(1, 3pts) This is a split-split-plot design with 4 blocks. Each block contains 3 whole plots, each whole plot contains 3 subplots, and each subplot contains 4 sub-subplots. This is a nesting struture. Specifically,

- a whole plot is a unit of antibiotic assigned to a technician,
- a subplot is a formulation of a dosage strength, and
- a sub-subplot is a quarter of a formulated dosage for a specific wall thickness,

with A being the whole-plot factor, B the subplot factor, and C is the sub-subplot factor. In this experiment, 3 units of antibiotics are used (1 unit for 1 technician) each day. Each technician must perform the formulation process 3 times and encapsulate the formulated medication 12 times daily. The graph you draw should incorporate these details.

(2, 2pts) Because only the factor C is the sub-subplot factor, any effects involving C would be tested against the smallest experimental errors, including main effect C, two-factor interactions $A : C$ and $B : C$, and three-factor interaction $A : B : C$.

(3, 2pts) The total degrees of freedom (dfs) in this experiment is the run size, calculated as $4 \times 3 \times 3 \times 4 = 144$. For the analysis at the sub-subplot level, we can consider it as a randomized block design with $4 \times 3 \times 3 = 36$ blocks. Therefore, the remaining dfs for the sub-subplot level is $144 - 36 = 108$. Considering the effects in the answer to problem (2), which consume $3\ (C) + 2 \times 3\ (A : C) + 2 \times 3\ (B : C) + 2 \times 2 \times 3\ (A : B : C) = 27$ dfs, there are $108 - 27 = 81$ dfs left for estimating the smallest experimental errors.

(4, 2pts) Below is a scenario of a split plot design. This design includes 4 blocks (days), each containing 3 whole plots (3 units of antibiotic), and each whole plot consists of 12 subplots. Factor A is a whole plot factor, while factors B and C are subplot factors. Notice that each dosage formulation is paired with only one wall thickness, unlike the 1st experiment where each formulation is linked to 4 wall thicknesses. In this experiment, 3 units of antibiotics are used (1 unit for 1 technician) each day. Each technician must perform the formulation process 12 times and encapsulate the formulated medication 12 times daily.

An alternative scenario of a split plot design can be outlined as follows. This design includes 4 blocks (days), each containing 9 whole plots, and each whole plot consists of 4 subplots. Factors A and B are whole plot factors, while factor C is a subplot factors. Notice that each unit of antibiotic is paired with only one dosage formulation, unlike the 1st experiment where each unit is linked to 3 dosage formulations. In this experiment, 9 units of antibiotics are used (3 units for 1 technician) each day. Each technician must perform the formulation process 3 times and encapsulate the formulated medication 12 times daily.

(5, 2pts) Here is a scenario of a randomized block design. This design includes 4 blocks (days), each with 36 homogeneous experimental units (EUs), where each EU represents a unit of antibiotic. Every day, every level combination of A, B, and C is randomly assigned to one EU, meaning that one unit of antibiotic is utilized for each execution of a level combination of A, B, and C. In this experiment, 36 units of antibiotics are used (9 units for 1 technician) each day. Each technician must perform the formulation process 12 times and encapsulate the formulated medication 12 times daily.

(6, 2pts) The first design plan requires only 12 units of antibiotic, whereas the third design plan requires 144 units. The first experiment conserves more resources, but sacrifices the accuracy of estimating main effect A (or effects invloving only whole-plot and/or sub-plot factors) and its testing power.

(7, 2pts) The design (i) is of resolution III, while the design (ii) is of resolution IV.

(8, 2pts) Whether evaluated by the resolution criterion or the aberration criterion, design (ii) is the preferable design.

(9, 2pts) The defining contrast subgroups of designs (i) and (ii) are $\{I, 12345, 1246, 356\}$ and $\{I, 1235, 1246, 3456\}$ respectively. After full foldover, the words with odd lengths are eliminated. Therefore, design (i) transitions into a 2^{6-1} design with $I = 1246$, whereas design (ii) remains the original 2^{6-2} design with 2 replicates. Because design (i) offers more dfs for effect estimation after foldover, it is the preferred option.

(10, 2pts) 2-factor interactions 14, 24, 34, and 45. FYI, the aliasings involving only 2-factor interactions in design (ii) are: $12 = 35 = 46$, $13 = 25$, $14 = 26$, $23 = 15$, $24 = 16$, $34 = 56$, $45 = 36$.

(11, 2pts) Among all alias sets generated by design (ii), only 2 of them contain no main effects or two-factor interactions, namely $134 = 245 = 236 = 156$ and $234 = 145 = 136 = 256$. If the block effect is confounded with either of these alias sets, no main effects or two-factor interactions will be sacrificed due to blocking.

(12, 2.5pts) 3 main effects: 1, 2, 4, and 6 two-factor interactions: 13, 15, 23, 25, 34, 45. FYI, the aliasings involving only main effects and 2-factor interactions in design (i) are: 1, 2, $3 = 56$, 4, $12 = 46$, 13, $14 = 26$, 23, $24 = 16$, 34, 45, $35 = 6$, 25, 15, $5 = 36$.

(13, 2pts) The 2-factor interactions sacrificed due to blocking are $B_1 = 16 = 24$, $B_2 = 26 = 14$, and $B_1B_2 = 12 = 46$. None of these 2-factor interactions are clear before blocking, so all effects listed in the answer to problem (12) remain clear.

(15, 2pts) It is a single array because, for every level combinations of A, B, C, and D, the noise factor E has only one setting, either $+1$ or -1. Another way to determine whether it is a cross array is by examining the part of the word $ABCDE$ that includes only control factors, namely $ABCD$. Since $ABCD$ is not included in the defining contrast subgroup of this design, it is a single array, not a cross array.

(16, 2pts) No. To use location-dispersion modeling, each level combination of A, B, C, and D must have multiple settings of E to generate multiple y-values for calculating

$\bar{y}_i$'s and $\log s_i^2$'s. However, in this design, each level combination of A, B, C, and D corresponds to only one setting of E, making it impossible to obtain multiple y-values to calculate $\log s_i^2$.

(17, 2.5pts) Assign probability 1/2 to each level, -1 and $+1$, of x_E and treat x_E as a random variable. This leads to $E(x_E) = 0$ and $Var(x_E) = 1$. The location model can subsequently be derived as follows:

$$\begin{aligned} E_{x_E}(y) &= E_{x_E}(7.64 + 0.11x_A - 0.09x_B + 0.05x_C + 0.13x_E + 0.08x_B x_E) \\ &= 7.64 + 0.11x_A - 0.09x_B + 0.05x_C + 0.13E_{x_E}(x_E) + 0.08x_B E_{x_E}(x_E) \\ &= 7.64 + 0.11x_A - 0.09x_B + 0.05x_C. \end{aligned} \tag{I}$$

The transmitted variance model (i.e., the dispersion model) can be derived as follows:

$$\begin{aligned} Var_{x_E}(y) &= Var_{x_E}(7.64 + 0.11x_A - 0.09x_B + 0.05x_C + 0.13x_E + 0.08x_B x_E) \\ &= Var_{x_E}[(0.13 + 0.08x_B)x_E] \\ &= (0.13 + 0.08x_B)^2 Var_{x_E}(x_E) \\ &= (0.13 + 0.08x_B)^2 = 0.0169 + 0.0208x_B + 0.0064x_B^2 = 0.0233 + 0.0208x_B, \end{aligned}$$

where the last equality holds because $x_B^2 = 1$. Following the 2-step procedure for a nominal-the-best problem, we begin by minimizing the dispersion model $Var_{x_E}(y)$, which indicates that x_B should be set at -1. After substituting $x_B = -1$ into the location model $E_{x_E}(y)$, we obtain the following result:

$$E(y) = 7.73 + 0.11x_A + 0.05x_C.$$

Using the equation above and noting that A and C are quantitative factors, any (x_A, x_C) combinations satisfying $E(Y) = 7.6$, for instance $(x_A, x_C) = (-1, -2/5)$, are suitable settings to consider. It's advisable to select settings that lie within the experimental region.

(18, 2pts) Since the defining contrast subgroup $\{I, ABCDE\}$ contains no word composed solely of the letters A, B, C, and E, the projected design is a 2^4 full factorial design, which can be represented as a $2^3 \times 2^1$ cross array, where 2^3 is the control array and 2^1 is the noise array. This cross array allows us to calculate $\bar{y}_i$'s and $\log s_i^2$'s at each level combinations of A, B, and C, enabling analysis through location-dispersion modeling.

(19, 2pts) For location-dispersion modeling, constructing a cross array is necessary. To estimate all effects simultaneously (3 main effects and 3 two-factor interactions) in either location or dispersion models, the control array needs to have a resolution of at least V. With only 3 control factors, achieving a resolution of at least V necessitates using a 2^3 full factorial design. For the noise array, with 2 noise factors, a 2^2 full factorial design is required; otherwise, the main effects of the 2 noise factors would be aliased in a 2-run 2^{2-1} design. Thus, the complete cross array is a $2^3 \times 2^2 = 2^5$ full factorial design, consisting of 32 runs.

(20, 2pts) In constructing a design for response modeling, both control and noise factors can be treated as treatment factors. To estimate all these effects (5 main effects and

10 2-factor interactions) simultaneously, the single array must have a resolution of at least V. With 5 factors and 16 runs (half fraction), a 2^{5-1} fractional factorial design needs one generator. Using $I = ABCDE$ as the generator ensures that all main effects and two-factor interactions are clear. For 5 factors and 8 runs (1/4 fraction), a 2^{5-2} fractional factorial design requires two generators, making it impossible to maintain all words in the defining contrast subgroup to have lengths of at least five. Hence, 16 runs is the minimal run size that meets these requirements.