Question

What is Statistics?

- A branch of math --- calculation, derivative, proof, …
- A collection of many statistics (formula)
- A useful tools for extracting information/knowledge from the data

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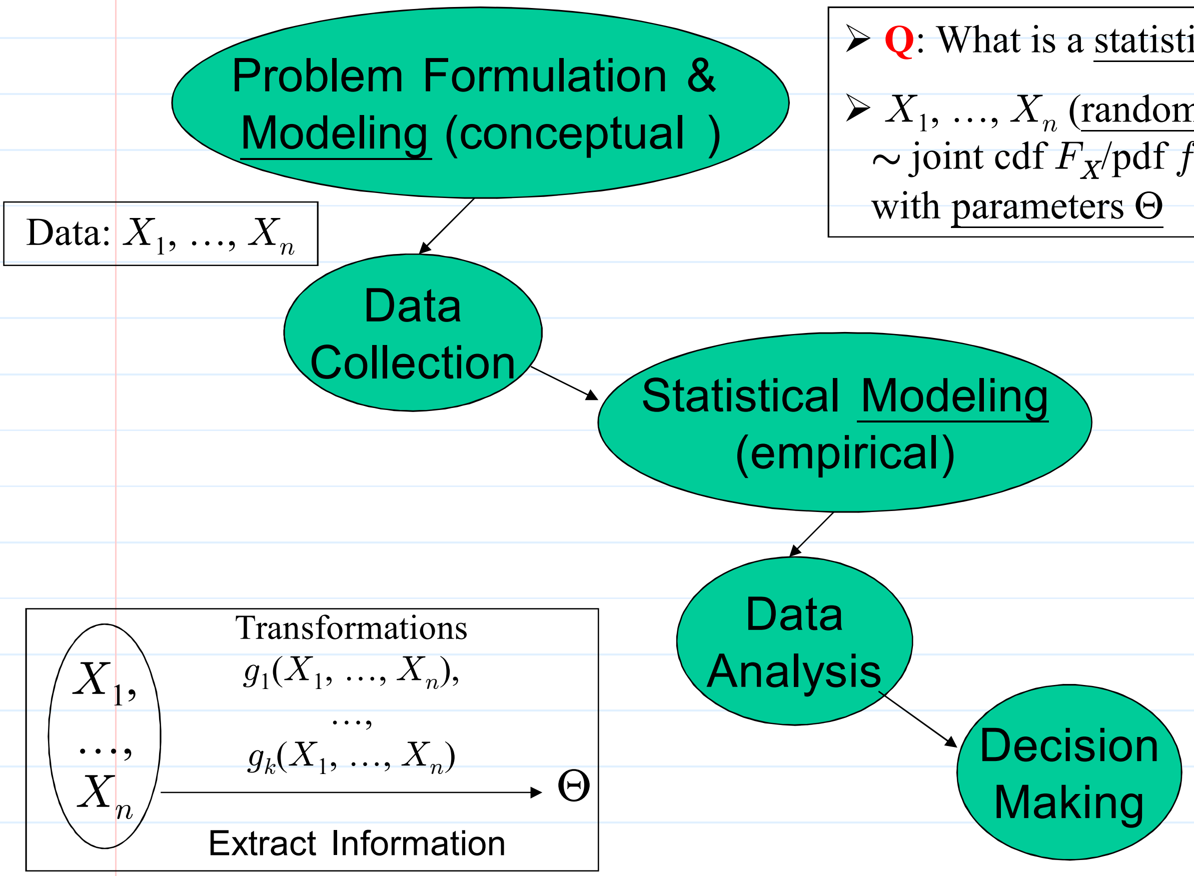
未來的資訊	Information

aim of statistics: provide ***insight*** by means of *data*

Basic Procedures of Statistics

- Statistics divides the study of data into *five* steps:

> **Q**: What is a statistical model?
> $X_1, \ldots, X_n$ (random variables) $\sim$ joint cdf F_X/pdf f_X/pmf p_X with parameters Θ



1. Problem formulation & modeling (conceptual approach)

- Problem formulation: use statistical/probabilistic/ mathematical language to "clearly" define the problem and the objective of study
- modeling (conceptual approach): use the information that we possessed *prior to obtaining data* to develop a *representation of the underlying system*, also account for uncertainty in data

2. Data collection: producing *representative* data for drawing correct information

- survey sampling
- design of experiment
- observational data

3. Statistical modeling (empirical approach): use empirical information contained in the data to build a model or to justify/adjust the (conceptual) model developed in **1.**, also account for uncertainty in data

- a statistical model is a description of the joint distribution of data
- a statistical model may contain the following components:
  - nonparametric component
  - parametric component: (fixed, random) effects
  - distribution component

4. data analysis: mining information from data

- graphical methods
- numerical methods
  - (point, interval) estimation
  - hypothesis testing

5. Inference/decision making: drawing conclusions & answering questions based on results obtained in **4**.

- Example (from Gilchrist, *Statistical Modelling*, 1984):

  "A range of problems related to the positioning of stores and the planning of delivery routes requires information on the distances by road, $\boldsymbol{y}$, between different places. Where a large number of such places are involved, finding these distances by driving or by direct measurement along the roads on a map is time-consuming."

"To avoid this problem, the usual approach is to relate the road distances $\boldsymbol{y}$ to the straight line distance, denoted by $\boldsymbol{x}$, as measured using a scale map. This relationship will be expressed mathematically and will enable us to predict a value of y given a corresponding value of x. This relationship will be our quantitative model of the situation. The fundamental question is: *how do we obtain this relationship (model)*."

Let's assume the following conditions (are they reasonable?):

a) $x=0 \Rightarrow y=0$

b) If there is a straight road between two points, then $x=y$; otherwise, $y \geq x$

c) Generally, y should increase with x. However, because of randomness in road patterns, places with same x's may have different y's.

d) Under similar situations, e.g. urban roads, the form of the relationship should not depend strongly on the distances involved, i.e., if x is, say, doubled, we would expect y is also approximately doubled.

Consider the following relationships (models):

1.	$y=x$	[satisfies a) and d), but not b) or c)]
2.	$y=x+\varepsilon$, ε: random component	[now allows c), but not b)]
3.	$y=\alpha+x+\varepsilon$, α: a constant	[helps with b), but a) fails]
4.	$y=\beta x+\varepsilon$, β: a constant ≥ 1	[satisfies all four conditions. true?]

5. distribution assumption can be added on the ε in *4*, e.g., $\varepsilon \sim N(0, \sigma^2)$

Note: The above (conceptual) model is derived without any data provided.

Problem formulation: Estimate and test parameters in $y=\beta x+\varepsilon$, where $\beta \geq 1$

Some Notes in Problem formulation & modeling (conceptual approach)

- understand the physical/social/political/biological/medical/... background to avoid the missing of important conditions that should be included in model
- understand the objective
- make sure you know what the client wants
- state the problem in "statistical language"

Albert Einstein. *The formulation of a problem is often more essential than its solution which may be merely a matter of mathematical or experimental skill.*

Example (cont.):

- the collected data are given in the tabular.
  Is it a "representative" data set?
- observational or experimental data?
- **Q**: If you can design the experiment, what are the data collection issues that should be concerned in the example?
- Consider the following situations:
  - if there are hundred/thousand of places, how to choose a small number of appropriate locations? geometrically uniform allocation? stratified sampling?
  - what if there are many routes that link any two places? replication required?
  - who should be assigned to measure these y's by driving? randomization? blocking?

y	x
10.7	9.5
6.5	5
29.4	23
17.2	15.2
18.4	11.4
19.7	11.8
16.6	12.1
29	22
40.5	28.2
14.2	12.1
11.7	9.8
25.6	19
16.3	14.6
9.5	8.3
28.8	21.6
31.2	26.5
6.5	4.8
25.7	21.7
26.5	18
33.1	28

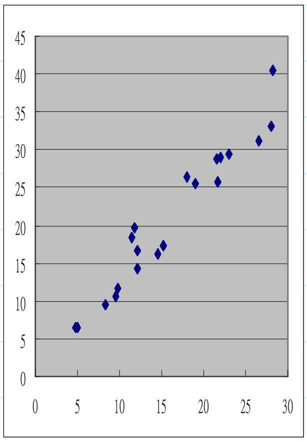
Some Notes in Data Collection

- are the data observational or experimental?
- how to collect a representative data?
- is there non-response?
- are there missing values?
- qualitative or quantitative?
- how are the data coded?
- what are the units of measurement?
- beware of data entry errors

Example (cont.):

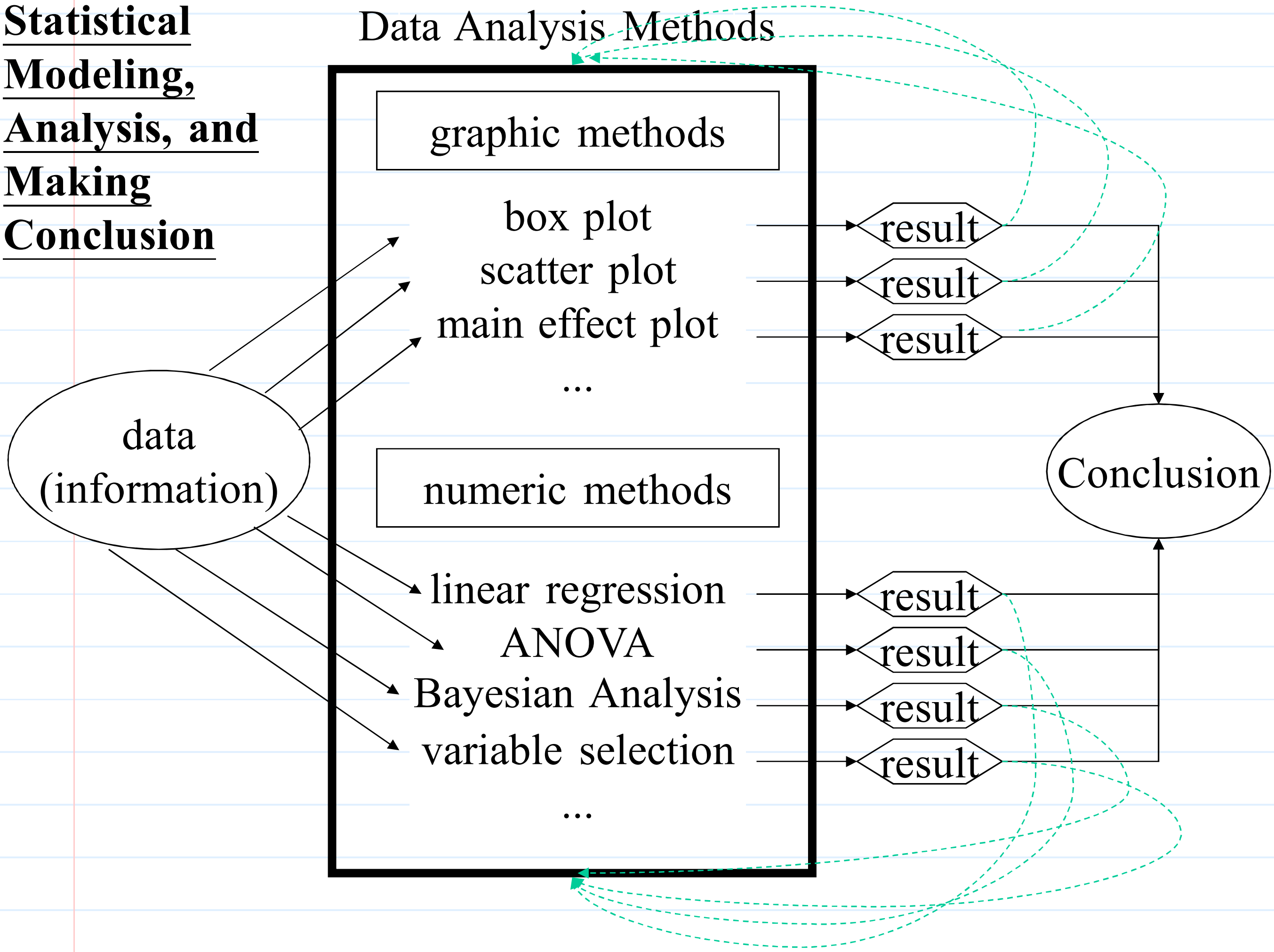
- What empirical model will you suggest after examining the plot?
- should empirical model be identical to conceptual model?
- if the plot (or numeric analysis) reveals different patterns ...
  - ➢ what if you find curvature or jump relationship existing between x and y?
  - ➢ what if you find non-constant variance?

  how should the conceptual model be adjusted?



- graphic analyses offer vivid and intuitive perception
- numeric analyses present numeric summaries (such as estimation and testing of parameters in the model) for making concrete conclusions. E.g., $\hat{\beta} = 1.29$ and is significant in t-test, and $\hat{\sigma} = 2.376$
- Conclusion: $\hat{y} = 1.29x$
  (or offer confident interval of $\hat{y}$)

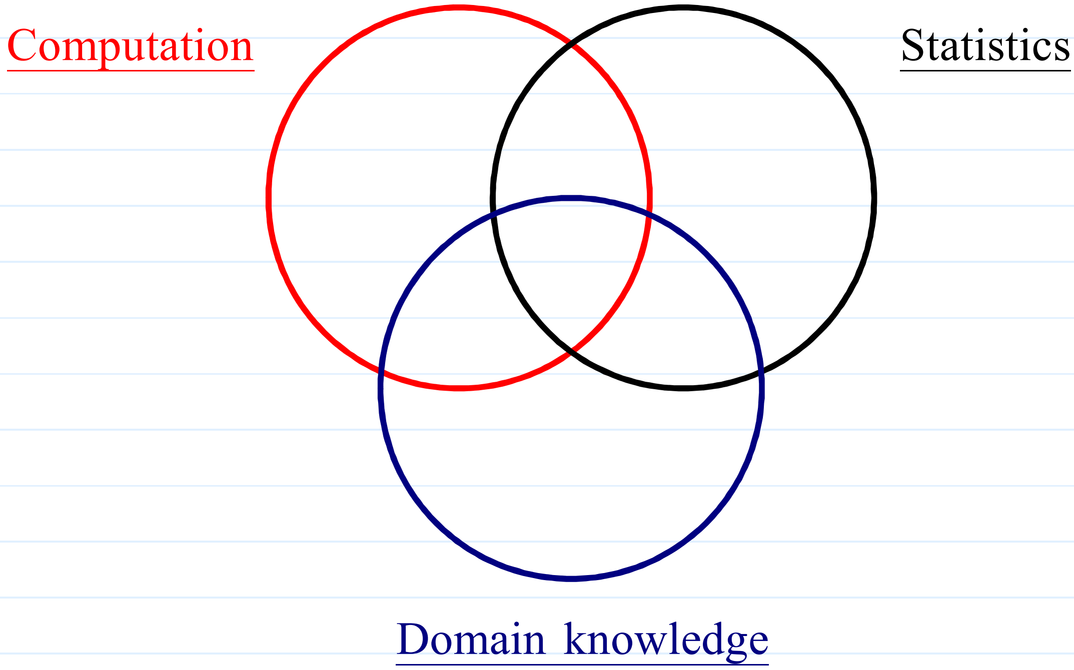
Statistical Modeling, Analysis, and Making Conclusion



Some Notes in Statistical modeling, Data analysis, and Decision making

- If possible, most available analysis methods should be performed.
- Assumptions and analysis results between different methods could be (slightly) different
- Data analysis is inherently interactive
- Conclusions should be summarized based on consistent results.
- Important information usually consistently appear in the results of every methods
- quantitative (定量) and qualitative (定性) conclusions

A successful data analysis usually requires a mix of the three components:



❖ **Reading**: Faraway (2005, 1st edition), 1.1

❖ **Further reading**:

- Statistical modelling (Gilchrist, 1984)
- Statistics: a guide to the unknown (edited by Tanur et al., 1972, 1978, 1989; Peck et al., 2005)
- Applied statistics: principles and examples (Cox & Snell, 1981)

❖ **Some other reading:**

- Lewis (2004), Moneyball (中譯：魔球).
- Kahneman (2011), Thinking, Fast and Slow (中譯：快思慢想).
- Silver (2012), The Signal and the Noise (中譯：精準預測).

What aspects you should focus on in this course?

1. **Understand** analysis methods
   - objective is ...?
   - for an estimator (parameter), what's its meaning?
   - for a test, what are its H_0 and H_1?
   - how to find statistically significant results in outputs?
   - assumptions and limitations in a statistical model?
   - ...
2. **Interpretation**: for those significant results, how to interpret them in the language that your clients use
3. **How to implement** the analysis method in softwares, such as R, Splus, SAS, ...?