NTHU STAT 3875, 2018

Lecture Notes

	$\begin{array}{c} \text{Ch 11, p. 38} \\ \hline \text{Test } \underline{H_0} : \underline{\pi_\Delta = 1/2} \ (\Leftrightarrow \underline{\Delta = 0}) \text{vs.} \underline{H_A} : \underline{\pi_\Delta \neq 1/2} \ (\Leftrightarrow \underline{\Delta \neq 0}) \end{array}$
	- intuitively, should reject H_0 if $\hat{\pi}_{\Delta}$ is too small (closer to 0) or too large (closer to 1)
	tost statistic IL (or IL)
	$\frac{U_{Y}}{* \text{ Define}} \xrightarrow{n} \sum_{i=1}^{m} \sum_{j=1}^{m} \sum_{i=1}^{m} \sum_{j=1}^{m} \frac{V_{ij}}{V_{ij}} \cdot \sum_{j=1}^{n} \sum_{j=1}^{m} \frac{V_{ij}}{2} \cdot \sum_{j=1}^{n} V_{ij$
	$\frac{i-1}{j-1}$ $\frac{i-1}{j-1}$
- VWY	$\overset{\text{cf.}}{\longrightarrow} \underbrace{\text{Reject } H_0}_{\text{Reject } H_0} \text{ if } \underbrace{U_Y}_{\text{Is too small or too large (closer to 0 or mn)}}_{\text{In the number of } M_0}.$
	* Let $\underline{R}_{\underline{Y_{(j)}}}$ be the rank of $\underline{Y_{(j)}}$ in the <i>pooled</i> sample. Then, $in \{\underline{Y_{i}, \cdots, \underline{Y_{m}}}\} \xrightarrow{\Phi} \{\underline{X_{i}, \cdots, \underline{X_{n}}}, \underline{Y_{i}, \cdots, \underline{Y_{m}}}\}$ $\sum_{j=1}^{\underline{m}} \underline{R}_{\underline{Y_{(j)}}} = \underline{\operatorname{rank \ sum}} \text{ of } \underline{Y_{j}}$'s $(\operatorname{or} \underline{Y_{(j)}}'s) = \underline{R}_{\underline{n+1}} + \cdots + \underline{R}_{\underline{n+m}} \xrightarrow{\Phi} \underline{W_{Y}}.$
	* Notice that check LNp 32-
	$\underline{U}_{\underline{Y}} = \sum_{\underline{j=1}}^{\underline{m}} \left(\sum_{\underline{i=1}}^{\underline{n}} V_{\underline{ij}} \right) = \sum_{\underline{j=1}}^{\underline{m}} \underbrace{(\underline{R}_{\underline{Y_{(j)}}} - \underline{j})}_{\#\{X_{\underline{(i)}} < Y_{\underline{(j)}}\}} = \left(\sum_{\underline{j=1}}^{\underline{m}} \underline{R}_{\underline{Y_{(j)}}} \right) - \frac{\underline{m(m+1)}}{2}$ $\#\{X_{\underline{i} \ge Y_{\underline{i}}}\} = \underbrace{W_{\underline{Y}} - \underline{[m(m+1)]/2}}_{\text{fixed}}$
	$\frac{\underline{j}^{-1}}{\underline{\mu}^{-1}} \xrightarrow{\underline{j}^{-1}} \underbrace{\#\{X_{(i)} < Y_{(j)}\}}_{\underline{\mu}^{-1}} = \underbrace{W_Y}_{\underline{\mu}^{-1}} - [m(m+1)]/2.$
	* Similarly, U_X can be defined by changing " $X_{(i)} \leq Y_{(j)}$ " in V_{ij} to
	$"X_{(i)} \ge Y_{(j)}", \text{ and } \blacktriangleleft \text{ check graph in Thm10 (LNp. 36)}$
	$\cdot \frac{1}{mn} \underbrace{U_X}{\stackrel{e}{\longrightarrow}} 1 - \pi_{\underline{\Delta}} = P_{\underline{\Delta}}(X \ge Y) \xleftarrow{cf.} \frac{1}{mn} \underbrace{T_Y}{\stackrel{e}{\longrightarrow}} \underbrace{\pi_{\Delta}} = P_{\underline{\Delta}}(X < Y)$
	$\cdot U_X = mn - U_Y \xleftarrow{\text{CF.}} W_x = \underbrace{(m+n)(m+n+1)}_{2} - W_Y$
	$ \underbrace{U_X}_{X} = \underline{mn} - \underline{U_Y} \stackrel{\text{(ff.)}}{\longrightarrow} W_x = \underbrace{(m+n)(m+n+1)}_{Z} - W_Y \\ \underbrace{U_X}_{X} = W_X - \frac{1}{2}\underline{n}(\underline{n}+1) \stackrel{\text{(ff.)}}{\longleftarrow} U_Y = W_Y - \frac{1}{2}\underline{m}(\underline{m}+1) $
	$\cdot \underline{U_X} = \underline{mn} - \underline{U_Y} \xleftarrow{CF.} \mathbf{W_x} = \underbrace{(\mathbf{m} + \mathbf{n})(\mathbf{m} + \mathbf{n} + 1)}_{\mathbf{Z}} - \mathbf{W_Y}$
	$ \begin{array}{l} \cdot \underline{U}_X = \underline{mn} - \underline{U}_Y < \stackrel{(f.)}{\longrightarrow} W_x = \underbrace{(m+n)(m+n+1)}_2 - W_Y \\ \cdot \underline{U}_X = \underline{W}_X - \frac{1}{2}\underline{n}(\underline{n}+1) < \stackrel{(f.)}{\longleftarrow} U_Y = W_Y - \frac{1}{2}\underline{m}(\underline{m}+1) \\ \cdot \underline{reject} \ H_0 \ \text{if} \ \underline{U}_X \ \text{is too small or too large} \\ - \ \text{null distribution of} \ U_Y: \ \text{the pmf of} \ U_Y \ \text{under} \ H_0 \ \text{can be obtained from} \end{array} $
ome	$ \begin{array}{l} \cdot \underline{U}_X = \underline{mn} - \underline{U}_Y < \stackrel{(f.)}{\longrightarrow} W_x = \underbrace{(m+n)(m+n+1)}_2 - W_Y \\ \cdot \underline{U}_X = \underline{W}_X - \frac{1}{2}\underline{n}(\underline{n}+1) < \stackrel{(f.)}{\longleftarrow} U_Y = W_Y - \frac{1}{2}\underline{m}(\underline{m}+1) \\ \cdot \underline{reject} \ H_0 \ \text{if} \ \underline{U}_X \ \text{is too small or too large} \\ - \ \text{null distribution of} \ U_Y: \ \text{the pmf of} \ U_Y \ \text{under} \ H_0 \ \text{can be obtained from} \end{array} $
pmfsymmetri about mr	$ \begin{array}{c} \cdot \underline{U}_{X} = \underline{mn} - \underline{U}_{Y} < \stackrel{\text{cf.}}{\longrightarrow} \\ W_{X} = \underbrace{W_{X}}_{2} - \underbrace{\frac{1}{2}\underline{n}(\underline{n}+1)}_{2} < \stackrel{\text{cf.}}{\longrightarrow} \\ \overline{U}_{Y} = W_{Y} - \underbrace{\frac{1}{2}\underline{m}(\underline{m}+1)}_{2} < \stackrel{\text{cf.}}{\longrightarrow} \\ \overline{U}_{Y} = W_{Y} - \underbrace{\frac{1}{2}\underline{m}(\underline{m}+1)}_{2} \\ \cdot \\ \text{reject } H_{0} \text{ if } \underline{U}_{X} \text{ is too small or too large} \\ - \\ \underline{\text{null distribution of } \underline{U}_{Y} \text{ is too small or too large}} \\ - \\ \underline{\text{null distribution of } \underline{W}_{Y} \text{ by } \\ \underline{\text{pnf symmetric about } \underline{\underline{m}(\underline{m}+n+1)}_{2}} \\ \end{array} $
symmetri about mr	$\begin{array}{c} \cdot \underline{U}_X = \underline{mn} - \underline{U}_Y < \stackrel{(f.)}{\longrightarrow} W_x = \underbrace{(m+n)(m+n+1)}_2 - W_Y \\ \cdot \underline{U}_X = \underline{W}_X - \frac{1}{2}\underline{n}(\underline{n}+1) < \stackrel{(f.)}{\longleftarrow} U_Y = W_Y - \frac{1}{2}\underline{m}(\underline{m}+1) \\ \cdot \underline{reject} \ H_0 \ \text{if} \ \underline{U}_X \ \text{is too small or too large} \\ - \underline{null \ distribution} \ \text{of} \ \underline{U}_Y : \ \text{the pmf of} \ \underline{U}_Y \ under \ \underline{H_0} \ \text{can be obtained from} \\ \text{the null \ distribution} \ \text{of} \ \underline{W}_Y \ by \ pmf \ symmetric \ about \ \underline{m}(\underline{m}+n+1) \\ \underline{2} \end{array}$
symmetri about 2 Check Note6	$ \begin{array}{c} \cdot \underline{U_X} = \underline{mn} - \underline{U_Y} < ^{cf.} \\ W_x = \underbrace{(m+n)(m+n+1)}_2 - W_y \\ \cdot \underline{U_X} = \underline{W_X} - \frac{1}{2}\underline{n}(\underline{n}+1) < ^{cf.} \\ \overline{U_Y} = W_y - \frac{1}{2}\underline{m}(\underline{m}+1) \\ \cdot \underline{n} \\ \text{reject } H_0 \text{ if } \underline{U_X} \text{ is too small or too large} \\ \hline \underline{null \ distribution} \ of \underline{U_Y} \text{ : the } \underline{pmf} \ of \underline{U_Y} \ under \ \underline{H_0} \ can \ be \ obtained \ from \\ \underline{m(m+n+1)} \\ \underline{null \ distribution} \ of \ \underline{W_Y} \ by \\ \underline{pmf} \ symmetric \ about \ \underline{m(m+n+1)} \\ \underline{2} \\ \hline \underline{P}(\underline{U_Y} = \underline{u}) = \underline{P}\Big(\underline{W_Y} - \frac{m(m+1)}{2} = \underline{u}\Big) = \underline{P}\Big(\underline{W_Y} = \underline{u} + \frac{m(m+1)}{2}\Big). \\ \hline - \text{ The } \underline{\text{tests based on } \underline{U_Y} \ and \ \underline{W_Y} \ (or \ \underline{U_X} \ and \ \underline{W_X}) \ are \ actually \ \underline{equivalent}. \end{array} $
symmetri about 2 Check Note 6 (UNp 25)	$ \begin{array}{l} \cdot \underline{U}_{X} = \underline{mn} - \underline{U}_{Y} \Leftarrow \overline{f} \Rightarrow W_{X} = \underbrace{(\mathbf{m}+\mathbf{n})(\mathbf{m}+\mathbf{n}+1)}_{2} - W_{Y} \\ \cdot \underline{U}_{X} = \underline{W}_{X} - \frac{1}{2}\underline{n}(\underline{n}+1) & \Leftarrow \overline{f} \Rightarrow \overline{U}_{Y} = W_{Y} - \frac{1}{2}\underline{m}(\underline{m}+1) \\ \cdot \underline{reject} \ H_{0} \ \text{if} \ \underline{U}_{X} \ \text{is too small or too large} \\ - \underline{null \ distribution \ of} \ \underline{U}_{Y} \ \text{is too small or too large} \\ - \underline{null \ distribution \ of} \ \underline{W}_{Y} \ \text{by} \qquad \underbrace{pmf \ symmetric \ about \ \underline{m}(\underline{m}+n+1)}_{2} \\ \underline{P}(\underline{U}_{Y} = \underline{u}) = \underline{P}\Big(\underline{W}_{Y} - \frac{m(m+1)}{2} = \underline{u}\Big) = \underline{P}\Big(\underline{W}_{Y} = \underline{u} + \frac{m(m+1)}{2}\Big). \\ - \text{The tests based on} \ \underline{U}_{Y} \ \text{and} \ \underline{W}_{Y} \ (\text{or} \ \underline{U}_{X} \ \text{and} \ \underline{W}_{X}) \ \text{are actually equivalent.} \\ \text{Note 8 (A comparison of t-test and Mann-Whitney (M-W) test)} \end{array}$
symmetri about 2 Check Note 6 (UNp 25)	$ \begin{array}{c} & \underbrace{U_X} = \underline{mn} - \underbrace{U_Y} \Leftarrow \overleftarrow{CF} & \forall \mathbf{W_x} = \underbrace{(\mathbf{m} + \mathbf{n})(\mathbf{m} + \mathbf{n} + 1)}_2 - \forall \mathbf{W_Y} \\ & \underbrace{U_X} = \underbrace{W_X} - \frac{1}{2}\underline{n}(\underline{n} + 1) & \Leftarrow \overleftarrow{CF} & \forall \mathbf{U_Y} = \mathbf{W_Y} - \frac{1}{2}\underline{\mathbf{m}}(\underline{\mathbf{m}} + \mathbf{i}) \\ & & \underline{\mathbf{reject}} \ H_0 \ \text{if} \ \underline{U_X} \ \text{is too small or too large} \\ & - \ \underline{\text{null distribution of}} \ \underline{U_Y} \ \text{: the pmf of } \underbrace{U_Y} \ \text{under } \underline{H_0} \ \text{can be obtained from} \\ & & \underline{\text{the null distribution of } \underline{W_Y} \ \text{by}} \ \begin{array}{c} \underline{pnf} \ symmetric \ about \ \underline{m}(\underline{m} + n + 1)} \\ & \underline{pnf} \ symmetric \ about \ \underline{m}(\underline{m} + n + 1)} \\ & \underline{p}(\underline{U_Y} = \underline{u}) = \underline{P}\Big(\underline{W_Y} - \frac{m(m+1)}{2} = \underline{u}\Big) = \underline{P}\Big(\underline{W_Y} = u + \frac{m(m+1)}{2}\Big). \\ & - \ \text{The tests based on } \underline{U_Y} \ \text{and } \underline{W_Y} \ (\text{or } \underline{U_X} \ \text{and } \underline{W_X}) \ \text{are actually equivalent.} \\ & \\ \hline \text{tote 8 (A comparison of t-test and Mann-Whitney (M-W) test)} \\ & \underline{\text{Unlike } t\text{-test, the } \underline{M}\text{-W test does not depend on normality assumption.} \end{array} \right)$
Symmetri about 2 Check Note6 (UNp25) Note7 (UNp.30)	$ \begin{array}{c} \cdot \underline{U_X} = \underline{mn} - \underline{U_Y} \xleftarrow{CF} & \mathbf{W_x} = \underbrace{(\mathbf{m+n})(\mathbf{m+n+1})}_{\mathbf{Z}} - \mathbf{W_Y} \\ \cdot \underline{U_X} = \underline{W_X} - \frac{1}{2}\underline{n}(\underline{n+1}) & \Leftarrow{CF} & \mathbf{U_Y} = \mathbf{W_Y} - \frac{1}{2}\underline{m}(\underline{m+1}) \\ \cdot \underline{neject} \ H_0 \ \text{if} \ \underline{U_X} \ \text{is too small or too large} \\ \hline \underline{null \ distribution} \ \text{of} \ \underline{U_Y} \ \text{is too small or too large} \\ \hline \underline{null \ distribution} \ \text{of} \ \underline{W_Y} \ \text{by} & \underbrace{pnf} \ symmetric \ about \ \underline{m}(\underline{m+n+1})}_{\mathbf{Z}} \\ \hline \underline{P}(\underline{U_Y} = \underline{u}) = \underline{P}\left(\underline{W_Y} - \frac{m(m+1)}{2} = \underline{u}\right) = \underline{P}\left(\underline{W_Y} = \underline{u} + \frac{m(m+1)}{2}\right). \\ \hline - \ \text{The tests based on} \ \underline{U_Y} \ \text{and} \ \underline{W_Y} \ \text{(or} \ \underline{U_X} \ \text{and} \ \underline{W_X}) \ \text{are actually equivalent.} \\ \hline \text{Mote states based on} \ \underline{t-test} \ \text{and} \ \text{Mann-Whitney} \ (M-W) \ \text{test} \\ \hline \underline{Unlike \ t-test}, \ \text{the} \ \underline{M-W} \ \text{test does not depend on normality assumption.} \\ \hline \underline{W-W} \ \text{test is insensitive} \ \text{to outliers, where as the} \ \underline{t-test} \ \text{is sensitive}. \\ \hline \end{array}$
symmetri about 2 Check Note6 (UNp25) Note7 (UNp.30)	$ \underbrace{U_X}_{X} = \underline{mn} - \underline{U_Y} \xleftarrow{cf} \\ W_X = \underbrace{(m+n)(m+n+1)}_{2} - W_Y \\ \underbrace{U_X}_{X} = \underbrace{W_X}_{X} - \frac{1}{2}\underline{n}(\underline{n}+1) \\ \underbrace{cf}_{X} = \underbrace{W_Y}_{Y} - \frac{1}{2}\underline{m}(\underline{m}+1) \\ \underline{v}_{X} = \underbrace{W_X}_{Y} - \frac{1}{2}\underline{n}(\underline{n}+1) \\ \underline{v}_{Y} = \underbrace{W_Y}_{Y} - \underbrace{W_Y}_{Y} = \underbrace{W_Y}_{Y} - \frac{1}{2}\underline{m}(\underline{m}+1) \\ \underline{m}(\underline{m}+\underline{n}+1) \\ \underline{m}(\underline{m}+\underline{n}+1) \\ \underline{m}(\underline{m}+\underline{n}+1) \\ \underline{m}(\underline{m}+\underline{n}+1) \\ \underline{m}(\underline{m}+\underline{n}+\underline{n}) \\ \underline{m}(\underline{m}+\underline{n}+\underline{n}) \\ \underline{m}(\underline{m}+\underline{n}+\underline{n}) \\ \underline{m}(\underline{m}+\underline{n}+\underline{n}) \\ \underline{m}(\underline{m}+\underline{n}) \\ \underline{m}$
symmetri about 2 Check Note 6 (UNp 25) Note 7 (UNp.30)	 U_X = mn - U_Y < K → W_x = (m+n)(m+n+1) - W_Y U_X = W_X - ½n(n + 1) < K → U_Y = W_Y - ½m(m+1) reject H₀ if U_X is too small or too large null distribution of U_Y: the pmf of U_Y under H₀ can be obtained from the null distribution of W_Y by pmf symmetric about m(m+n+1)/2 P(U_Y = u) = P(W_Y - m(m+1)/2 = u) = P(W_Y = u + m(m+1)/2). The tests based on U_Y and W_Y (or U_X and W_X) are actually equivalent. Note 8 (A comparison of t-test and Mann-Whitney (M-W) test) Unlike t-test, the M-W test does not depend on normality assumption. c an be applied when F & G are not normal, e.g., Cauchy When the normality assumption holds, the t-test is more powerful.
symmetri about 2 Check Note6 (UNp25) Note7 (UNp.30) Check 6 Check 6 Check 6	 U_X = mn - U_Y < E > W_x = (m+n)(m+n+1) - W_Y U_X = W_X - ½n(n+1) < E > U_Y = W_Y - ½m(m+1) reject H₀ if U_X is too small or too large null distribution of U_Y: the pmf of U_Y under H₀ can be obtained from the null distribution of W_Y by pmf symmetric about m(m+n+1) P(U_Y = u) = P(W_Y - m(m+1)/2 = u) = P(W_Y = u + m(m+1)/2). The tests based on U_Y and W_Y (or U_X and W_X) are actually equivalent. Note 8 (A comparison of t-test and Mann-Whitney (M-W) test) Unlike t-test, the M-W test does not depend on normality assumption. Locan be applied when F & G are not normal, e.g. Cauchy The M-W test is insensitive to outliers, where as the t-test is sensitive. based on ranks based on X, Y - I When the normality assumption holds, the t-test is nearly as powerful. However, under normality assumption, the M-W test is nearly as powerful.
symmetri about 2 Check Note 6 (UNp 25) Note 7 (UNp 30) Check • comparison (LNp 34)	 U_X = mn - U_Y < E → W_x = (m+n)(m+n+1) - W_Y U_X = W_X - ½n(n+1) < F → U_Y = W_Y - ½m(m+1) reject H₀ if U_X is too small or too large null distribution of U_Y: the pmf of U_Y under H₀ can be obtained from the null distribution of W_Y by pmf symmetric about m(m+n+1) P(U_Y = u) = P(W_Y - m(m+1)/2 = u) = P(W_Y = u + m(m+1)/2). The tests based on U_Y and W_Y (or U_X and W_X) are actually equivalent. Note 8 (A comparison of t-test and Mann-Whitney (M-W) test) Unlike t-test, the M-W test does not depend on normality assumption. c an be applied when F & G are not normal, e.g., Cauchy The M-W test is insensitive to outliers, where as the t-test is sensitive. based on maks based on X, Y → When the normality assumption holds, the t-test is nearly as powerful.
symmetri about 2 Check Note 6 (UNp 25) Note 7 (UNp.30) Check 6 Check 6 Check 6 Check 6	$ \begin{array}{c} \underbrace{U_X} = \underline{mn} - \underbrace{U_Y} \leftarrow \underbrace{E} \forall W_X = \underbrace{(m+n)(m+n+1)}_2 - \underbrace{W_Y}_2 \\ \underbrace{U_X} = \underbrace{W_X}_2 - \frac{1}{2}\underline{n}(\underline{n}+1) \leftarrow \underbrace{CF} \forall U_Y = \underbrace{W_Y}_2 - \frac{1}{2}\underline{m}(\underline{m}+1) \\ \cdot \underbrace{\mathrm{reject}}_{H_0} \text{ if } \underbrace{U_X} \text{ is too small or too large} \\ - \underline{\mathrm{null distribution of }}_{V_Y} \text{ is the pmf of } \underbrace{U_Y}_2 \text{ under } \underbrace{H_0}_0 \text{ can be obtained from the null distribution of } \underbrace{W_Y}_Y \text{ by } \underbrace{pmf symmetric about \underbrace{m}(\underline{m}+1)}_2 \\ \underline{P}(\underbrace{U_Y}_2 = \underline{u}) = \underbrace{P}\left(\underbrace{W_Y}_2 - \frac{m(m+1)}{2} = \underline{u}\right) = \underbrace{P}\left(\underbrace{W_Y}_2 = u + \frac{m(m+1)}{2}\right). \\ - \text{ The tests based on } \underbrace{U_Y}_2 \text{ and } \underbrace{W_Y}_2 \text{ (or } \underbrace{U_X}_2 \text{ and } \underbrace{W_X}_2) \text{ are actually equivalent.} \\ \hline \text{ tote 8 (A comparison of t-test and Mann-Whitney (M-W) test)} \\ \underline{Unlike t-test}_1 \text{ the } \underbrace{M-W \text{ test does not depend on normality assumption.} \\ \underline{L} \text{ can be applied when } \underbrace{F \& G}_2 \text{ are not normal, e.g., Cauchy}_2 \\ \hline \text{ When the normality assumption holds, the t-test is more powerful.} \\ \hline \text{However, under normality assumption, the } \underbrace{M-W \text{ test is nearly as powerful}_2 \\ as the \underline{t-test}_1 It has been shown that to attain the same power < different cannot cannot be applied to a train the same power < different cannot be applied to a train the same power < different cannot be applied to a train the same power < different cannot be applied to a train the same power < different cannot be applied to a train the same power < different cannot be applied to a train the same power < different cannot be applied to a train the same power < different cannot be applied to a train the same power < different cannot be applied to a train the same power < different cannot be applied to a train the same power < different cannot be applied to a train the same power < different cannot be applied to a train the same power < different cannot be applied to a train the same power < different cannot be applied to a train the same power < different cannot be applied to a train the same power < different cannot $

made by S.-W. Cheng (NTHU, Taiwan)

NTHU STAT 3875, 2018

Lecture Notes

	1 3875, 2018	Lecture Note
	• Theorem 12 (means and variances of \underline{U}_Y and \underline{W}_Y under \underline{H}_0 \underline{H}_0	Ch 11, p. 40
	Consider the nonparametric model (\Diamond) in LNp.35. If $\Delta = 0$ ($\Leftrightarrow \pi_{\Delta} =$	
pmfs	• $E(\underline{W}_{\underline{Y}}) = [\underline{m}(m+n+1)]/2$ and $Var(\underline{W}_{\underline{Y}}) = [mn(m+n+1)]/1$	2
	their $(\Leftrightarrow \underline{E(W_X)} = [\underline{n}(m+n+1)]/2 \text{ and } \underline{Var(W_X)} = [\underline{mn}(m+n+1)]/2$	
means	$1 \qquad 1 \qquad$	
		ame ariance
$\frac{U_{Y}}{m(m)}$	V_X ($\Leftrightarrow \underline{E(U_X)} = \underline{mn/2}$ and $\underline{Var(U_X)} = \underline{[mn(m+n+1)]/12}$	
2	since $U_X = \underline{mn} - \underline{U_Y}$ Note. $\forall \pi_0 = 1$	Jy/mn
	Proof. It is enough to prove the case of W_Y . Under H_0, $E(\hat{\pi})$, H_0, H	
	• Note that $W_{P} = R_{\underline{n+1}} + \dots + R_{\underline{m+n}}$. $Var(\widehat{\pi}_{\underline{A}}) = \underline{m+n}$	$\rightarrow \infty$
	Under $\underline{H_0}: \underline{\Delta} = 0, (\underline{R_{n+1}, \cdots, R_{m+n}})$ can be viewed as a	
	without-replacement simple random sample from the population	
	• Let $\underline{N} = \underline{m+n}$. Since $\{\underline{1}, \dots, \underline{n}, \underline{n+1}, \dots, \underline{m+n}\}$.	
popula	ation $N(N+1)$, $N(N+1)(2N+1)$	
<u>Si z</u>		
	the population mean μ and variance σ^2 of this population distribution	
	$\underline{\mu} = \frac{1}{N} \left(\sum_{k=1}^{N} k \right) = \frac{N+1}{2} \text{and} \underline{\sigma^2} = \frac{1}{N} \left(\sum_{k=1}^{N} k^2 \right) - \underline{\mu^2} = \frac{N^2}{12}$	$\frac{-1}{2}$.
		Ch 11 - 11
Sample mean Lsamj		12m Immection
menn	sample $(R_{n+1}, \ldots, R_{m+n})$. Then (by Thms 1 & 3 in LN, Ch7, p.16-1 ple size = $\underbrace{R}_{(N+1)/2} = \underbrace{\mu}_{(N+1)/2} = \underbrace{M}_{(N-1)(N+1)/12} = \underbrace{M}_{(N-1)/12} = \underbrace{M}_{(N-1)/1$.8), 1(n+m+1) 12m mection
menn	sample $(R_{n+1}, \ldots, R_{m+n})$. Then (by Thms 1 & 3 in LN, Ch7, p.16-1 ple size = $\underbrace{E(\overline{R}) = \mu}_{(N+1)/2}$ and $\underbrace{Var(\overline{R}) = (\sigma^2/m)[(N-m)/(N-1)]}_{(N-1)/12} = \underbrace{Var(N-1)/2}_{(N-1)/12}$. • The results follows from $E(W_Y) = m E(\overline{R})$ and $\underbrace{Var(W_Y) = m^2 Var}_{(N-1)/12}$.	(.8), (.1 + m + 1) 12 m (.12 m) (.12 m)
menn	sample $(R_{n+1}, \ldots, R_{m+n})$. Then (by Thms 1 & 3 in LN, Ch7, p.16-1 ple size = $\underbrace{R}_{(N+1)/2} = \underbrace{\mu}_{(N+1)/2} = \underbrace{M}_{(N-1)(N+1)/12} = \underbrace{M}_{(N-1)/12} = \underbrace{M}_{(N-1)/1$	(1), $(1) + (1) + (1)$ $(1) + (1) + (1)$ $(1) + (1) + (1)$ $(1) + (1) + (1)$ $(1) + (1) + (1)$ $(1) + (1) + (1)$ $(1) + (1) + (1)$ $(1) + (1) + (1)$ $(1) + (1) + (1)$ $(1) + (1) + (1)$ $(1) + (1) + (1)$ $(1) + (1) + (1)$ $(1) + (1) + (1)$ $(1) + (1) + (1)$ $(1) + (1) + (1)$ $(1) + (1) + (1)$ $(1) + (1) + (1)$ $(1) + (1) + (1)$ $(1) + (1) + (1)$ $(1) + (1) + (1)$ $(1) + (1) + (1)$ $(1) + (1) + (1)$ $(1) + (1) + (1)$ $(1) + (1) + (1)$ $(1) + (1) + (1)$ $(1) + (1) + (1)$ $(1) + (1) + (1)$
1 samj	sample $(R_{n+1}, \ldots, R_{m+n})$. Then (by Thms 1 & 3 in LN, Ch7, p.16-1 $E(\overline{R}) = \mu$ and $Var(\overline{R}) = (\sigma^2/m)[(N-m)/(N-1)] = \frac{M}{(N+1)/2}$. • The results follows from $E(W_Y) = m E(\overline{R})$ and $Var(W_Y) = m^2 Var$. • Theorem 13 (Asymptotic null distribution of U_Y) Consider the nonparametric model (\Diamond) in LNp.35 and the null H_0 : ($\Leftrightarrow \pi_{\Delta} = 1/2$). For m, n both greater than 10, the null distribution (or U_X) is well approximated by a normal distribution, i.e.,	(A), $(A) + (A) + (A)$
1 samj	sample $(R_{n+1}, \ldots, R_{m+n})$. Then (by Thms 1 & 3 in LN, Ch7, p.16-1 $E(\overline{R}) = \mu$ and $Var(\overline{R}) = (\sigma^2/m)[(N-m)/(N-1)] = \frac{M}{(N+1)/2}$. • The results follows from $E(W_Y) = m E(\overline{R})$ and $Var(W_Y) = m^2 Var$. • Theorem 13 (Asymptotic null distribution of U_Y) Consider the nonparametric model (\Diamond) in LNp.35 and the null H_0 : ($\Leftrightarrow \pi_{\Delta} = 1/2$). For m, n both greater than 10, the null distribution (or U_X) is well approximated by a normal distribution, i.e.,	(A), $(A) + (A) + (A)$
1 samj	sample $(R_{n+1}, \ldots, R_{m+n})$. Then (by Thms 1 & 3 in LN, Ch7, p.16-1 $E(\overline{R}) = \mu$ and $Var(\overline{R}) = (\sigma^2/m)[(N-m)/(N-1)] = \frac{M}{(N-1)/2}$. • The results follows from $E(W_Y) = m E(\overline{R})$ and $Var(W_Y) = m^2 Var$. • Theorem 13 (Asymptotic null distribution of U_Y) Consider the nonparametric model (\Diamond) in LNp.35 and the null H_0 : ($\Leftrightarrow \pi_{\Delta} = 1/2$). For m, n both greater than 10, the null distribution (or U_Y) is well apprecimated by a permed distribution is a	(A), $(A) + (A) + (A)$
1 samj	sample $(\underline{R}_{n+1}, \ldots, \underline{R}_{m+n})$. Then (by Thms 1 & 3 in LN, Ch7, p.16-1 $E(\underline{R}) = \mu$ and $Var(\underline{R}) = (\sigma^2/m)[(\underline{N-m})/(\underline{N-1})] = \frac{M}{(\underline{N-1})/2}$. • The results follows from $E(W_Y) = \underline{m} E(\overline{R})$ and $Var(W_Y) = \underline{m^2} Var$. • Theorem 13 (Asymptotic null distribution of \underline{U}_Y) Consider the nonparametric model (\Diamond) in LNp.35 and the null \underline{H}_0 : ($\Leftrightarrow \pi_{\Delta} = 1/2$). For $\underline{m, n}$ both greater than 10, the null distribution (or \underline{U}_X) is well approximated by a normal distribution, i.e., • $(\Phi - \frac{U_Y}{\sqrt{Var(U_Y)}}) \stackrel{D}{\approx} N(0, 1)$ (or $\frac{U_X - E(U_X)}{\sqrt{Var(U_X)}} \stackrel{D}{\approx} N(0, 1)$)	(A), $(A) + (A) + (A)$
1 samj	sample $(R_{n+1}, \ldots, R_{m+n})$. Then (by Thms 1 & 3 in LN, Ch7, p.16-1 $R = Size = m$ $\frac{E(\overline{R})}{(N+1)/2} = \mu$ and $Var(\overline{R}) = (\sigma^2/m)[(N-m)/(N-1)] = m$ • The results follows from $E(W_Y) = m E(\overline{R})$ and $Var(W_Y) = m^2 Var$ Theorem 13 (Asymptotic null distribution of U_Y) Consider the nonparametric model (\Diamond) in LNp.35 and the null H_0 : ($\Leftrightarrow \pi_{\Delta} = 1/2$). For m, n both greater than 10, the null distribution (or U_X) is well approximated by a normal distribution, i.e., $(\Rightarrow -\frac{V_X}{\sqrt{Var(U_Y)}} \approx N(0, 1)$ (or $\frac{U_X - E(U_X)}{\sqrt{Var(U_X)}} \approx N(0, 1)$) The proof is omitted, but some notes are given below. • This Thm does not follow immediatedly from the ordinary CLT alth	$(n+m+1)$ $I2m$ (\overline{R}) (\overline{R}) $\Delta = 0$ $Of U_{Y}$ $De used to the second to the se$
1 samj	sample $(R_{n+1}, \ldots, R_{m+n})$. Then (by Thms 1 & 3 in LN, Ch7, p.16-1 $R = Size = m$ $\frac{E(\overline{R})}{(N+1)/2} = \mu$ and $Var(\overline{R}) = (\sigma^2/m)[(N-m)/(N-1)] = m$ • The results follows from $E(W_Y) = m E(\overline{R})$ and $Var(W_Y) = m^2 Var$ Theorem 13 (Asymptotic null distribution of U_Y) Consider the nonparametric model (\Diamond) in LNp.35 and the null H_0 : ($\Leftrightarrow \pi_{\Delta} = 1/2$). For m, n both greater than 10, the null distribution (or U_X) is well approximated by a normal distribution, i.e., $(\Rightarrow -\frac{V_X}{\sqrt{Var(U_Y)}} \approx N(0, 1)$ (or $\frac{U_X - E(U_X)}{\sqrt{Var(U_X)}} \approx N(0, 1)$) The proof is omitted, but some notes are given below. • This Thm does not follow immediatedly from the ordinary CLT alth	$(n+m+1)$ $I2m$ (\overline{R}) (\overline{R}) $\Delta = 0$ $Of U_{Y}$ $De used to the second to the se$
1 samj	sample $(R_{n+1}, \ldots, R_{m+n})$. Then (by Thms 1 & 3 in LN, Ch7, p.16-1 $R = Size = m$ $\frac{E(\overline{R})}{(N+1)/2} = \mu$ and $Var(\overline{R}) = (\sigma^2/m)[(N-m)/(N-1)] = m$ • The results follows from $E(W_Y) = m E(\overline{R})$ and $Var(W_Y) = m^2 Var$ Theorem 13 (Asymptotic null distribution of U_Y) Consider the nonparametric model (\Diamond) in LNp.35 and the null H_0 : ($\Leftrightarrow \pi_{\Delta} = 1/2$). For m, n both greater than 10, the null distribution (or U_X) is well approximated by a normal distribution, i.e., $(\Rightarrow -\frac{V_X}{\sqrt{Var(U_Y)}} \approx N(0, 1)$ (or $\frac{U_X - E(U_X)}{\sqrt{Var(U_X)}} \approx N(0, 1)$) The proof is omitted, but some notes are given below. • This Thm does not follow immediatedly from the ordinary CLT alth	$(n+m+1)$ $I2m$ (\overline{R}) (\overline{R}) $\Delta = 0$ $Of U_{Y}$ $De used to the second to the se$
	sample $(R_{n+1}, \ldots, R_{m+n})$. Then (by Thms 1 & 3 in LN, Ch7, p.16-1 le size = $\underbrace{E(\overline{R}) = \mu}_{(N+1)/2}$ and $\underbrace{Var(\overline{R}) = (\sigma^2/m)[(N-m)/(N-1)]}_{(N-1)/2} = \underbrace{M}_{(N-1)/2} = \underbrace{M}_{(N-1)/2$	$(n+m+1)$ $I \ge m$ (\overline{R}) $($
	sample $(R_{n+1}, \ldots, R_{m+n})$. Then (by Thms 1 & 3 in LN, Ch7, p.16-1 le size = $\underbrace{E(\overline{R}) = \mu}_{(N+1)/2}$ and $\underbrace{Var(\overline{R}) = (\sigma^2/m)[(N-m)/(N-1)]}_{(N-1)/2} = \underbrace{I}_{(N+1)/2} = \underbrace{I}_{(N+1)/2$	$(R),$ $(n+m+1)$ $I \ge m$ $(R).$ $(R).$ $(R).$ $(L) = 0$

made by S.-W. Cheng (NTHU, Taiwan)

NTHU STAT 3875, 2018

Lecture Notes

	Lecture Note
	Ch 11, p. 42
	Example 5 (Asymptotic null dist. of $\underline{W}_{\underline{Y}}$, heat of fusion of ice, cont. Ex.4 in LNp.34)
	• $\underline{n=13} \pmod{\underline{A}}, \underline{m=8} \pmod{\underline{B}}, \underline{W_B=51}.$
	• Under the <u>null</u> , $\mu_{W_B} = \underline{E(W_B)} = [\underline{8}(\underline{8}+\underline{13}+\underline{1})]/\underline{2} = \underline{88},$
	known constant, $\sigma_{W_B} = \sqrt{Var(W_B)} = \sqrt{[(8 \times 13)(8+13+1)]/12} = 13.8.$
	$ \begin{array}{c} \textbf{known constant.} \\ \textbf{not parameter} \\ \bullet \\ \textbf{Because} \\ \textbf{N(0.1)} \\ \end{array} \begin{array}{c} \sigma_{W_B} = \sqrt{Var(W_B)} = \sqrt{[(8 \times 13)(8 + 13 + 1)]/12} = 13.8. \\ \hline \textbf{W}_B - \mu_{W_B} \\ \hline \textbf{w}_B - \mu_{W_B} \\ \hline \textbf{w}_B $
	the approximate <u>p</u> -value is $P(N(0,1) > 2.68) = 2 \times [1 - \Phi(2.68)] = 0.0074$
	$(\Rightarrow \underline{\text{reject } H_0} \text{ at } \alpha = 0.01 \Rightarrow \underline{\text{consistent}} \text{ with the } \underline{\text{testing result using } \underline{\text{exact}}}$
	null distribution in Ex.4) check UVp.33
	Theorem 14 (Nonparametric confidence interval for $\underline{\Delta}$)
	Consider the nonparametric model (\Diamond) in LNp.35.
betwee	where A is a known constant?
C.L.&	
testin	
	$\begin{array}{c} \text{Then,} \\ \hline X_1^* & X_0^* \\ \hline \text{new data} \\ \hline X_1, \dots, \\ \hline X_n, \\ \hline Y_1 - \underline{\Delta_0}, \dots, \\ \hline Y_m - \underline{\Delta_0} \\ \hline Y_m \\ \hline \\ \hline Y_m - \underline{\Delta_0} \\ \hline \\ \hline \\ i.i.d. \\ \hline F \\ \hline \\ \hline \\ H_{\Delta}: \Delta \neq 0 \\ \hline \end{array}$
	- The test of \underline{H}_0^* vs. \underline{H}_A^* using the data \underline{X}_i 's and \underline{Y}_j 's is equivalent to
	testing $\underline{H}_0: \underline{\Delta} = 0$ vs. $\underline{H}_A: \underline{\Delta} \neq 0$ using the data \underline{X}_i 's and $(\underline{Y}_j - \underline{\Delta}_0)$'s.
	- To test $\underline{H_0^*}$: $\underline{\Delta = \Delta_0}$, can use
V8	* the <u>test statistic</u> : $U_Y(\Delta_0) = \#\{X_i < Y_j - \Delta_0\} = \#\{Y_j - X_i > \Delta_0\},$
	* the <u>test statistic</u> : $U_Y(\underline{\Delta_0}) = \#\{X_i \leq Y_j - \underline{\Delta_0}\} = \#\{Y_j - X_i > \underline{\Delta_0}\},\$ * the acceptance region: $k(\alpha) \leq U_Y(\Delta_0) \leq mn - k(\alpha),$ [What if $\Delta_0 = 0$?]
it	* the <u>test statistic</u> : $U_{Y}(\underline{\Delta}_{0}) = \#\{X_{i} \leq Y_{j} - \underline{\Delta}_{0}\} = \#\{Y_{j} - X_{i} > \underline{\Delta}_{0}\},\$ * the acceptance region: $\underline{k}(\underline{\alpha}) \leq U_{Y}(\underline{\Delta}_{0}) \leq \underline{mn} - \underline{k}(\underline{\alpha}),$ What if $\underline{\Delta}_{0} = 0$? where $\underline{k}(\underline{\alpha})$ is the critical value determined by the significance level $\underline{\alpha}$
it int	* the acceptance region: $\underline{k(\alpha)} \leq \underline{U_Y(\Delta_0)} \leq \underline{mn - k(\alpha)}$, What if $\Delta_0 = 0$? where $\underline{k(\alpha)}$ is the critical value determined by the significance level α (Note. The null distribution of $\underline{U_Y(\Delta_0)}$ check of module dist.)
it int	* the acceptance region: $\underline{k(\alpha)} \leq \underline{U_Y(\Delta_0)} \leq \underline{mn - k(\alpha)}$, What if $\Delta_0 = 0$? where $\underline{k(\alpha)}$ is the critical value determined by the significance level α
it int	* the acceptance region: $\underline{k(\alpha)} \leq \underline{U_Y(\Delta_0)} \leq \underline{mn} - \underline{k(\alpha)}$, What if $\Delta_0 = 0$? where $\underline{k(\alpha)}$ is the critical value determined by the significance level α (Note. The null distribution of $\underline{U_Y(\Delta_0)}$ check is symmetric about $\underline{mn/2}$.) \underline{Check} by the duality of test and C.I., a $100(1-\alpha)\%$ confidence interval for Δ is
k(a) cau	* the acceptance region: $\underline{k}(\underline{\alpha}) \leq \underline{U}_{Y}(\Delta_{0}) \leq \underline{mn} - \underline{k}(\underline{\alpha})$, What if $\Delta_{0} = 0$? where $\underline{k}(\underline{\alpha})$ is the critical value determined by the significance level $\underline{\alpha}$ (Note. The null distribution of $\underline{U}_{Y}(\Delta_{0})$ check is symmetric about $\underline{mn/2}$.) check \underline{mn} modelist. • By the duality of test and C.I., a $\underline{100(1 - \alpha)\%}$ confidence interval for $\underline{\Delta}$ is $\underline{C} = \{ \Delta \mid k(\alpha) \leq U_{Y}(\Delta) \leq \underline{mn} - k(\alpha) \}$
k(a) can bbtained the crit	* the acceptance region: $\underline{k}(\underline{\alpha}) \leq \underline{U}_{Y}(\Delta_{0}) \leq \underline{mn} - \underline{k}(\underline{\alpha})$, What if $\Delta_{0} = 0$? where $\underline{k}(\underline{\alpha})$ is the critical value determined by the significance level $\underline{\alpha}$ (Note. The null distribution of $\underline{U}_{Y}(\Delta_{0})$ check is symmetric about $\underline{mn/2}$.) check \underline{mn} modelist. • By the duality of test and C.I., a $\underline{100(1 - \alpha)\%}$ confidence interval for $\underline{\Delta}$ is $\underline{C} = \{ \Delta \mid k(\alpha) \leq U_{Y}(\Delta) \leq \underline{mn} - k(\alpha) \}$
k(a) cau obtained the <u>Crit</u>	* the acceptance region: $\underline{k}(\underline{\alpha}) \leq \underline{U}_{Y}(\Delta_{0}) \leq \underline{mn} - \underline{k}(\underline{\alpha})$, What if $\Delta_{0} = 0$? where $\underline{k}(\underline{\alpha})$ is the critical value determined by the significance level $\underline{\alpha}$ (Note. The null distribution of $\underline{U}_{Y}(\Delta_{0})$ check is symmetric about $\underline{mn/2}$.) check \underline{mn} modelist. • By the duality of test and C.I., a $\underline{100(1 - \alpha)\%}$ confidence interval for $\underline{\Delta}$ is $\underline{C} = \{ \Delta \mid k(\alpha) \leq U_{Y}(\Delta) \leq \underline{mn} - k(\alpha) \}$
k(a) can obtained the crit value o Wy (ch Thmil, U	* the acceptance region: $\underline{k}(\underline{\alpha}) \leq \underline{U}_{Y}(\Delta_{0}) \leq \underline{mn} - \underline{k}(\underline{\alpha})$, What if $\Delta_{0} = 0$? where $\underline{k}(\underline{\alpha})$ is the critical value determined by the significance level $\underline{\alpha}$ (Note. The null distribution of $\underline{U}_{Y}(\Delta_{0})$ check is symmetric about $\underline{mn/2}$.) check \underline{mn} modelist. • By the duality of test and C.I., a $\underline{100(1 - \alpha)\%}$ confidence interval for $\underline{\Delta}$ is $\underline{C} = \{ \Delta \mid k(\alpha) \leq U_{Y}(\Delta) \leq \underline{mn} - k(\alpha) \}$
k(a) cau obtained the <u>Crit</u>	* the acceptance region: $\underline{k}(\underline{\alpha}) \leq \underline{U}_{Y}(\Delta_{0}) \leq \underline{mn} - \underline{k}(\underline{\alpha})$, What if $\Delta_{0} = 0$? where $\underline{k}(\underline{\alpha})$ is the critical value determined by the significance level $\underline{\alpha}$ (Note. The null distribution of $\underline{U}_{Y}(\Delta_{0})$ check is symmetric about $\underline{mn/2}$.) check \underline{mn} modelist. • By the duality of test and C.I., a $\underline{100(1 - \alpha)\%}$ confidence interval for $\underline{\Delta}$ is $\underline{C} = \{ \Delta \mid k(\alpha) \leq U_{Y}(\Delta) \leq \underline{mn} - k(\alpha) \}$
k(a) cau obtained the crit value Wy (ch Thmil, u in Tab	* the acceptance region: $k(\underline{\alpha}) \leq U_Y(\Delta_0) \leq \underline{mn} - k(\underline{\alpha})$, What if $\Delta_0 = 0$? where $k(\underline{\alpha})$ is the critical value determined by the significance level $\underline{\alpha}$ (Note. The null distribution of $U_Y(\Delta_0)$ check is symmetric about $\underline{mn/2}$.) Check \underline{mn} models is $\underline{mn} - \underline{k(\alpha)}$ • By the duality of test and C.I., a $100(1 - \alpha)\%$ confidence interval for Δ is $\underline{C} = \{\Delta \mid \underline{k(\alpha)} \leq U_Y(\Delta) \leq \underline{mn} - \underline{k(\alpha)}\}.$ Fixed • Let $\underline{D_{(1)}, D_{(2)}, \dots, D_{(mn)}}$ denote the pivotal guantity $\underline{crdered} \ \underline{mn} \ \underline{differences} \ (\underline{Y_j - X_i})$'s. • Then, $\underline{C} = [\underline{D_{(k(\alpha))}}, \ \underline{D_{(mn-k(\alpha)+1)}}].$ What $\underline{C} = [\underline{D_{(k(\alpha))}}, \ \underline{D_{(mn-k(\alpha)+1)}}].$ What $\underline{C} = [\underline{D_{(k(\alpha))}}, \ \underline{D_{(mn-k(\alpha)+1)}}].$
k(a) cau obtained the crit value Wy (ch Thmil, u in Tab	* the acceptance region: $k(\underline{\alpha}) \leq U_Y(\Delta_0) \leq \underline{mn} - k(\underline{\alpha})$, What if $\Delta_0 = 0$? where $k(\underline{\alpha})$ is the critical value determined by the significance level $\underline{\alpha}$ (Note. The null distribution of $U_Y(\Delta_0)$ Check is symmetric about $\underline{mn/2}$.) • By the duality of test and C.I., a $100(1 - \alpha)\%$ confidence interval for Δ is $C = \{\Delta \mid \underline{k}(\alpha) \leq U_Y(\Delta) \leq \underline{mn} - k(\alpha)\}.$ • Let $\underline{D}_{(1)}, \underline{D}_{(2)}, \dots, \underline{D}_{(mn)}$ denote the $\underline{\text{ordered}} \ \underline{mn} \ \underline{\text{differences}} \ (\underline{Y}_j - X_i)$'s. Then, $\underline{C} = [\underline{D}_{(k(\alpha))}, \ \underline{D}_{(mn-k(\alpha)+1)}].$ • To see this, * if $\underline{\Delta}_0 \equiv \underline{D}_{(k(\alpha))}$, then $\underline{U}_Y(\Delta_0) = \#\{\underline{Y}_j - X_i > \Delta_0\} \equiv \underline{mn} - \underline{k}(\alpha), \ast accept$
k(a) cau obtained the crit value Wy (ch Thmil, u in Tab	* the acceptance region: $k(\underline{\alpha}) \leq U_Y(\Delta_0) \leq \underline{mn} - k(\underline{\alpha})$, What if $\Delta o = 0$? where $k(\underline{\alpha})$ is the critical value determined by the significance level $\underline{\alpha}$ (Note. The null distribution of $U_Y(\Delta_0)$ is symmetric about $\underline{mn/2}$.) • By the duality of test and C.I., a $100(1 - \alpha)\%$ confidence interval for Δ is $\underline{C} = \{\Delta \mid \underline{k}(\alpha) \leq U_Y(\Delta) \leq \underline{mn} - k(\alpha)\}.$ Fixed • Let $\underline{D_{(1)}}, \underline{D_{(2)}}, \dots, \underline{D_{(mn)}}$ denote the given and the given an
k(a) cau obtained the crit value Wy (ch Thmil, u in Tab	* the acceptance region: $k(\underline{\alpha}) \leq U_Y(\Delta_0) \leq \underline{mn} - k(\underline{\alpha})$, What if $\Delta o = 0$? where $k(\underline{\alpha})$ is the critical value determined by the significance level $\underline{\alpha}$ (Note. The null distribution of $U_Y(\Delta_0)$ is symmetric about $\underline{mn/2}$.) • By the duality of test and C.I., a $100(1 - \alpha)\%$ confidence interval for Δ is $\underline{C} = \{\Delta \mid \underline{k}(\alpha) \leq U_Y(\Delta) \leq \underline{mn} - k(\alpha)\}.$ Fixed • Let $\underline{D_{(1)}}, \underline{D_{(2)}}, \dots, \underline{D_{(mn)}}$ denote the given and the given an
k(a) cau obtained the crit value Wy (ch Thmil, u in Tab	* the acceptance region: $k(\underline{\alpha}) \leq \underline{U}_{Y}(\Delta_{0}) \leq \underline{mn} - k(\underline{\alpha})$, What if $\Delta_{0} = 0$? where $k(\underline{\alpha})$ is the critical value determined by the significance level $\underline{\alpha}$ (Note. The null distribution of $U_{Y}(\Delta_{0})$ check is symmetric about $\underline{mn/2}$.) • By the duality of test and C.I., a $100(1 - \alpha)\%$ confidence interval for Δ is $C = \{\Delta \mid k(\alpha) \leq U_{Y}(\Delta) \leq \underline{mn} - k(\alpha)\}$. • Let $\underline{D}_{(1)}, \underline{D}_{(2)}, \dots, \underline{D}_{(mn)}$ denote the pivotal $\underline{pivotal}$ $\underline{mn \cdot k(\alpha)} + \underline{l}$ $\underline{k(\alpha)} \leq \underline{U}_{Y}(\Delta) \leq \underline{mn - k(\alpha)}\}$. • To see this, * if $\Delta_{0} \equiv \underline{D}_{(k(\alpha))}$, then $\underline{U}_{Y}(\Delta_{0}) = \#\{\underline{Y}_{j} - X_{i} > \Delta_{0}\} \equiv \underline{mn - k(\alpha)} + 1$, * if $\underline{\Delta}_{0} \leq \underline{D}_{(k(\alpha))}$, then $\underline{U}_{Y}(\Delta_{0}) = \#\{\underline{Y}_{j} - X_{i} > \Delta_{0}\} \geq \underline{mn - k(\alpha)} + 1$, * if $\underline{\Delta}_{0} \leq \underline{D}_{(k(\alpha))}$, then $\underline{U}_{Y}(\Delta_{0}) = \#\{\underline{Y}_{j} - X_{i} > \Delta_{0}\} \geq \underline{mn - k(\alpha)} + 1$, * if $\underline{\Delta}_{0} \leq \underline{D}_{(k(\alpha))}$, then $\underline{U}_{Y}(\Delta_{0}) = \#\{\underline{Y}_{j} - X_{i} > \Delta_{0}\} \geq \underline{mn - k(\alpha)} + 1$, * if $\underline{\Delta}_{0} \leq \underline{D}_{(k(\alpha))}$, then $\underline{U}_{Y}(\Delta_{0}) = \#\{\underline{Y}_{j} - X_{i} > \Delta_{0}\} \geq \underline{k(\alpha)}$, \underline{accept} * if $\underline{\Delta}_{0} \leq \underline{D}_{(mn-k(\alpha)+1)}$, then $\underline{U}_{Y}(\Delta_{0}) = \#\{\underline{Y}_{j} - X_{i} > \Delta_{0}\} \geq \underline{k(\alpha)}$, \underline{accept}
k(a) cau obtained the crit value Wy (ch Thmil, u in Tab	* the acceptance region: $k(\underline{\alpha}) \leq U_Y(\Delta_0) \leq \underline{mn} - k(\underline{\alpha})$, What if $\Delta o = 0$? where $k(\underline{\alpha})$ is the critical value determined by the significance level $\underline{\alpha}$ (Note. The null distribution of $U_Y(\Delta_0)$ is symmetric about $\underline{mn/2}$.) • By the duality of test and C.I., a $100(1 - \alpha)\%$ confidence interval for Δ is $\underline{C} = \{\Delta \mid \underline{k}(\alpha) \leq U_Y(\Delta) \leq \underline{mn} - k(\alpha)\}.$ Fixed • Let $\underline{D_{(1)}}, \underline{D_{(2)}}, \dots, \underline{D_{(mn)}}$ denote the given and the given an

made by S.-W. Cheng (NTHU, Taiwan)