Point Estimation —點估計

• What is point estimation?

Recall. The 4 steps of statistics (Introduction, LNp.2)

Example 6.1 (current across muscle cell membrane, TBp. 257-258)

- Bevan, Kullberg, and Rice (1979) studied random fluctuations of current across a muscle cell membrane. The cell membrane contained a large number of channels, which opened and closed at random and were assumed to operate independently. The net current resulted from ions flowing through open channels.
- They obtained 49,152 observations of the net current, $x_1, \ldots, x_{49152}$. observed data
- The net current was the sum of a large number of roughly independent small currents. statistical modeling CLT meaning? 規律 random variables
- It seems appropriate to model the net current data, $X_1, \ldots, X_{49152}$ as i.i.d. $N(\mu, \sigma^2)$, where μ and σ^2 represent the mean and variance of net current. Note that the values of μ and σ^2 are unknown. systematic pattern parameters
- Question: how to use the observed data, $x_1, \ldots, x_{49152}$ to gain knowledge about the values of μ and σ^2? transformation = ?

cf. cf.

Example 6.2 (emission of alpha particles, TBp. 255-256)

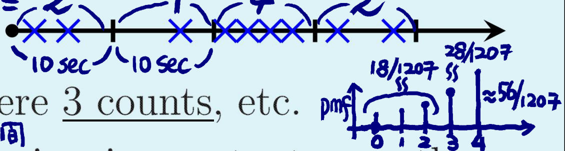
- Berkson (1966) conducted an experiment about emission of alpha particles from radioactive sources. The number of emissions per unit of time is not constant but fluctuates in a random fashion.
- The experimenter recorded 10,220 times between successive emissions. The numbers of emissions, x_i, $i = 1, \ldots, 1027$, observed in 1207 time intervals, each of length 10 sec, are summarized in the following table:

observed data

$x_i \in \{0, 1, 2\}$	$x_i = 3$	$x_i = 4$	$\cdots$
18	28	56	$\cdots$

histogram

$x_i = 2 + 1 + 4 + 2 + \cdots = 10220$



e.g., in 28 of the 1207 intervals, there were 3 counts, etc.

- Assume (1) the underlying rate of emission is constant over the period of observation (2) the particles come from a very large number of independent sources 次/單位時間 Recall. Poisson approximation to $B(n,p)$ when $n \uparrow \infty$, $p \downarrow 0$, $np \to \lambda$ meaning? statistical modeling
- It seems appropriate to model the numbers of emissions $X_1, \ldots, X_{1027}$ as i.i.d. $P(\lambda)$, where λ represents the underlying rate of emission. Note that the value of λ is unknown. r.v.'s systematic pattern cf.
- Question: how to use the observed data, $x_1, \ldots, x_{1207}$, to gain knowledge about the value of λ? transformation = ?

Example 6.3 (rainfall amount, TBp. 258-259)

- Le Cam and Neyman (1967) studied rainfall amounts from storms. random 該圖
- They obtained rainfall amount data, see the graphs for the histogram of the data. Let us denote $x_1, \ldots, x_{227}$ as the 227 rainfall amounts. observed data
- statistical modeling: The family of $\Gamma(\alpha, \lambda)$, where $\alpha > 0, \lambda > 0$, provides a flexible set of pdfs for non-negative random variable. We may model the rainfall amount data, $X_1, \ldots, X_{227}$ as i.i.d. $\sim \Gamma(\alpha, \lambda)$. Note that the values of α and λ are unknown. random variables transformation=? systematic pattern
- Question: how to use $x_1, \ldots, x_{227}$ to find a *particular* Gamma distribution $\Gamma(\alpha_0, \lambda_0)$ that can "best" fit the observed data, i.e., which pdf of Gamma is "mostly similar" to the histogram? c.f. Ex 6.1 & Ex. 6.2

histogram (TBp. 74~75, 200~201): a surrogate of pdf/pmf

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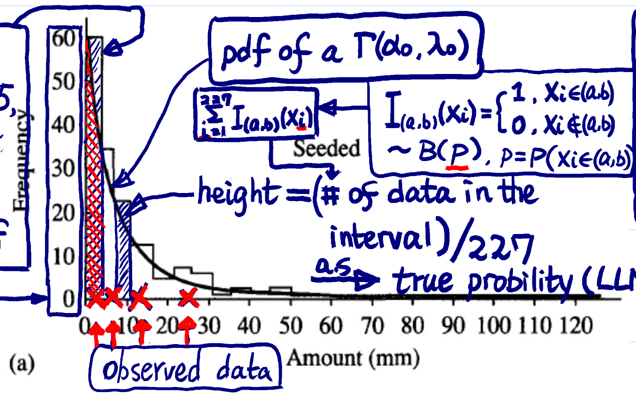


observed data

pdf of a $\Gamma(\alpha_0, \lambda_0)$

$\sum_{i=1}^{227} I_{(a,b)}(X_i)$

$I_{(a,b)}(X_i) = \begin{cases} 1, & X_i \in (a,b) \\ 0, & X_i \notin (a,b) \end{cases}$ $\sim B(P)$, $P = P(X_i \in (a,b))$

height = (# of data in the interval)/227 $\xrightarrow{a.s.}$ true probility (LLN)

Q: What information does a histogram carry?

Q: What's the difference between the 3 statistical modelings?

Ans conceptual (Ex. 6-1 & 6-2) versus empirical (Ex. 6-3)

Summary (procedure of fitting a particular distribution to data, i.e. point estimation)

1. **observed data.** $x_1, \ldots, x_n$
   - **Ex 6.1:** 49152 net currents; **Ex 6.2:** 1027 numbers of emissions; **Ex 6.3:** 227 rainfall amounts
2. **statistical modeling.** Regard $x_1, \ldots, x_n$ as a ☆realization (呈現值) of random variables $X_1, \ldots, X_n$, and assign $X_1, \ldots, X_n$ a joint distribution:

$$\text{a joint cdf } F(\cdot|\Theta), \quad \text{or a joint pdf } f(\cdot|\Theta), \quad \text{or a joint pmf } p(\cdot|\Theta),$$

This is a distribution family, ∵ Θ have many plausible values, i.e., $\Theta \in$ parameter space

cf.

parameters

where $\Theta = (\theta_1, \ldots, \theta_k)$, and θ_i's are **fixed constants**, but their values are **unknown**.

Note. With the assumptions, indep.: marginal → joint; identical: accumulate information for same pattern,

   - **Ex 6.1:** i.i.d. Normal, $\Theta = (\mu, \sigma^2)$ ← $\mu \in \mathbb{R}$, $\sigma > 0$
   - **Ex 6.2:** i.i.d. Poisson, $\Theta = \lambda$ ← $\lambda > 0$
   - **Ex 6.3:** i.i.d. Gamma, $\Theta = (\alpha, \lambda)$ ← $\alpha > 0, \lambda > 0$

a r.v. hat

3. **point estimation.** Find **a function of** $X_1, \ldots, X_n$, denoted by $\widehat{\Theta}$, to estimate Θ or a function of Θ, and substitute $x_1, \ldots, x_n$ to get an estimate. Why?

The solution to the questions in Ex. 6-1 ~ 6-3

Notes

1. In statistical modeling, we define (Objective of Data Analysis: identify systematic pattern by removing random disturb.)

   — (unknown) systematic pattern ← θ | 不變 | 規律 | certain | signal | 靜 | 陰

   — random disturbance ← r.v.'s | 變 | 隨機 | uncertain | noise | 動 | 陽

   in the data.

   隨機 → 萬變不離其宗 ← 不變, 規律 (unknown, 規律)

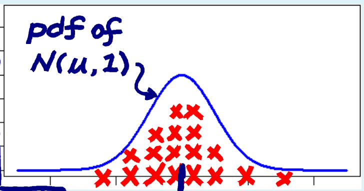
   - For example, unknown ($\mu \in \mathbb{R}^1$)

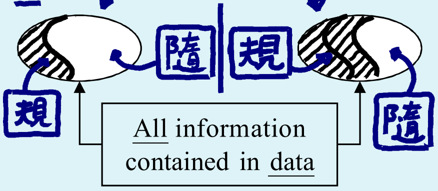
     Say, X_i's heights → (1) $X_1, \ldots, X_n$ i.i.d. $\sim N(\mu, 1)$ 規律 (2) $X_1, \ldots, X_n$ i.i.d. $\sim N(\mu, \sigma^2)$ (規律, known, unknown)

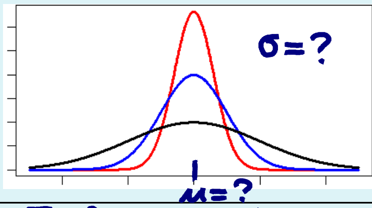
     $X_1 = \mu + \varepsilon_1 \cdots X_n = \mu + \varepsilon_n$ 規律 different modeling $X_1 = \mu + \sigma\varepsilon_1 \cdots X_n = \mu + \sigma\varepsilon_n$

     隨機 → $\varepsilon_1, \ldots, \varepsilon_n$ iid $\sim N(0, 1)$ $\varepsilon_1, \ldots, \varepsilon_n$ iid $\sim N(0, 1)$

Information about μ & ε_i's are mixed in X_i's and indistingable







68% → $\mu \pm \sigma$
95% → $\mu \pm 2\sigma$
99.7% → $\mu \pm 3\sigma$

Compare their difference: $\bar{X}_n \xrightarrow{P} \mu$ (WLLN)

— prediction of μ (parameter) $\Rightarrow E(\bar{X}_n) = \mu$, $Var(\bar{X}_n) = \sigma^2/n$

— prediction of X_{n+1}(r.v.) $= \mu + \epsilon_{n+1} \Leftarrow \epsilon_{n+1} \sim N(0, 1)$

- The assumptions given in statistical modeling (i.e., joint distribution) need to be examined.