NTHU STAT 5510 Final Examination June 19, 2024

Instructions: Attempt all questions. Short and specific answers are preferred. Given explanation when requested to do so, but keep it as short and simple as possible. Give only one answer to each question – if you give alternative answers, the worst answer will be graded.

Question A.

A researcher is studying the absorption times of a particular type of antibiotic capsule. There are 3 technicians (factor A), 3 dosage strengths (factor B), and four capsule wall thicknesses (factor C) of interest to the researcher so that each replicate of a (full) factorial experiment would require 36 observations. The experimenter has decided on 4 replicates, and it is decided to run each replicate on a different day. Note that the days can be considered as blocks. Within a replicate (or a block) (day), the experiment is performed by randomly assigning *a unit* of antibiotic to a technician who conducts the experiment on the three dosage strengths and the four wall thicknesses. When a particular dosage strength is formulated *once*, all four wall thicknesses are tested at that strength. Then another dosage strength is selected and all four wall thicknesses are tested. Meanwhile, two other laboratory technicians also follow this plan, each starting with a unit of antibiotic. There are two randomization restrictions in the experiment. The following figure illustrates the randomization restrictions and experimental layout in this design for a block:

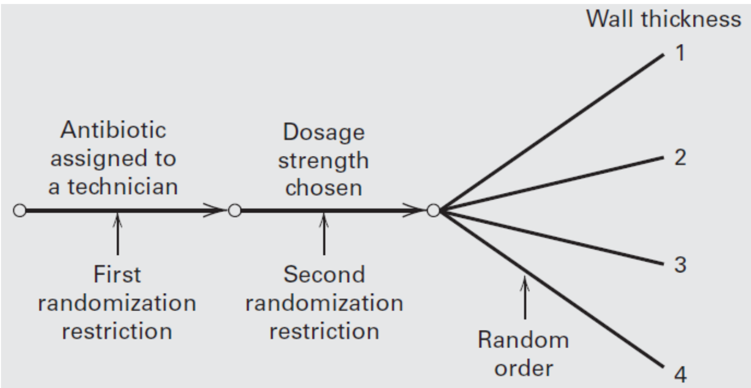

Figure 1.

(1) (3 pts) Draw a graph to demonstrate the structure of experimental units, and how the treatments are assigned to the experimental units.

(2) (2 pts) Under the above design plan, list all the factorial effects that are tested against the smallest experimental errors in ANOVA.

(3) (2 pts) How many degrees of freedom left for estimating the smallest experimental errors? Explain.

For this experiment, demonstrate (you might use a flowchart similar to Figure 1 or a graph like the one in your answer to problem (1)) how the order in which the treatment level combinations are run would be determined if this experiment were run as each one of the following 2 design plans given in problems (4) and (5):

(4) (2 pts) a split-plot design (i.e., consisting of only 2 kinds of experimental units, whole plots and subplots, in a block),
(5) (2 pts) a factorial design in a randomized block.
(6) (2 pts) In the first design plan (as shown in Figure 1) and the third design plan (refer to problem (5)), *how many units* of antibiotics are needed for each entire experiment? Which design plan saves more resources? What are sacrificed due to saving these resources?

Question B.

Consider the following two 2^{6-2} fractional factorial designs and denote the six factors by 1, 2, …, 6:

(i). 2^{6-2} with 5=1234, 6=124

(ii). 2^{6-2} with 5=123, 6=124

(7) (2 pts) What is the resolution of each of the two fractional factorial designs?
(8) (2 pts) Which design do you prefer? Justify your answer.
(9) (2 pts) Considering the foldover approach, i.e., allowing for a *full* foldover follow-up experiment when necessary, which design would you prefer? Justify your answer.
(10) (2 pts) For the design in (ii), if we further know that any two-factor interaction involving factor 6 is negligible, which two-factor interactions are estimable under the usual assumptions that three-factor and higher interactions are negligible?
(11) (2 pts) Under the same assumptions as in problem (10), find a scheme to arrange the design in (ii) in two blocks each of size 8, and explain why your choice is the best.
(12) (2.5 pts) Find the nine clear effects of the design in (i).
(13) (2 pts) If the design in (i) is arranged in four blocks with B_1 =16 and B_2 =26, which of the clear effects identified in problem (12) are still clear? Explain.

Question C.

In an experiment, five 2-level factors were studied to determine their effect on the free height of a leaf spring used in an automotive application. The five factors were A=furnace temperature, B=heating time, C=transfer time, D=hold time, and E=quench oil temperature, where A, B, C, D were treated as control factors and E as a noise factor. The two levels of each factors were coded as −1 and +1. A 16-run fractional factorial design with the generator $E = ABCD$ was conducted. A fitted model with free height as response (y) was obtained as:

$$y = 7.64 + 0.11x_A - 0.09x_B + 0.05x_C + 0.13x_E + 0.08x_B x_E.$$

(15) (2 pts) Is the design matrix a cross array or a single array? Explain.

(16) (2 pts) Can we apply location-dispersion modeling on this data? Explain.

(17) (2.5 pts) Use the fitted model to suggest a set of conditions where the mean free height is as close as possible to 7.6 and the variance of free height as small as possible.

(18) (2 pts) We can find from the fitted model that factor D was not important. Suppose that we project the design matrix onto factors A, B, C, E (i.e., removing the column of factor D from the design matrix). Can location-dispersion modeling be applied on this projected design? Explain.

(19) (2 pts) Consider A, B, C as control factors, and D and E as noise factors. Considering all two-factor interactions involving the control factors are important, design an experiment for analysis using location-dispersion modeling. What type of design have you created? Explain.

(20) (2 pts) Consider A, B, C as control factors, and D and E as noise factors. If a response modeling approach will be used and all two-factor interactions are considered important, what is the *smallest run size* fractional factorial design that can meet these requirements? Explain.