(1, 3pts) There are 24 whole-plot experimental units (replications) and 48 sub-plot experimental units (repetitions). Both M and T are whole-plot factors. So, the ANOVA table should look like:

Source	d.f.	SS	MS
M	3		
T	2		
$M \times T$	6		
Whole-plot error	12		
Sub-plot error	24		
Total	47		

(2, 2pts) *response* $\sim M + T + M \times T + \epsilon^{WP} + \epsilon^{SP}$, where ϵ^{WP} is a 48×1 vector with 24 distinct whole-plot errors, and ϵ^{SP} a 48×1 vector with 48 distinct sub-plot errors.

(3, 4pts) Because $E(\text{sub-plot-error MS}) = \sigma_S^2$, where σ_S^2 is the variance of sub-plot errors, the estimate of the sampling error variance component due to repetitions (i.e., 1) can be treated as the sub-plot-error MS. So, the ANOVA table is:

Source	d.f.	SS	MS
M	3	60	20
T	2	50	25
$M \times T$	6	30	5
Whole-plot error	12	58.8	4.9
Sub-plot error	24	24	1
Total	47	222.8	

(4, 2pts) The $M \times T$ interaction is a whole-plot effect, so it must be tested against whole-plot error MS. The test statistic is $5/4.9 = 1.02$, and the null distribution is an $F_{6,12}$ whose mean is 1.2. Because $1.02 < 1.2$, we do not reject the null of no $M \times T$ interaction.

(5, 3pts) For a main-effect-only (i.e., no interaction) model, the ANOVA table becomes:

Source	d.f.	SS	MS
M	3	60	20
T	2	50	25
Whole-plot error	18	88.8	4.933
Sub-plot error	24	24	1
Total	47	222.8	

So, the standard error is

$$\sqrt{\frac{1}{n} + \frac{1}{n}} \times \sqrt{\text{whole-plot-error MS}} = \sqrt{\frac{2}{48/3}} \times \sqrt{4.933} \approx 0.785,$$

where $n = 48/3 = 16$ is the number of observations of a treatment. The whole-plot-error MS is used in the calculation because T is a whole-plot effect. An alternative way to derive the answer is given below. For treatments A and B,

$$\begin{aligned}
Var(\bar{y}_A - \bar{y}_B) =& Var(\bar{y}_A) + Var(\bar{y}_B) = 2 \times Var(\bar{y}_A) \\
=& 2 \times \frac{1}{(48/3)^2} \\
& \times Var[2 \times (\text{sum of 8 whole-plot errors}) + (\text{sum of 16 sub-plot errors})] \\
=& 2 \times \frac{1}{16^2} \times (4 \times 8\sigma_W^2 + 16\sigma_S^2) = 2 \times \frac{1}{16} \times (2\sigma_W^2 + \sigma_S^2)
\end{aligned}$$

where σ_W^2 and σ_S^2 are the variances of whole-plot and sub-plot errors respectively. Because E(whole-plot-error MS) $= 2\sigma_W^2 + \sigma_S^2$, we can use whole-plot-error MS as an unbiased estimate of $2\sigma_W^2 + \sigma_S^2$.

(6, 2pts) In the defining contrast subgroup of d_1, only the words 459, 4678, and 56789 are composed of factors 4, 5, 6, 7, 8, 9. So, the defining contrast subgroup of the projected design is $I = 459 = 4678 = 56789$, which is a 2^{6-2}_{III} design.

(7, 1pt) No, because all the main effects appear in at least one word of length 3. Any main effects is aliased with at least one 2-factor interaction.

(8, 2pts) The main effect 9 has a more serious aliasing than the other main effects because the former is aliased with *four* 2-factor interactions while any of the latter is aliased with only *one* two-factor interaction.

(9, 2pts) The defining contrast subgroup of D_1 can be obtained from that of d_1 by removing all the words involving 5. The resolution of D_1 is III because a word like 369 (or 279, 189) is still retained in its defining contrast subgroup.

(10, 2pts) The words of length three in the defining contrast subgroup of D_1 are 369, 279, 189. The clear main effects are 4 and 5. All the 2-factor interactions involving 5 are clear because there is no three-letter or four-letter words involving 5 in the defining contrast subgroup of D_1.

(11, 3pts) Yes, it is achievable. To make all main effects clear, the resolution must be at least IV. That is, the fold-over design (the 2nd design) must have negative signs on all the length-three words. This can be achieved by switching the signs in every columns of d_1 (i.e., $1 \to -1$, $2 \to -2$, ..., $9 \to -9$) to generate the fold-over design. The resulting augmented design by combing d_1 and the fold-over design would have a defining contrast subgroup containing only the even-length words in the defining contrast subgroup of d_1. In this case, the augmented design would be a resolution IV design.

(12, 2pts) Treat x_F as a random variable and assign probability 1/2 on each of the levels -1 and $+1$ of x_F. Then, we have $E(x_F) = 0$ and $Var(x_F) = 1$. The location model can be derived by

$$\begin{aligned}
E_{x_F}(y) &= E_{x_F}(7.64 + 0.11x_A - 0.09x_B + 0.05x_C - 0.13x_F + 0.08x_Bx_F) \\
&= 7.64 + 0.11x_A - 0.09x_B + 0.05x_C - 0.13E_{x_F}(x_F) + 0.08x_BE_{x_F}(x_F) \\
&= 7.64 + 0.11x_A - 0.09x_B + 0.05x_C. \qquad \text{(I)}
\end{aligned}$$

(13, 3pts) Treat x_F as a random variable and assign probability 1/2 on each of the levels -1 and $+1$ of x_F. Then, we have $E(x_F) = 0$ and $Var(x_F) = 1$. The transmitted variance model (i.e., the dispersion model) can be derived by

$$\begin{aligned} Var_{x_F}(y) &= Var_{x_F}(7.64 + 0.11x_A - 0.09x_B + 0.05x_C - 0.13x_F + 0.08x_Bx_F) \\ &= Var_{x_F}[-(0.13 - 0.08x_B)x_F] \\ &= (0.13 - 0.08x_B)^2 Var_{x_F}(x_F) \\ &= (0.13 - 0.08x_B)^2 = 0.0169 - 0.0208x_B + 0.0064x_B^2 = 0.0233 - 0.0208x_B, \end{aligned}$$

where the last equality holds because $x_B^2 = 1$.

(14, 3pts) We start by minimizing the dispersion model $Var_{x_F}(y)$ to determine the value of x_B to be set at $+1$. Substituting $x_B = +1$ into the location model $E_{x_F}(y)$, we obtain

$$E(y) = 7.55 + 0.11x_A + 0.05x_C.$$

In the above equation, all (x_A, x_C) combinations that satisfy $E(Y) = 7.6$, such as $(x_A, x_C) = (5/11, 0)$, are valid settings to consider. Notice that it is preferable to choose settings that fall within the experimental region.

(15, 2pts) To simultaneously estimate all these effects, both the resolutions of the control array and the noise array needs to be at least V. For the control array, since there are only 4 control factors, to achieve resolution $\geq$ V, only a 2^4 full factorial design can be used. Similarly, for the noise array, with only 2 noise factors, to achieve a resolution $\geq$ V, only a 2^2 full factorial design can be used. Therefore, the entire cross array is a $2^4 \times 2^2 = 2^6$ full factorial design with 64 runs.

(16, 2pts) To simultaneously estimate all these effects, the resolution of the single array needs to be at least V. For 6 factors and 32 runs, a 2^{5-1} fractional factorial design can be used, which requires one generator. We can choose a word of length 5 as the generator, such as $I = ABCDE$, or a word of length 6, such as $I = ABCDEF$. Both of them have a resolution $\geq$ V, ensuring that all main effects and two-factor interactions are clear. Therefore, all the effects of interest can be simultaneously estimated using these single arrays.

(17, 2pts) Among all the 2^{6-1} FFDs with resolution at least V, the generator with the longest wordlength is $ABCDEF$. The generators for the other FFDs are words of length 5. Therefore, the FFD with $I = ABCDEF$ is the minimum abberation design.