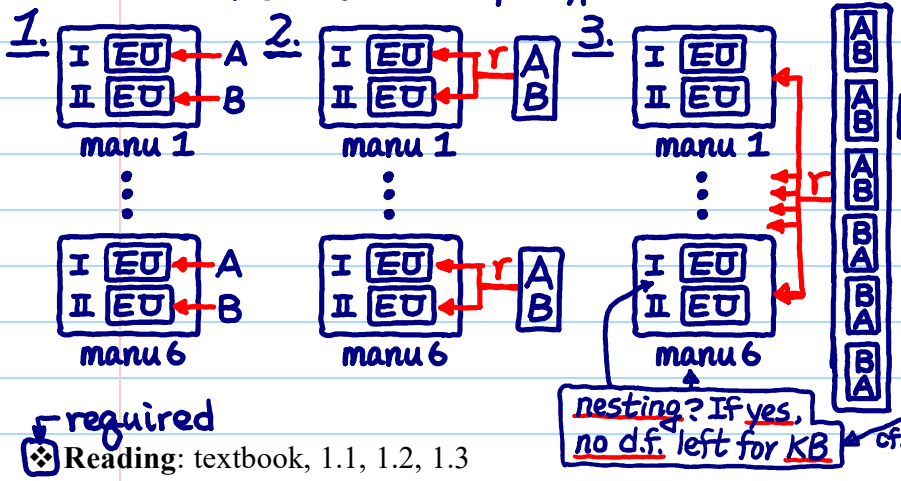
slightly confounded

3. (known: manu., order; known: KB)

KB	manu.	order
A	1	I
B	1	II
B	2	I
A	2	II
B	3	I
A	3	II
A	4	I
B	4	II
B	5	I
A	5	II
A	6	I
B	6	II

randomly chosen

an EU = a manuscript typed once



★typist: block factor
an EU = a manuscript typed once by a typist

crossing	typist 1	...	typist k
manu 1	I EU, II EU	...	I EU, II EU
⋮	⋮	⋱	⋮
manu 6	I EU, II EU	...	I EU, II EU

Q: How to assign A, B to EUs?

required

Reading: textbook, 1.1, 1.2, 1.3