NTHU STAT 5510 Final Examination June 15, 2023

Instructions: Attempt all questions. Short and specific answers are preferred. Given explanation when requested to do so, but keep it as short and simple as possible. Give only one answer to each question – if you give alternative answers, the worst answer will be graded.

Question A.

A chemist wants to compare 3 treatments. The experimental material he plans to use comes from 4 different manufacturers. He expects systematic differences among the material from the different manufactures. Moreover, he is also interested in finding out whether differences between treatments depend on the manufacturer. For each manufacturer, there is sufficient experimental material for 12 experimental units. Suppose he comes to you with the following partial ANOVA table:

Source	d.f.	SS	MS
Manufacturers (M)			20.0
Treatments (T)			25.0
Interaction (M×T)			5.0
Error			2.3
Total	47	222.8	

The chemist claims that he has used 4 replications per treatment, per manufacturer. After some questioning you find out that the four "replications" are actually two replications (i.e., two experimental units) and two repetition measurements for each experimental unit (i.e., two readings from the same experimental units).

(1) (3 pts) In this case, what should the ANOVA really look like? Give only its sources of variation and degrees of freedom.

(2) (2 pts) Give the linear model of the ANOVA in question (1). **[Hint**. The error structure of this ANOVA model is different from the one for the partial ANOVA table above. The ANOVA model in this question has a split-plot structure.**]**

Unfortunately, the computer file with his original data was destroyed. In order to correct the situation and come up with a reasonable ANOVA table, you devise a small experiment which will allow you to obtain an estimate of the sampling error variance component due to repetitions. Suppose this estimate equals 1.

(3) (4 pts) Assuming that 1 is a reasonable estimate for the actual experiment, i.e., assuming that this value was obtained from the actual experiment, complete now the ANOVA table given in problem (1) by filling in SSs and MSs.

(4) (2 pts) Based on the ANOVA in problem (3), test the null hypothesis that there is no interaction between M and T. [**Hint**. The mean of an F_{d_1,d_2} distribution is $d_2/(d_2-2)$.]

(5) (3 pts) Suppose it was known before the experiment that there is no interaction between M and T. Based on a *no interaction version* of the ANOVA in problem (3), give the standard error for a simple treatment comparison (i.e., the differences between the estimates of the means of any two treatments).

Question B.

A 16 run experiment employs the 2^{9-5}_{III} design with **5=123**, **6=124**, **7=134**, **8=234**, and **9=1234**. Denote the design by d_1. Its defining contrast subgroup is:

{**I, 459, 369, 279, 189, 1235, 1246, 3456, 1347, 2457, 2367, 1567, 2348, 1458, 1368, 2568, 1278, 3578, 4678, 12349, 12569, 13579, 14679, 23589, 24689, 34789, 56789, 2345679, 1345689, 1245789, 1236789, 12345678**}

(6) (2 pts) When d_1 is projected onto factors 4, 5, 6, 7, 8, 9, what is the defining contrast subgroup of the projected design and what is its resolution?

(7) (1 pt) Are there any clear main effects in the design d_1? Explain.

(8) (2 pts) The factor 9 appears in all the words of length three but the other factors only appear once in these length-three words. What does this imply about the severity of aliasing for the main effects? Explain.

(9) (2 pts) The fold-over technique can be applied as follows. Suppose that factor 5 is the most important factor. Obtain the second design d_2 by switching the sign in the column for factor 5 in d_1. Combine d_1 and d_2 to obtain an augmented 2^{9-4} design, denoted by D_1. Describe the defining contrast subgroup of D_1. What is the resolution of D_1? Explain.

(10) (2 pts) Identify the clear main effects in D_1. Which of the two-factor interactions involving 5 are clear in D_1? Explain.

(11) (3 pts) Is it possible to construct an augmented design in which all the main effects are clear using the fold-over technique? If possible, please proceed. If not, explain why.

Question C.

In an experiment, six 2-level factors A, B, C, D, E, F were studied to determine their effects. The five factors A, B, C, D, E were treated as control factors and the factor F as a noise factor. The two levels of each factors were coded as −1 and +1. A cross-array design was conducted with the inner array for the control factors being a 2^{5-1} fractional factorial design with generator $E = ABCD$, and the outer array for the single noise factor being a 2^1 design. A fitted model of the response y was obtained as:

$$y = 7.64 + 0.11x_A - 0.09x_B + 0.05x_C - 0.13x_F + 0.08x_B x_F.$$

(12) (2 pts) Use the fitted model to obtain a location model and explain how you obtain it.

(13) (3 pts) Use the fitted model to obtain a transmitted variance model (i.e., a dispersion model) and explain how you obtain it.

(14) (3 pts) Suppose all the control factors are quantitative factors. Use two-step procedure to suggest a set of conditions where the mean response is as close as possible to 7.6 and the variance of y over the noise factor as small as possible. Explain how you obtain the conditions.

Suppose that factors A, B, C, D are control factors and factors E and F are noise factors. It is necessary to estimate all the main effects of control factors, all the 2-factor interactions between control factors, all the main effects of the noise factors, and all the 2-factor interactions between control and noise factors.

(15) (2 pts) Set up a cross-array design to investigate this problem. What type of design have you obtained? Explain.

(16) (2 pts) Show how a single-array design can be employed to investigate this problem and require only 32 runs.

(17) (2 pts) What is the best 32-run single-array design in terms of minimum aberration? Explain.