NTHU MATH 2820, 2025



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• 8 method • 8 method • 8 method • 9 method • 10 % method for univariate case, TBp. 162) • 10 % method for univariate case, TBp. 163) • 10 % method for univariate case, TBp.			CIT~6. D.2-58
Let $Y = g(X)$. Suppose we only know the mean μ_X and variance σ_X^2 of X , but not the entire distribution (i.e., do not know cdf, pdf/pmf of X). Can we derive the distribution of Y ? If not, can we "roughly" describe the mean and variance of Y ? (Note. $E[g(X)] \neq g[E(X)]$.) Theorem 3.9 (8 method for univariate case, TBp. 162) Thm3.1 (LV_{b} 41) We hold? $Y = g(X) \boxtimes g(\mu_X) + (X - \mu_X)g'(\mu_X)$ (by Taylor expansion) $\Rightarrow E[g(X)] \approx g(\mu_X) + (X - \mu_X)g'(\mu_X)$ (by Taylor expansion) $\Rightarrow E[g(X)] \approx g(\mu_X) + (X - \mu_X)g'(\mu_X) + \frac{1}{2}(X - \mu_X)^2g''(\mu_X)$ $\Rightarrow E[g(X)] \approx g(\mu_X) + \frac{1}{2}\sigma_X^2g''(\mu_X)$ Note. How good these approximations are depends on whether g can be reasonably well approximated by the 1st or 2nd order polynomials in a neigh- borhood of μ_X and on the size of σ_X for each Chebyshev's inequality (LV_{b} 42) Function of two univariate random variables $Z = g(X, Y)$: Let $\mu = (\mu_X, \mu_Y)$. $Z = g(X, Y) \cong g(\mu) + (X - \mu_X) \frac{\partial g(\mu)}{\partial x} + (Y - \mu_Y) \frac{\partial g(\mu)}{\partial y}$ $\Rightarrow E(Z) \approx g(\mu)$ $\forall x_X M_X$ $\forall x_X M_X$ $\Rightarrow E(Z) \approx g(\mu)$ $\forall x_X (Z) \approx \sigma_X^2 \left[\frac{\partial g(\mu)}{\partial x} \right]^2 + \sigma_X^2 \left[\frac{\partial g(\mu)}{\partial y} \right]^2 + 2\sigma_{XY} \left[\frac{\partial g(\mu)}{\partial x} \right] \left[\frac{\partial g(\mu)}{\partial y} \right]$ $\Rightarrow E[g(X,Y)] \approx g(\mu) + (X - \mu_X) \frac{\partial g(\mu)}{\partial x^2} + (Y - \mu_Y) \frac{\partial g(\mu)}{\partial y} \right]$ $\Rightarrow E[g(X,Y)] \approx g(\mu) + (X - \mu_X) \frac{\partial g(\mu)}{\partial x^2} + (Y - \mu_Y) \frac{\partial g(\mu)}{\partial y} + \frac{1}{2}(X - \mu_X)^2 \frac{\partial^2 g(\mu)}{\partial y^2} + \frac{1}{2}(Y - \mu_Y)^2 \frac{\partial^2 g(\mu)}{\partial y^2} + \frac{1}{2}(Y - \mu_Y)^2 \frac{\partial^2 g(\mu)}{\partial y^2} + \frac{1}{2}(Y - \mu_Y)^2 \frac{\partial^2 g(\mu)}{\partial x^2} + \frac{1}{2}(Y - \mu_Y)^2 \partial^2 $	• δ method	Question 3.3	Recall Chebyshev's ineguality (LNp.43)-
and variance σ_X^2 of X , but not the entire distribution (i.e., do not know cdf, pdf/pmf of X). Can we derive the distribution of Y ? If not, can we "roughly" describe the mean and variance of Y ? (Note. $E[g(X)] \neq g[E(X)]$.) Theorem 3.9 (6 method for univariate case, TBp. 162) Thm3.1 (LN_p 441) Were down $Y = g(X) \bigotimes g(\mu_X) + (X - \mu_X)g'(\mu_X)$ (by Taylor expansion) $\Rightarrow E[g(X)] \approx g(\mu_X) + (X - \mu_X)g'(\mu_X) + \frac{1}{2}(X - \mu_X)^2g''(\mu_X)$ $\Rightarrow E[g(X)] \approx g(\mu_X) + (X - \mu_X)g'(\mu_X) + \frac{1}{2}(X - \mu_X)^2g''(\mu_X)$ $\Rightarrow E[g(X)] \approx g(\mu_X) + \frac{1}{2}\sigma_X^2g''(\mu_X)$ Note. How good these approximated by the lst or 2nd order polynomials in a neigh- borhood of μ_X and on the size of σ_X check Chebyshevs inequality (LN_p 42) Function of two univariate case, TBp. 165) Function of two univariate random variables $Z = g(X, Y)$: Let $\mu = (\mu_X, \mu_Y)$. $Z = g(X, Y) \cong g(\mu) + (X - \mu_X) \frac{\partial g(\mu)}{\partial x} + (Y - \mu_Y) \frac{\partial g(\mu)}{\partial y}$ $\Rightarrow E(Z) \approx g(\mu)$ $Var(Z) \approx \sigma_X^2 \left[\frac{\partial g(\mu)}{\partial x} \right]^2 + \sigma_X^2 \left[\frac{\partial g(\mu)}{\partial y} \right]^2 + 2\sigma_{XY} \left[\frac{\partial g(\mu)}{\partial x} \right] \left[\frac{\partial g(\mu)}{\partial y} \right]$ $\Rightarrow E[g(X, Y]) \approx g(\mu) + (X - \mu_X) \frac{\partial g(\mu)}{\partial x} + (Y - \mu_Y) \frac{\partial g(\mu)}{\partial y} \right]$ $\Rightarrow E(Z) \approx g(\mu)$ $Var(Z) \approx \sigma_X^2 \left[\frac{\partial g(\mu)}{\partial x^2} \right]^2 + (X - \mu_X)(Y - \mu_Y) \frac{\partial g(\mu)}{\partial x} \right]$ $\Rightarrow E[g(X, Y]] \approx g(\mu) + \frac{1}{2}\sigma_X^2 \frac{\partial^2 g(\mu)}{\partial x^2} + \sigma_X Y \frac{\partial^2 g(\mu)}{\partial x \partial y} + \frac{1}{2}\sigma_X^2 \frac{\partial^2 g(\mu)}{\partial x^2} \right]$ Note. The general case of a function of \underline{n} random variables can be worked out similarly. a Reading: textbook, Chapter 4 \rightarrow required b Enderive function of \underline{n} random variables can be worked out similarly.	Y	Let $Y = g$	$\mu(X)$. Suppose we only know the mean μ_X
(i.e., do not know cdf, pdf/pmf of X). Can we derive the distribution of Y? If not, can we "roughly" describe the mean and variance of Y? (Note. $E[g(X)] \neq g[E(X)]$.) Theorem 3.9 (8 method for univariate case, TBp. 162) The TAM 3.1 (LVb 41) Were day $Y = g(X) \cong g(\mu_X) + (X - \mu_X)g'(\mu_X)$ (by Taylor expansion) $\Rightarrow E[g(X)] \approx g(\mu_X) + (X - \mu_X)g'(\mu_X)$ (by Taylor expansion) $\forall ar[g(X)] \approx Yar(X)[g'(\mu_X)]^2$ or $Y = g(X) \cong g(\mu_X) + (X - \mu_X)g'(\mu_X) + \frac{1}{2}(X - \mu_X)^2g''(\mu_X)$ $\Rightarrow E[g(X)] \approx g(\mu_X) + \frac{1}{2}\sigma_X^2g''(\mu_X)$ Note. How good these approximations are depends on whether g can be reasonably well approximated by the lst or 2nd order polynomials in a neighborhood of μ_X and on the size of σ_X check Chebyshev's inequality (LVb, 4.2) Function of two univariate random variables $Z = g(X, Y)$: Let $\mu = (\mu_X, \mu_Y)$. $Z = g(X, Y) \cong g(\mu) + (X - \mu_X) \frac{\partial g(\mu)}{\partial x} + (Y - \mu_Y) \frac{\partial g(\mu)}{\partial y}$ $\Rightarrow E(Z) \approx g(\mu)$ $\forall x(Z) \approx \sigma_X^2 \left[\frac{\partial g(\mu)}{\partial x} \right]^2 + \sigma_Y^2 \left[\frac{\partial g(\mu)}{\partial y} \right]^2 + 2\sigma_{XY} \left[\frac{\partial g(\mu)}{\partial x} \right] \left[\frac{\partial g(\mu)}{\partial y} \right]$ $\Rightarrow E[g(X,Y)] \cong g(\mu) + (X - \mu_X) \frac{\partial g(\mu)}{\partial x} + (Y - \mu_Y) \frac{\partial g(\mu)}{\partial y} \right]$ $\Rightarrow E[g(X,Y)] \approx g(\mu) + (X - \mu_X) \frac{\partial g(\mu)}{\partial x} + (Y - \mu_Y) \frac{\partial g(\mu)}{\partial y} \right]$ $\Rightarrow E[g(X,Y)] \approx g(\mu) + (X - \mu_X) \frac{\partial g(\mu)}{\partial x} + (Y - \mu_Y) \frac{\partial g(\mu)}{\partial y} \right]$ $\Rightarrow E[g(X,Y)] \approx g(\mu) + \frac{1}{2}\sigma_X^2 \frac{\partial^2 g(\mu)}{\partial x^2} + \frac{\sigma_X \partial^2 g(\mu)}{\partial x} + \frac{1}{2}\sigma_X^2 \frac{\partial^2 g(\mu)}{\partial y^2} + \frac{1}{2}(Y - \mu_Y)^2 \frac{\partial^2 g(\mu)}{\partial y^2} + \frac{1}{2}\sigma_X^2 \frac{\partial^2 g(\mu)}{\partial x^2} + \frac{1}{2}\sigma_X^2 \partial^2 g($		and variance	e σ_X^2 of <u>X</u> , but not the entire distribution
distribution of Y? If not, can we "roughly" describe the mean and variance of Y? (Note. $E[g(X)] \neq g[E(X)]$.) Theorem 3.9 (8 method for univariate case, TBp. 162) The 3.1 (LVb.41) When dot? Y = $g(X) \gtrsim g(\mu_X) + (X - \mu_X)g'(\mu_X)$ (by Taylor expansion) $\Rightarrow E[g(X)] \approx g(\mu_X) + (X - \mu_X)g'(\mu_X) + \frac{1}{2}(X - \mu_X)^2g''(\mu_X)$ $\Rightarrow E[g(X)] \approx g(\mu_X) + \frac{1}{2}\sigma_X^2g''(\mu_X)$ Note. How good these approximations are depends on whether g can be reasonably well approximated by the 1st or 2nd order polynomials in a neigh- borhood of μ_X and on the size of σ_X . Check Chebyshev's inequality (LNp.43) Function of two univariate case, TBp. 165) Function of two univariate case, TBp. 162) Function of two univariate case, TBp. 165) Function of two univariate case, TBp. 162) Function of two univariate case, TB		(i.e., do not	know cdf, pdf/pmf of X). Can we derive the
$\frac{\text{mean and variance of } Y? \text{ (Note. } E[g(X)] \neq g[E(X)].\text{)}}{\text{Theorem 3.9 (6 method for univariate case, TBp. 162)} \text{Thm 3.1 } (LN_{b}441)}$ $\frac{\text{Theorem 3.9 (6 method for univariate case, TBp. 162)}{\text{Theorem 3.9 (6 method for univariate case, TBp. 162)} \text{Thm 3.1 } (LN_{b}441)}$ $\frac{\text{Wer. dow}}{\text{Hold?}} Y = g(\underline{X}) \approx g(\mu_{X}) + (X - \mu_{X})g'(\mu_{X}) \text{ (by Taylor expansion)}}{(\mathbf{C}, \mathbf{F})}$ $\frac{\text{Var}[g(X)]}{\text{Var}[g(X)]} \approx y(\mu_{X}) + (X - \mu_{X})g'(\mu_{X}) + \frac{1}{2}(X - \mu_{X})^{2}g''(\mu_{X})}$ $\Rightarrow E[g(X)] \approx g(\mu_{X}) + (X - \mu_{X})g'(\mu_{X}) + \frac{1}{2}(X - \mu_{X})^{2}g''(\mu_{X})$ Note. How good these approximations are depends on whether \underline{g} can be reasonably well approximated by the 1st or 2nd order polynomials in a neighborhood of $\underline{\mu_{X}}$ and on the size of σ_{X} . Check Chebyshev's inequality (LN_{P}, 43)} $\frac{\text{Cut-e. p.257}}{\text{Theorem 3.10 (6 method for multivariate case, TBp. 165)}$ Function of two univariate random variables $\underline{Z} = g(X, Y)$: Let $\underline{\mu} = (\underline{\mu_{X}, \mu_{Y}})$. $Z = g(X, Y) \approx g(\mu) + (X - \mu_{X})\frac{\partial g(\mu)}{\partial x} + (Y - \mu_{Y})\frac{\partial g(\mu)}{\partial y}$ $\Rightarrow E(Z) \approx g(\mu)$ $\text{Var}(Z) \approx \sigma_{X}^{2} \left[\frac{\partial g(\mu)}{\partial x}\right]^{2} + \sigma_{Y}^{2} \left[\frac{\partial g(\mu)}{\partial y}\right]^{2} + 2\sigma_{XY} \left[\frac{\partial g(\mu)}{\partial y}\right] \left[\frac{\partial g(\mu)}{\partial y}\right]$ $\Rightarrow E[g(X,Y)] \approx g(\mu) + (X - \mu_{X})\frac{\partial g(\mu)}{\partial x} + (Y - \mu_{Y})\frac{\partial g(\mu)}{\partial y}$ $\frac{1}{2}(X - \mu_{X})^{2}\frac{\partial^{2} g(\mu)}{\partial y^{2}} + (X - \mu_{X})(Y - \mu_{Y})\frac{\partial g(\mu)}{\partial y}$ $\frac{1}{2}(Y - \mu_{Y})^{2}\frac{\partial^{2} g(\mu)}{\partial y^{2}} + (X - \mu_{X})(Y - \mu_{Y})\frac{\partial^{2} g(\mu)}{\partial x^{2}} + \frac{1}{2}(Y - \mu_{Y})^{2}\frac{\partial^{2} g(\mu)}{\partial y^{2}} + \frac{1}{2}(Y - \mu_{Y})^{2}\frac{\partial^{2} g(\mu)}{\partial y$	A A X	distribution	<u>of Y? If not</u> , can we <u>"roughly" describe</u> the
Theorem 3.9 (§ method for univariate case, TBp. 162) Thm 3.1 (LWp 441) Were deep $Y = g(\underline{X}) \cong g(\mu_X) + (X - \mu_X)g'(\mu_X)$ (by Taylor expansion) $\Rightarrow E[g(X)] \approx g(\mu_X)$ (\overline{C} , $Var[g(X)] \approx Var(X)[g'(\mu_X)]^2$ or $Y = g(X) \boxtimes g(\mu_X) + (X - \mu_X)g'(\mu_X) + \frac{1}{2}(X - \mu_X)^2g''(\mu_X)$ $\Rightarrow E[g(X)] \approx g(\mu_X) + \frac{1}{2}\sigma_X^2g''(\mu_X)$ Note. How good these approximations are depends on whether \underline{g} can be reasonably well approximated by the 1st or 2nd order polynomials in a neighborhood of μ_X and on the size of σ_X , check Chebyshev's inequality (LNp. 43) Function of two univariate case, TBp. 165) Function of two univariate random variables $\underline{Z} = g(X,Y)$: Let $\underline{\mu} = (\mu_X, \mu_Y)$. $Z = g(X,Y) \cong g(\mu) + (X - \mu_X)\frac{\partial g(\mu)}{\partial x} + (Y - \mu_Y)\frac{\partial g(\mu)}{\partial y}$ ($\underline{\lambda}$, A_Y ϕ_X , ϕ_Y but act the goint dist. $\sigma_Y = g(X,Y) \cong g(\mu) + (X - \mu_X)\frac{\partial g(\mu)}{\partial x} + (Y - \mu_Y)\frac{\partial g(\mu)}{\partial y}$ ($\underline{\lambda}$, $\frac{1}{2}(X - \mu_X)^2\frac{\partial^2 g(\mu)}{\partial y^2} + \frac{1}{2}(X - \mu_X)^2\frac{\partial^2 g(\mu)}{\partial y^2} + \frac{1}{2}(Y - \mu_Y)^2\frac{\partial^2 g(\mu)}{\partial y^2} + (X - \mu_X)(Y - \mu_Y)\frac{\partial^2 g(\mu)}{\partial x \partial y} + \frac{1}{2}(Y - \mu_Y)^2\frac{\partial^2 g(\mu)}{\partial y^2}$ $\Rightarrow E[g(X,Y)] \approx g(\mu) + \frac{1}{2}\sigma_X^2\frac{\partial^2 g(\mu)}{\partial x^2} + \sigma_XY\frac{\partial^2 g(\mu)}{\partial x \partial y} + \frac{1}{2}\sigma_Y^2\frac{\partial^2 g(\mu)}{\partial y^2}$ $\Rightarrow E[g(X,Y)] \approx g(\mu) + \frac{1}{2}\sigma_X^2\frac{\partial^2 g(\mu)}{\partial x^2} + \sigma_XY\frac{\partial^2 g(\mu)}{\partial x \partial y} + \frac{1}{2}\sigma_Y^2\frac{\partial^2 g(\mu)}{\partial y^2}$ Note. The general case of a function of \underline{n} random variables can be worked out similarly. * Reading: textbook, Chapter 4 * reguired	What if Var(X)	$\underline{\text{mean}}$ and $\underline{\mathbf{v}}$	<u>variance</u> of \underline{Y} ? (Note. $\underline{E[g(X)] \neq g[E(X)]}$.)
When does $Y = g(X) \cong g(\mu_X) + (X - \mu_X)g'(\mu_X)$ (by Taylor expansion) $E[g(X)] \approx g(\mu_X)$ (cf. $Var[g(X)] \approx Var(X)[g'(\mu_X)]^2$ or $Y = g(X) \boxtimes g(\mu_X) + (X - \mu_X)g'(\mu_X) + \frac{1}{2}(X - \mu_X)^2g''(\mu_X)$ $\Rightarrow E[g(X)] \approx g(\mu_X) + \frac{1}{2}\sigma_X^2g''(\mu_X)$ Note. How good these approximations are depends on whether g can be reasonably well approximated by the 1st or 2nd order polynomials in a neigh- borhood of μ_X and on the size of σ_X check Chebyshev's inequality (LNp U2) Function of two univariate case. TBp. 165) Function of two univariate case. TBp. 165) Function of two univariate random variables $Z = g(X, Y)$: Let $\mu = (\mu_X, \mu_Y)$. $Z = g(X, Y) \cong g(\mu) + (X - \mu_X)\frac{\partial g(\mu)}{\partial x} + (Y - \mu_Y)\frac{\partial g(\mu)}{\partial y}$ $\Rightarrow E(Z) \approx g(\mu)$ $Var(Z) \approx \sigma_X^2 \left[\frac{\partial g(\mu)}{\partial x^2}\right]^2 + \sigma_Y^2 \left[\frac{\partial g(\mu)}{\partial y}\right]^2 + 2\sigma_{XY} \left[\frac{\partial g(\mu)}{\partial x}\right] \left[\frac{\partial g(\mu)}{\partial y}\right]$ $\Rightarrow E[g(X,Y)] \approx g(\mu) + (X - \mu_X)\frac{\partial g(\mu)}{\partial x^2} + (Y - \mu_Y)\frac{\partial g(\mu)}{\partial y}$ $= \frac{1}{2}(X - \mu_X)^2\frac{\partial^2 g(\mu)}{\partial y^2} + (X - \mu_X)(Y - \mu_Y)\frac{\partial^2 g(\mu)}{\partial x \partial y} + \frac{1}{2}(Y - \mu_Y)^2\frac{\partial^2 g(\mu)}{\partial y^2} + \frac{1}{2}(Y - \mu_Y)^2\frac{\partial^2 g(\mu)}{\partial y^2} + \frac{1}{2}(X - \mu_X)^2\frac{\partial^2 g(\mu)}{\partial y^2} + \frac{1}{2}(Y - \mu_Y)^2\frac{\partial^2 g(\mu)}{\partial y^2} + \frac{1}{2}(Y - \mu_$	Theorem 3.9 (δ method for <u>univ</u>	rariate case, TBp. 162) Thm 3.1 (LNp. 41)
$ \begin{array}{l} \hline \begin{array}{l} \hline e \\ \hline e$	When does $Y =$	$g(X) \ge g(\mu$	$(\mu_X) + (X - \mu_X)g'(\mu_X)$ (by Taylor expansion)
$Var[g(X)] \approx Var(X)[g'(\mu_X)]^2$ or $Y = g(X) \bigotimes g(\mu_X) + (X - \mu_X)g'(\mu_X) + \frac{1}{2}(X - \mu_X)^2g''(\mu_X)$ $\Rightarrow E[g(X)] \approx g(\mu_X) + \frac{1}{2}\sigma_X^2g''(\mu_X)$ Note. How good these approximations are depends on whether \underline{g} can be reasonably well approximated by the 1st or 2nd order polynomials in a neighborhood of $\underline{\mu}_X$ and on the size of σ_X , \underline{f} check Chebyshev's inequality (LNp. 43) Theorem 3.10 (δ method for multivariate case, TBp. 165) Function of two univariate random variables $\underline{Z} = \underline{g}(X,Y)$: Let $\underline{\mu} = (\underline{\mu}_X, \underline{\mu}_Y)$. $Z = g(X,Y) \cong g(\mu) + (X - \mu_X) \frac{\partial g(\mu)}{\partial x} + (Y - \mu_Y) \frac{\partial g(\mu)}{\partial y}$ $\Rightarrow E(Z) \approx g(\mu)$ $Var(Z) \approx \sigma_X^2 \left[\frac{\partial g(\mu)}{\partial x} \right]^2 + \sigma_Y^2 \left[\frac{\partial g(\mu)}{\partial y} \right]^2 + 2\sigma_{XY} \left[\frac{\partial g(\mu)}{\partial x} \right] \left[\frac{\partial g(\mu)}{\partial y} \right]$ or $g(X,Y) \cong g(\mu) + (X - \mu_X) \frac{\partial g(\mu)}{\partial x^2} + (Y - \mu_Y) \frac{\partial g(\mu)}{\partial x} \right]$ $\Rightarrow E[g(X,Y)] \approx g(\mu) + \frac{1}{2}\sigma_X^2 \frac{\partial^2 g(\mu)}{\partial x^2} + \frac{1}{2}\sigma_X \frac{\partial^2 g(\mu)}{\partial x \partial y} + \frac{1}{2}\sigma_X^2 \frac{\partial^2 g(\mu)}{\partial y^2} \right]$ $\Rightarrow E[g(X,Y)] \approx g(\mu) + \frac{1}{2}\sigma_X^2 \frac{\partial^2 g(\mu)}{\partial x^2} + \sigma_X \frac{\partial^2 g(\mu)}{\partial x \partial y} + \frac{1}{2}\sigma_X^2 \frac{\partial^2 g(\mu)}{\partial y^2} \right]$ $\Rightarrow E[g(X,Y)] \approx g(\mu) + \frac{1}{2}\sigma_X^2 \frac{\partial^2 g(\mu)}{\partial x^2} + \sigma_X \frac{\partial^2 g(\mu)}{\partial x \partial y} + \frac{1}{2}\sigma_X^2 \frac{\partial^2 g(\mu)}{\partial y^2} \right]$ Note. The general case of a function of \underline{n} random variables can be worked out similarly. \diamond Reading: textbook, Chapter 4 \Rightarrow reguired	$t \to E[q]$	$g(X) \approx g(\mu$	
or $Y = g(X) \bigotimes g(\mu_X) + (X - \mu_X)g'(\mu_X) + \frac{1}{2}(X - \mu_X)^2 g''(\mu_X)$ $\Rightarrow E[g(X)] \approx g(\mu_X) + \frac{1}{2}\sigma_X^2 g''(\mu_X)$ Note. How good these approximations are depends on whether \underline{g} can be reasonably well approximated by the 1st or 2nd order polynomials in a neighborhood of μ_X and on the size of σ_X . $-$ check Chebyshev's inequality (LNp. 43) Theorem 3.10 (∂ method for multivariate case, TBp. 165) Function of two univariate random variables $\underline{Z} = g(X, Y)$: Let $\underline{\mu} = (\mu_X, \mu_Y)$. $Z = g(X, Y) \cong g(\mu) + (X - \mu_X) \frac{\partial g(\mu)}{\partial x} + (Y - \mu_Y) \frac{\partial g(\mu)}{\partial y}$ $\Rightarrow E(Z) \approx g(\mu)$ $\forall xr(Z) \approx \sigma_X^2 \left[\frac{\partial g(\mu)}{\partial x} \right]^2 + \sigma_Y^2 \left[\frac{\partial g(\mu)}{\partial y} \right]^2 + 2\sigma_{XY} \left[\frac{\partial g(\mu)}{\partial x} \right] \left[\frac{\partial g(\mu)}{\partial y} \right]$ or $g(X, Y) \cong g(\mu) + (X - \mu_X) \frac{\partial g(\mu)}{\partial x} + (Y - \mu_Y) \frac{\partial g(\mu)}{\partial y}$ $+ \frac{1}{2}(X - \mu_X)^2 \frac{\partial^2 g(\mu)}{\partial x^2} + (X - \mu_X)(Y - \mu_Y) \frac{\partial^2 g(\mu)}{\partial x \partial y}$ $+ \frac{1}{2}(Y - \mu_Y)^2 \frac{\partial^2 g(\mu)}{\partial y^2}$ $\Rightarrow E[g(X, Y)] \approx g(\mu) + \frac{1}{2}\sigma_X^2 \frac{\partial^2 g(\mu)}{\partial x^2} + \sigma_{XY} \frac{\partial^2 g(\mu)}{\partial x \partial y} + \frac{1}{2}\sigma_Y^2 \frac{\partial^2 g(\mu)}{\partial y^2}$ Note. The general case of a function of \underline{n} random variables can be worked out similarly. \diamond Reading: textbook, Chapter 4 \Rightarrow reguired	$ V\overline{ar[g]}$	$\overline{q(X)} \approx Va$	$r(X)[g'(\mu_X)]^2$
of $Y = g(X) = g(\mu_X) + \frac{1}{2} \sigma_X^2 g''(\mu_X)$ $\Rightarrow E[g(X)] \approx g(\mu_X) + \frac{1}{2} \sigma_X^2 g''(\mu_X)$ Note. How good these approximations are depends on whether \underline{g} can be reasonably well approximated by the 1st or 2nd order polynomials in a neighborhood of $\underline{\mu_X}$ and on the size of σ_X , the characteristic case, TBp. 165) Function of two univariate random variables $\underline{Z} = g(X, Y)$: Let $\underline{\mu} = (\mu_X, \mu_Y)$. Function of two univariate random variables $\underline{Z} = g(X, Y)$: Let $\underline{\mu} = (\mu_X, \mu_Y)$. $E(Z) \approx g(\mu) + (X - \mu_X) \frac{\partial g(\mu)}{\partial x} + (Y - \mu_Y) \frac{\partial g(\mu)}{\partial y}$ $\Rightarrow E(Z) \approx g(\mu)$ $\forall xar(Z) \approx \sigma_X^2 \left[\frac{\partial g(\mu)}{\partial x} \right]^2 + \sigma_Y^2 \left[\frac{\partial g(\mu)}{\partial y} \right]^2 + 2\sigma_{XY} \left[\frac{\partial g(\mu)}{\partial x} \right] \left[\frac{\partial g(\mu)}{\partial y} \right]$ or $g(X, Y) \cong g(\mu) + (X - \mu_X) \frac{\partial g(\mu)}{\partial x^2} + (Y - \mu_Y) \frac{\partial g(\mu)}{\partial y}$ $\Rightarrow E[g(X, Y)] \approx g(\mu) + \frac{1}{2} \sigma_X^2 \frac{\partial^2 g(\mu)}{\partial x^2} + \frac{1}{2} \sigma_Y^2 \frac{\partial^2 g(\mu)}{\partial x \partial y} + \frac{1}{2} (Y - \mu_Y)^2 \frac{\partial^2 g(\mu)}{\partial y^2}$ $\Rightarrow E[g(X, Y)] \approx g(\mu) + \frac{1}{2} \sigma_X^2 \frac{\partial^2 g(\mu)}{\partial x^2} + \sigma_X \frac{\partial^2 g(\mu)}{\partial x \partial y} + \frac{1}{2} \sigma_Y^2 \frac{\partial^2 g(\mu)}{\partial y^2}$ Note. The general case of a function of \underline{n} random variables can be worked out similarly. \Leftrightarrow Reading: textbook, Chapter 4 \Rightarrow reguired \Leftrightarrow For the proving Partiene	$\int \int Or V -$	$a(X) \square a(u)$	$\frac{1}{(X - \mu_x)a'(\mu_x)} + \frac{1}{(X - \mu_x)^2a''(\mu_x)} + \frac{1}{(X - \mu_x)^2a''(\mu_x)}$
$\Rightarrow E[g(X)] \approx g(\mu_X) + \frac{1}{2} \sigma_X^2 g''(\mu_X)$ Note. How good these approximations are depends on whether <u>g</u> can be reasonably well approximated by the 1st or 2nd order polynomials in a neighborhood of μ_X and on the size of σ_X , <u>check Chebyshev's inequality (LNp 43)</u> Theorem 3.10 (δ method for multivariate case, TBp. 165) Function of two univariate random variables $\underline{Z} = \underline{g}(X, Y)$: Let $\underline{\mu} = (\mu_X, \mu_Y)$. Function of two univariate random variables $\underline{Z} = \underline{g}(X, Y)$: Let $\underline{\mu} = (\mu_X, \mu_Y)$. $Z = g(X, Y) \cong g(\mu) + (X - \mu_X) \frac{\partial g(\mu)}{\partial x} + (Y - \mu_Y) \frac{\partial g(\mu)}{\partial y}$ (δ_X) $\varphi = E(Z) \approx g(\mu)$ $\varphi = E(Z) \approx g(\mu)$ $\varphi = (X, Y) \cong g(\mu) + (X - \mu_X) \frac{\partial g(\mu)}{\partial y}^2 + 2\sigma_{XY} \left[\frac{\partial g(\mu)}{\partial x} \right] \left[\frac{\partial g(\mu)}{\partial y} \right]$ or $g(X, Y) \cong g(\mu) + (X - \mu_X) \frac{\partial g(\mu)}{\partial x^2} + (Y - \mu_Y) \frac{\partial g(\mu)}{\partial y}$ $+ \frac{1}{2}(X - \mu_X)^2 \frac{\partial^2 g(\mu)}{\partial x^2} + (X - \mu_X)(Y - \mu_Y) \frac{\partial^2 g(\mu)}{\partial x \partial y}$ $+ \frac{1}{2}(Y - \mu_Y)^2 \frac{\partial^2 g(\mu)}{\partial y^2}$ $\Rightarrow E[g(X, Y)] \approx g(\mu) + \frac{1}{2} \frac{\sigma_X^2}{\Delta x^2} + \frac{\sigma_{XY}}{\Delta x \partial y} + \frac{1}{2} \frac{\sigma_Y^2}{\Delta y} \frac{\partial^2 g(\mu)}{\partial y^2}$ Note. The general case of a function of <u>n</u> random variables can be worked out similarly. φ Furthor Backing Back		$g(\Lambda) \sim g(\mu)$	$f(X) + (\underline{X - \mu_X})g(\mu_X) + \frac{1}{2}(\underline{X - \mu_X})g(\mu_X)$
Note. How good these approximations are depends on whether \underline{g} can be reasonably well approximated by the 1st or 2nd order polynomials in a neighborhood of μ_X and on the size of σ_X . — check Chebyshev's inequality (LNp. 43) Theorem 3.10 (δ method for multivariate case, TBp. 165) Function of two univariate random variables $\underline{Z} = \underline{g}(X, Y)$: Let $\underline{\mu} = (\underline{\mu}_X, \mu_Y)$. Let $\underline{\mu} = (\underline{\mu}_X, \mu_Y)$. Let $\underline{\mu} = (\underline{\mu}_X, \mu_Y)$. $Z = g(X, Y) \cong g(\mu) + (X - \mu_X) \frac{\partial g(\mu)}{\partial x} + (Y - \mu_Y) \frac{\partial g(\mu)}{\partial y}$ (Δx) Δx (Δy) $\cong E(Z) \approx g(\mu)$ Var($Z) \approx \sigma_X^2 \left[\frac{\partial g(\mu)}{\partial x} \right]^2 + \sigma_Y^2 \left[\frac{\partial g(\mu)}{\partial y} \right]^2 + 2\sigma_{XY} \left[\frac{\partial g(\mu)}{\partial x} \right] \left[\frac{\partial g(\mu)}{\partial y} \right]$ or $g(X, Y) \cong g(\mu) + (X - \mu_X) \frac{\partial g(\mu)}{\partial x} + (Y - \mu_Y) \frac{\partial g(\mu)}{\partial y}$ $+ \frac{1}{2}(X - \mu_X)^2 \frac{\partial^2 g(\mu)}{\partial y^2} + (X - \mu_X)(Y - \mu_Y) \frac{\partial^2 g(\mu)}{\partial x \partial y}$ $+ \frac{1}{2}(Y - \mu_Y)^2 \frac{\partial^2 g(\mu)}{\partial y^2}$ $\Rightarrow E[g(X, Y)] \approx g(\mu) + \frac{1}{2}\sigma_X^2 \frac{\partial^2 g(\mu)}{\partial x^2} + \sigma_{XY} \frac{\partial^2 g(\mu)}{\partial x \partial y} + \frac{1}{2}\sigma_X^2 \frac{\partial^2 g(\mu)}{\partial y^2}$ Note. The general case of a function of \underline{n} random variables can be worked out similarly.	$\Rightarrow E[g$	$g(X)] \approx g(\mu$	$u_X) + \frac{1}{2} \sigma_X^2 g''(\mu_X)$
reasonably well approximated by the 1st or 2nd order polynomials in a neighborhood of μ_X and on the size of σ_X , check Chebyshev's inequality (LNp. 43) Cht-6, p.247 Theorem 3.10 (§ method for multivariate case, TBp. 165) Function of two univariate random variables $\underline{Z} = \underline{g}(X, Y)$: Let $\mu = (\mu_X, \mu_Y)$. Let $\mu = (\mu_X, \mu_Y)$. $Z = g(X, Y) \cong g(\mu) + (X - \mu_X) \frac{\partial g(\mu)}{\partial x} + (Y - \mu_Y) \frac{\partial g(\mu)}{\partial y}$ $\Rightarrow E(Z) \approx g(\mu)$ $\forall x_X, \psi_Y$ $\Rightarrow E(Z) \approx g(\mu)$ $\forall x_X(Z) \approx \sigma_X^2 \left[\frac{\partial g(\mu)}{\partial x}\right]^2 + \sigma_Y^2 \left[\frac{\partial g(\mu)}{\partial y}\right]^2 + 2\sigma_{XY} \left[\frac{\partial g(\mu)}{\partial x}\right] \left[\frac{\partial g(\mu)}{\partial y}\right]$ or $g(X, Y) \cong g(\mu) + (X - \mu_X) \frac{\partial g(\mu)}{\partial x^2} + (Y - \mu_Y) \frac{\partial g(\mu)}{\partial y}$ $+ \frac{1}{2}(X - \mu_X)^2 \frac{\partial^2 g(\mu)}{\partial x^2} + (X - \mu_X)(Y - \mu_Y) \frac{\partial^2 g(\mu)}{\partial x \partial y}$ $+ \frac{1}{2}(Y - \mu_Y)^2 \frac{\partial^2 g(\mu)}{\partial y^2}$ $\Rightarrow E[g(X, Y)] \approx g(\mu) + \frac{1}{2}\sigma_X^2 \frac{\partial^2 g(\mu)}{\partial x^2} + \sigma_X \frac{\partial^2 g(\mu)}{\partial x \partial y} + \frac{1}{2}\sigma_Y^2 \frac{\partial^2 g(\mu)}{\partial y^2}$ Note. The general case of a function of \underline{n} random variables can be worked out similarly. \Rightarrow Reading: textbook, Chapter 4 \Rightarrow regwired	Note. How	good these ap	pproximations are depends on whether q can be
borhood of $\underline{\mu}_X$ and on the size of σ_X , check Chebyshev's inequality (LNp. 43) Theorem 3.10 (δ method for multivariate case, TBp. 165) Suppose that we only know $\underline{\mu}_X$ and $\underline{\mu}_Y$ $\underline{\mu}_Y$. Let $\underline{\mu} = (\underline{\mu}_X, \underline{\mu}_Y)$. $Z = g(X, Y) \cong g(\mu) + (X - \underline{\mu}_X) \frac{\partial g(\mu)}{\partial x} + (Y - \underline{\mu}_Y) \frac{\partial g(\mu)}{\partial y}$ $\underline{\lambda}$ $\nabla xr(Z) \approx \frac{\sigma_X^2}{\sigma_X^2} \left[\frac{\partial g(\mu)}{\partial x} \right]^2 + \frac{\sigma_Y^2}{\sigma_X^2} \left[\frac{\partial g(\mu)}{\partial y} \right]^2 + 2\sigma_{XY} \left[\frac{\partial g(\mu)}{\partial x} \right] \left[\frac{\partial g(\mu)}{\partial y} \right]$ or $g(X, Y) \cong g(\mu) + (X - \underline{\mu}_X) \frac{\partial g(\mu)}{\partial x} + (Y - \underline{\mu}_Y) \frac{\partial g(\mu)}{\partial y}$ $+ \frac{1}{2} (X - \underline{\mu}_X)^2 \frac{\partial^2 g(\mu)}{\partial x^2} + (X - \underline{\mu}_X)(Y - \underline{\mu}_Y) \frac{\partial^2 g(\mu)}{\partial x \partial y}$ $+ \frac{1}{2} (Y - \underline{\mu}_Y)^2 \frac{\partial^2 g(\mu)}{\partial y^2}$ $\Rightarrow E[g(X, Y)] \approx g(\mu) + \frac{1}{2} \frac{\sigma_X^2}{\sigma_X^2} + \frac{\sigma_{XY}}{\sigma_X^2} \frac{\partial^2 g(\mu)}{\partial x \partial y} + \frac{1}{2} \frac{\sigma_Y^2}{\sigma_Y^2} \frac{\partial^2 g(\mu)}{\partial y^2}$ Note. The general case of a function of \underline{n} random variables can be worked out similarly. * Reading: textbook, Chapter 4 * required	reasonably w	vell approximat	ed by the 1st or 2nd order polynomials in a neigh-
Theorem 3.10 (δ method for multivariate case, TBp. 165) Suppose that we only know $\frac{dx}{dx}$ $\frac{dy}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$ $\frac{dx}{dx}$	borhood of μ	ι_X and on the s	size of σ_{X} , $-$ check Chebyshev's inequality (LNp. 43)
Theorem 3.10 (§ method for multivariate case, TBp. 165) Suppose that we only know M_X, M_Y G_X, G_Y G_X, G_Y G_Y G_X, G_Y G_Y G_Y G_X, G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y G_Y			
Function of <u>two</u> univariate random variables $\underline{Z} = \underline{g}(X, Y)$: Let $\underline{\mu} = (\underline{\mu}_X, \underline{\mu}_Y)$. $Z = g(X, Y) \cong g(\mu) + (\underline{X} - \underline{\mu}_X) \frac{\partial g(\mu)}{\partial x} + (\underline{Y} - \underline{\mu}_Y) \frac{\partial g(\mu)}{\partial y}$ \longrightarrow $\exists X = g(X, Y) \cong g(\mu) + (\underline{X} - \underline{\mu}_X) \frac{\partial g(\mu)}{\partial x} + (\underline{Y} - \underline{\mu}_Y) \frac{\partial g(\mu)}{\partial y}$ \longrightarrow $\forall x (Z) \approx \underline{\sigma}_X^2 \left[\frac{\partial g(\mu)}{\partial x} \right]^2 + \underline{\sigma}_Y^2 \left[\frac{\partial g(\mu)}{\partial y} \right]^2 + 2\underline{\sigma}_{XY} \left[\frac{\partial g(\mu)}{\partial x} \right] \left[\frac{\partial g(\mu)}{\partial y} \right]$ $\forall x (Z) \approx \underline{\sigma}_X^2 \left[\frac{\partial g(\mu)}{\partial x} \right]^2 + \underline{\sigma}_Y^2 \left[\frac{\partial g(\mu)}{\partial y} \right]^2 + 2\underline{\sigma}_{XY} \left[\frac{\partial g(\mu)}{\partial x} \right] \left[\frac{\partial g(\mu)}{\partial y} \right]$ or $g(X, Y) \cong g(\mu) + (X - \mu_X) \frac{\partial g(\mu)}{\partial x^2} + (Y - \mu_Y) \frac{\partial g(\mu)}{\partial y}$ $+ \frac{1}{2} (\underline{X} - \mu_X)^2 \frac{\partial^2 g(\mu)}{\partial x^2} + (\underline{X} - \mu_X) (Y - \mu_Y) \frac{\partial^2 g(\mu)}{\partial x \partial y}$ $+ \frac{1}{2} (\underline{Y} - \mu_Y)^2 \frac{\partial^2 g(\mu)}{\partial y^2}$ $\Rightarrow E[g(X, Y)] \approx g(\mu) + \frac{1}{2} \underline{\sigma}_X^2 \frac{\partial^2 g(\mu)}{\partial x^2} + \underline{\sigma}_{XY} \frac{\partial^2 g(\mu)}{\partial x \partial y} + \frac{1}{2} \underline{\sigma}_Y^2 \frac{\partial^2 g(\mu)}{\partial y^2}$ Note. The general case of a function of \underline{n} random variables can be worked out similarly. \Leftrightarrow Reading: textbook, Chapter 4 \Leftarrow reguired \Leftrightarrow Further Bondian Burgers 51 52 54 55 (1102) (4.65 + 102)			Ch1~6, p.2-5
$Let \underline{\mu} = (\underline{\mu}_{X}, \underline{\mu}_{Y}).$ $Z = g(X, Y) \cong g(\mu) + (\underline{X} - \underline{\mu}_{X}) \frac{\partial g(\mu)}{\partial x} + (\underline{Y} - \underline{\mu}_{Y}) \frac{\partial g(\mu)}{\partial y}$ $\Rightarrow E(Z) \approx g(\mu)$ $Var(Z) \approx \frac{\sigma_{X}^{2}}{\sigma_{X}^{2}} \left[\frac{\partial g(\mu)}{\partial x} \right]^{2} + \frac{\sigma_{Y}^{2}}{\sigma_{Y}^{2}} \left[\frac{\partial g(\mu)}{\partial y} \right]^{2} + 2 \underline{\sigma}_{XY}} \left[\frac{\partial g(\mu)}{\partial x} \right] \left[\frac{\partial g(\mu)}{\partial y} \right]$ or $g(X, Y) \cong g(\mu) + (X - \mu_{X}) \frac{\partial g(\mu)}{\partial x} + (Y - \mu_{Y}) \frac{\partial g(\mu)}{\partial y}$ $+ \frac{1}{2} (\underline{X} - \mu_{X})^{2} \frac{\partial^{2} g(\mu)}{\partial y^{2}} + (\underline{X} - \mu_{X}) (Y - \mu_{Y}) \frac{\partial^{2} g(\mu)}{\partial x \partial y}$ $+ \frac{1}{2} (\underline{Y} - \mu_{Y})^{2} \frac{\partial^{2} g(\mu)}{\partial y^{2}}$ $\Rightarrow E[g(X, Y)] \approx g(\mu) + \frac{1}{2} \frac{\sigma_{X}^{2}}{\sigma_{X}^{2}} \frac{\partial^{2} g(\mu)}{\partial x^{2}} + \frac{\sigma_{XY}}{\sigma_{X} \partial y} \frac{\partial^{2} g(\mu)}{\partial x \partial y} + \frac{1}{2} \frac{\sigma_{Y}^{2}}{\sigma_{Y}^{2}} \frac{\partial^{2} g(\mu)}{\partial y^{2}}$ Note. The general case of a function of <u>n</u> random variables can be worked out similarly. $ Reading: textbook, Chapter 4 - reguired$	Theorem	3.10 (δ method for	Ch1~6, p.2-57 or <u>multivariate</u> case, TBp. 165)
only know $Z = g(X, Y) \cong g(\mu) + (X - \mu_X) \frac{\partial g(\mu)}{\partial x} + (Y - \mu_Y) \frac{\partial g(\mu)}{\partial y}$ $\Rightarrow E(Z) \approx g(\mu)$ $Var(Z) \approx \frac{\sigma_X^2}{\sigma_X^2} \left[\frac{\partial g(\mu)}{\partial x} \right]^2 + \frac{\sigma_Y^2}{\sigma_Y^2} \left[\frac{\partial g(\mu)}{\partial y} \right]^2 + 2 \frac{\sigma_{XY}}{\sigma_X} \left[\frac{\partial g(\mu)}{\partial x} \right] \left[\frac{\partial g(\mu)}{\partial y} \right]$ or $g(X, Y) \cong g(\mu) + (X - \mu_X) \frac{\partial g(\mu)}{\partial x} + (Y - \mu_Y) \frac{\partial g(\mu)}{\partial y}$ $+ \frac{1}{2} (X - \mu_X)^2 \frac{\partial^2 g(\mu)}{\partial x^2} + (X - \mu_X) (Y - \mu_Y) \frac{\partial^2 g(\mu)}{\partial x \partial y}$ $+ \frac{1}{2} (Y - \mu_Y)^2 \frac{\partial^2 g(\mu)}{\partial y^2}$ $\Rightarrow E[g(X, Y)] \approx g(\mu) + \frac{1}{2} \frac{\sigma_X^2}{\sigma_X^2} \frac{\partial^2 g(\mu)}{\partial x^2} + \frac{\sigma_{XY}}{\sigma_X^2} \frac{\partial^2 g(\mu)}{\partial x \partial y} + \frac{1}{2} \frac{\sigma_Y^2}{\sigma_Y^2} \frac{\partial^2 g(\mu)}{\partial y^2}$ Note. The general case of a function of <u>n</u> random variables can be worked out similarly. $\Rightarrow Reading: textbook, Chapter 4 - reguired$	Theorem	3.10 (δ method for a notion of two under the second sec	Ch1~6, p.2-57 for <u>multivariate</u> case, TBp. 165) nivariate random variables $\underline{Z} = \underline{g(X, Y)}$:
$\begin{array}{cccc} & \underbrace{\mathbf{M}_{\mathbf{Y}}}{\mathbf{G}_{\mathbf{X}}^{\mathbf{X}}, \mathbf{G}_{\mathbf{Y}}^{\mathbf{Y}}} & \Rightarrow & \mathrm{E}(Z) \approx g(\mu) \\ & \mathrm{Var}(Z) \approx \underline{\sigma_{X}^{2}} \left[\frac{\partial g(\mu)}{\partial x} \right]^{2} + \underline{\sigma_{Y}^{2}} \left[\frac{\partial g(\mu)}{\partial y} \right]^{2} + 2\underline{\sigma_{XY}} \left[\frac{\partial g(\mu)}{\partial x} \right] \left[\frac{\partial g(\mu)}{\partial y} \right] \\ & \mathrm{Var}(Z) \approx \underline{\sigma_{X}^{2}} \left[\frac{\partial g(\mu)}{\partial x} \right]^{2} + \underline{\sigma_{Y}^{2}} \left[\frac{\partial g(\mu)}{\partial y} \right]^{2} + 2\underline{\sigma_{XY}} \left[\frac{\partial g(\mu)}{\partial x} \right] \left[\frac{\partial g(\mu)}{\partial y} \right] \\ & \mathrm{or} & g(X,Y) \approx g(\mu) + (X - \mu_{X}) \frac{\partial g(\mu)}{\partial x^{2}} + (Y - \mu_{Y}) \frac{\partial g(\mu)}{\partial y} \\ & + \frac{1}{2} (X - \mu_{X})^{2} \frac{\partial^{2} g(\mu)}{\partial x^{2}} + (X - \mu_{X})(Y - \mu_{Y}) \frac{\partial^{2} g(\mu)}{\partial x \partial y} \\ & + \frac{1}{2} (Y - \mu_{Y})^{2} \frac{\partial^{2} g(\mu)}{\partial y^{2}} \end{array} \\ & \Rightarrow & E[g(X,Y)] \approx g(\mu) + \frac{1}{2} \underline{\sigma_{X}^{2}} \frac{\partial^{2} g(\mu)}{\partial x^{2}} + \underline{\sigma_{XY}} \frac{\partial^{2} g(\mu)}{\partial x \partial y} + \frac{1}{2} \underline{\sigma_{Y}^{2}} \frac{\partial^{2} g(\mu)}{\partial y^{2}} \\ & \text{Note. The general case of a function of } \underline{n} \text{ random variables can be worked out } \underline{similarly.} \end{array}$	Theorem Suppose Fu that we	3.10 (δ method for a method of $\underline{\text{two}}$ under the method of $\underline{\text{two}}$ under the method $\underline{\mu} = (\underline{\mu}_X, \underline{\mu}_Y).$	Ch1~6, p.2-57 for <u>multivariate</u> case, TBp. 165) nivariate random variables $\underline{Z} = \underline{g(X, Y)}$: $\partial a(u)$ $\partial a(u)$
Var $(Z) \approx \underline{\sigma_X^2} \left[\frac{\partial g(\mu)}{\partial x} \right]^2 + \underline{\sigma_Y^2} \left[\frac{\partial g(\mu)}{\partial y} \right]^2 + 2\underline{\sigma_{XY}} \left[\frac{\partial g(\mu)}{\partial x} \right] \left[\frac{\partial g(\mu)}{\partial y} \right]$ or $g(X,Y) \approx g(\mu) + (X - \mu_X) \frac{\partial g(\mu)}{\partial x} + (Y - \mu_Y) \frac{\partial g(\mu)}{\partial y} + \frac{1}{2} \frac{(X - \mu_X)^2}{\partial x^2} \frac{\partial^2 g(\mu)}{\partial x^2} + \frac{(X - \mu_X)(Y - \mu_Y)}{\partial x \partial y} \frac{\partial^2 g(\mu)}{\partial x \partial y} + \frac{1}{2} \frac{(Y - \mu_Y)^2}{\partial y^2} \frac{\partial^2 g(\mu)}{\partial x^2} + \frac{1}{2} \frac{\sigma_X^2}{\partial x^2} \frac{\partial^2 g(\mu)}{\partial x \partial y} + \frac{1}{2} \frac{\sigma_Y^2}{\partial y^2} \frac{\partial^2 g(\mu)}{\partial y^2}$ Note. The general case of a function of <u>n</u> random variables can be worked out similarly. Reading: textbook, Chapter 4 \rightarrow required	Theorem Suppose that we only know	3.10 (δ method for notion of two un t $\underline{\mu} = (\underline{\mu}_X, \underline{\mu}_Y)$. Z = g(X, Y)	Ch1~6, p.2-57 or <u>multivariate</u> case, TBp. 165) nivariate random variables $\underline{Z} = \underline{g(X, Y)}$: $(\underline{X} - \mu_X) \frac{\partial g(\mu)}{\partial x} + (\underline{Y} - \mu_Y) \frac{\partial g(\mu)}{\partial y}$
or $g(X,Y) \approx g(\mu) + (X - \mu_X) \frac{\partial g(\mu)}{\partial x} + (Y - \mu_Y) \frac{\partial g(\mu)}{\partial y}$ $+ \frac{1}{2} (X - \mu_X)^2 \frac{\partial^2 g(\mu)}{\partial x^2} + (X - \mu_X)(Y - \mu_Y) \frac{\partial^2 g(\mu)}{\partial x \partial y}$ $+ \frac{1}{2} (Y - \mu_Y)^2 \frac{\partial^2 g(\mu)}{\partial y^2}$ $\Rightarrow E[g(X,Y)] \approx g(\mu) + \frac{1}{2} \frac{\sigma_X^2}{\partial x^2} \frac{\partial^2 g(\mu)}{\partial x^2} + \frac{\sigma_{XY}}{\partial x \partial y} \frac{\partial^2 g(\mu)}{\partial x \partial y} + \frac{1}{2} \frac{\sigma_Y^2}{\partial y^2} \frac{\partial^2 g(\mu)}{\partial y^2}$ Note. The general case of a function of \underline{n} random variables can be worked out similarly. \Rightarrow Reading: textbook, Chapter 4 \rightarrow required	Theorem Suppose that we only know Mx My Fu Le	3.10 (δ method for nction of two un t $\underline{\mu} = (\underline{\mu}_X, \underline{\mu}_Y)$. Z = g(X, Y) $\Rightarrow E(Z) \approx g(\mu)$	Ch1-6, p.2-5 for <u>multivariate</u> case, TBp. 165) nivariate random variables $\underline{Z} = \underline{g(X,Y)}$: $g(\mu) + (\underline{X - \mu_X}) \frac{\partial g(\mu)}{\partial x} + (\underline{Y - \mu_Y}) \frac{\partial g(\mu)}{\partial y}$
$ \begin{array}{c} \bullet & \bullet $	Theorem Suppose that we only know $\frac{M_{X}}{S_{X}^{2}}, S_{Y}^{2}$ $\overline{S_{X}}, S_{Y}^{2}$	3.10 (δ method for nction of two un t $\underline{\mu} = (\underline{\mu}_X, \underline{\mu}_Y)$. Z = g(X, Y) $\Rightarrow E(Z) \approx g(\mu)$ $Var(Z) \approx g(\mu)$	Ch1~6, p.2-57 or <u>multivariate</u> case, TBp. 165) nivariate random variables $\underline{Z} = \underline{g(X,Y)}$: $Y) \cong g(\mu) + (\underline{X - \mu_X}) \frac{\partial g(\mu)}{\partial x} + (\underline{Y - \mu_Y}) \frac{\partial g(\mu)}{\partial y}$ $\mu)$ $\sigma_X^2 \left[\frac{\partial g(\mu)}{\partial x} \right]^2 + \sigma_Y^2 \left[\frac{\partial g(\mu)}{\partial y} \right]^2 + 2 \sigma_{XY} \left[\frac{\partial g(\mu)}{\partial x} \right] \left[\frac{\partial g(\mu)}{\partial y} \right]$
$+\frac{1}{2}(\underline{Y} - \mu_{\underline{Y}})^{2}\frac{\partial^{2}g(\mu)}{\partial y^{2}} \longrightarrow$ $\Rightarrow E[g(X, Y)] \approx g(\mu) + \frac{1}{2}\underline{\sigma_{\underline{X}}^{2}}\frac{\partial^{2}g(\mu)}{\partial x^{2}} + \underline{\sigma_{\underline{XY}}}\frac{\partial^{2}g(\mu)}{\partial x\partial y} + \frac{1}{2}\underline{\sigma_{\underline{Y}}^{2}}\frac{\partial^{2}g(\mu)}{\partial y^{2}}$ Note. The general case of a function of <u>n</u> random variables can be worked out similarly. $\bullet \text{ Reading: textbook, Chapter 4 } \bullet \text{ required}$	Theorem Suppose that we only know M_{X}, M_{Y} G_{X}^{2}, G_{Y}^{2} O_{XY}, but not the joint dist. or	3.10 (δ method for nction of two un t $\underline{\mu} = (\underline{\mu}_X, \underline{\mu}_Y)$. Z = g(X, Y) $\Rightarrow E(Z) \approx g(\mu)$ $Var(Z) \approx g(X, Y) \approx g(X, Y) \approx g(X, Y)$	Ch1-6, p.2-5 or <u>multivariate</u> case, TBp. 165) nivariate random variables $\underline{Z} = \underline{g(X,Y)}$: $Y) \cong g(\mu) + (\underline{X} - \mu_X) \frac{\partial g(\mu)}{\partial x} + (\underline{Y} - \mu_Y) \frac{\partial g(\mu)}{\partial y}$ \swarrow $\mu)$ $\sigma_X^2 \left[\frac{\partial g(\mu)}{\partial x} \right]^2 + \sigma_Y^2 \left[\frac{\partial g(\mu)}{\partial y} \right]^2 + 2\sigma_{XY} \left[\frac{\partial g(\mu)}{\partial x} \right] \left[\frac{\partial g(\mu)}{\partial y} \right]$ $g(\mu) + (X - \mu_X) \frac{\partial g(\mu)}{\partial x} + (Y - \mu_Y) \frac{\partial g(\mu)}{\partial y}$
$\Rightarrow E[g(X,Y)] \approx g(\mu) + \frac{1}{2} \frac{\sigma_X^2}{\partial x^2} \frac{\partial^2 g(\mu)}{\partial x^2} + \frac{\sigma_{XY}}{\partial x \partial y} \frac{\partial^2 g(\mu)}{\partial x^2} + \frac{1}{2} \frac{\sigma_Y^2}{\partial y^2} \frac{\partial^2 g(\mu)}{\partial y^2}$ Note. The general case of a function of <u>n</u> random variables can be worked out <u>similarly</u> . * Reading: textbook, Chapter 4 * required * Further Beading: Beyong 51, 52, 54, 55, (1) (2) (4, (5, r), pational)	Theorem Suppose that we only know M_X, M_Y G_X^2, G_Y^2 G_{XY}, but not the joint dist. of X, Y	3.10 (δ method for nction of two un t $\underline{\mu} = (\underline{\mu}_X, \underline{\mu}_Y)$. Z = g(X, Y) $\Rightarrow E(Z) \approx g(\mu)$ $Var(Z) \approx \frac{1}{2}$	Ch1-6, p.2-57 or <u>multivariate</u> case, TBp. 165) nivariate random variables $\underline{Z} = \underline{g(X,Y)}$: $Y) \cong g(\mu) + (\underline{X} - \mu_X) \frac{\partial g(\mu)}{\partial x} + (\underline{Y} - \mu_Y) \frac{\partial g(\mu)}{\partial y}$ $\underbrace{\swarrow}$ $\mu)$ $\sigma_X^2 \left[\frac{\partial g(\mu)}{\partial x} \right]^2 + \sigma_Y^2 \left[\frac{\partial g(\mu)}{\partial y} \right]^2 + 2\sigma_{XY} \left[\frac{\partial g(\mu)}{\partial x} \right] \left[\frac{\partial g(\mu)}{\partial y} \right]$ $g(\mu) + (X - \mu_X) \frac{\partial g(\mu)}{\partial x} + (Y - \mu_Y) \frac{\partial g(\mu)}{\partial y}$ $\frac{-\mu_X}{2} \frac{\partial^2 g(\mu)}{\partial x^2} + (X - \mu_X)(Y - \mu_Y) \frac{\partial^2 g(\mu)}{\partial x \partial y}$
 Note. The general case of a function of <u>n</u> random variables can be worked out <u>similarly</u>. * Reading: textbook, Chapter 4 - required * Further Beading: Baugass 51 52 54 55 (1) (2) (4 (5 in patient)) 	Theorem Suppose that we only know M_X, M_Y G_X^2, G_Y^2 G_{XY}, but not the joint dist. of X, Y	3.10 (δ method for nction of two un t $\underline{\mu} = (\underline{\mu}_X, \underline{\mu}_Y)$. Z = g(X, Y) $\Rightarrow E(Z) \approx g(\overline{\mu}_X)$ $Var(Z) \approx g(\overline{\mu}_X)$ $= \frac{1}{2}(X - \overline{\mu}_X)$	Ch1-6, p.2-5 or <u>multivariate</u> case, TBp. 165) nivariate random variables $\underline{Z} = \underline{g}(X, Y)$: $Y) \cong g(\mu) + (\underline{X} - \underline{\mu}_X) \frac{\partial g(\mu)}{\partial x} + (\underline{Y} - \underline{\mu}_Y) \frac{\partial g(\mu)}{\partial y}$ $\underbrace{\swarrow}$ $\mu)$ $\sigma_X^2 \left[\frac{\partial g(\mu)}{\partial x} \right]^2 + \underline{\sigma}_Y^2 \left[\frac{\partial g(\mu)}{\partial y} \right]^2 + 2\underline{\sigma}_{XY} \left[\frac{\partial g(\mu)}{\partial x} \right] \left[\frac{\partial g(\mu)}{\partial y} \right]$ $\underline{g}(\mu) + (X - \underline{\mu}_X) \frac{\partial g(\mu)}{\partial x} + (Y - \underline{\mu}_Y) \frac{\partial g(\mu)}{\partial y}$ $\underline{g}(\mu) + (X - \underline{\mu}_X) \frac{\partial g(\mu)}{\partial x^2} + (X - \underline{\mu}_X)(Y - \underline{\mu}_Y) \frac{\partial^2 g(\mu)}{\partial x \partial y}$ $\underline{g}(\mu)^2 \frac{\partial^2 g(\mu)}{\partial y^2}$ $\underbrace{\swarrow}$
 out similarly. ★ Reading: textbook, Chapter 4 → reguired ★ Further Bouding: Bougges 51 52 54 55 (1) (2) (4 (5) + optional) 	Theorem Suppose that we only know M_X, M_Y G_X^2, G_Y^2 G_{XY}, but not the joint dist. of X, Y	3.10 (δ method for nction of two un t $\underline{\mu} = (\underline{\mu}_X, \underline{\mu}_Y)$. Z = g(X, Y) $\Rightarrow E(Z) \approx g(\mu)$ $Var(Z) \approx g(\mu)$ $q(X, Y) \approx \frac{1}{2}$ $+\frac{1}{2}(X - \mu)$ $+\frac{1}{2}(Y - \mu)$ $\Rightarrow E[g(X, Y)]$	Ch1-6, p.2-5: or multivariate case, TBp. 165) nivariate random variables $\underline{Z} = \underline{g}(X, Y)$: $Y) \cong g(\mu) + (\underline{X} - \mu_X) \frac{\partial g(\mu)}{\partial x} + (\underline{Y} - \mu_Y) \frac{\partial g(\mu)}{\partial y}$ \swarrow μ) $\sigma_X^2 \left[\frac{\partial g(\mu)}{\partial x}\right]^2 + \sigma_Y^2 \left[\frac{\partial g(\mu)}{\partial y}\right]^2 + 2\sigma_{XY} \left[\frac{\partial g(\mu)}{\partial x}\right] \left[\frac{\partial g(\mu)}{\partial y}\right]$ $g(\mu) + (X - \mu_X) \frac{\partial g(\mu)}{\partial x} + (Y - \mu_Y) \frac{\partial g(\mu)}{\partial y}$ $-\mu_X)^2 \frac{\partial^2 g(\mu)}{\partial x^2} + (X - \mu_X)(Y - \mu_Y) \frac{\partial^2 g(\mu)}{\partial x \partial y}$ $g(\mu) + \frac{1}{2} \sigma_X^2 \frac{\partial^2 g(\mu)}{\partial x^2} + \sigma_{XY} \frac{\partial^2 g(\mu)}{\partial x \partial y} + \frac{1}{2} \sigma_Y^2 \frac{\partial^2 g(\mu)}{\partial y^2}$
 Reading: textbook, Chapter 4 reguired Eurther Booding: Developed 	Theorem Suppose that we only know M_X, M_Y G_X^2, G_Y^2 G_{XY}, but not the joint dist. of X, Y Note. 7	3.10 (δ method for netion of two un t $\underline{\mu} = (\underline{\mu}_X, \underline{\mu}_Y)$. Z = g(X, Y) $\Rightarrow E(Z) \approx g(\mu)$ $Var(Z) \approx g(\mu)$ $q(X, Y) \approx \frac{1}{2}$ $+\frac{1}{2}(X - \mu)$ $+\frac{1}{2}(Y - \mu)$ $\Rightarrow E[g(X, Y)]$ The general case	Cht-6, p.2-5: or <u>multivariate case</u> , TBp. 165) nivariate random variables $\underline{Z} = \underline{g}(X, Y)$: $Y) \cong g(\mu) + (\underline{X} - \mu_X) \frac{\partial g(\mu)}{\partial x} + (\underline{Y} - \mu_Y) \frac{\partial g(\mu)}{\partial y}$ \swarrow $\mu)$ $\sigma_X^2 \left[\frac{\partial g(\mu)}{\partial x} \right]^2 + \sigma_Y^2 \left[\frac{\partial g(\mu)}{\partial y} \right]^2 + 2\sigma_{XY} \left[\frac{\partial g(\mu)}{\partial x} \right] \left[\frac{\partial g(\mu)}{\partial y} \right]$ $g(\mu) + (X - \mu_X) \frac{\partial g(\mu)}{\partial x} + (Y - \mu_Y) \frac{\partial g(\mu)}{\partial y}$ $g(\mu) + (X - \mu_X) \frac{\partial g(\mu)}{\partial x^2} + (X - \mu_X)(Y - \mu_Y) \frac{\partial^2 g(\mu)}{\partial x \partial y}$ $g(\mu) + \frac{1}{2} \sigma_X^2 \frac{\partial^2 g(\mu)}{\partial x^2} + \sigma_{XY} \frac{\partial^2 g(\mu)}{\partial x \partial y} + \frac{1}{2} \sigma_Y^2 \frac{\partial^2 g(\mu)}{\partial y^2}$ $g(\mu) + \frac{1}{2} \sigma_X^2 \frac{\partial^2 g(\mu)}{\partial x^2} + \sigma_{XY} \frac{\partial^2 g(\mu)}{\partial x \partial y} + \frac{1}{2} \sigma_Y^2 \frac{\partial^2 g(\mu)}{\partial y^2}$ $g(\mu) + \frac{1}{2} \sigma_X^2 \frac{\partial^2 g(\mu)}{\partial x^2} + \sigma_{XY} \frac{\partial^2 g(\mu)}{\partial x \partial y} + \frac{1}{2} \sigma_Y^2 \frac{\partial^2 g(\mu)}{\partial y^2}$ $g(\mu) + \frac{1}{2} \sigma_X^2 \frac{\partial^2 g(\mu)}{\partial x^2} + \sigma_{XY} \frac{\partial^2 g(\mu)}{\partial x \partial y} + \frac{1}{2} \sigma_Y^2 \frac{\partial^2 g(\mu)}{\partial y^2}$ $g(\mu) + \frac{1}{2} \sigma_X^2 \frac{\partial^2 g(\mu)}{\partial x^2} + \sigma_{XY} \frac{\partial^2 g(\mu)}{\partial x \partial y} + \frac{1}{2} \sigma_Y^2 \frac{\partial^2 g(\mu)}{\partial y^2}$
AN RUPHER REMAIND. RUPESAS TELEVINE VELOVE PLANE VELOVE VE	Theorem Suppose that we only know $M_X M_Y$ G_X^2, G_Y^2 G_{XY} , but not the joint dist. of X, Y Note. To out simi	3.10 (δ method for nction of two un t $\mu = (\mu_X, \mu_Y)$. Z = g(X, Y) $\Rightarrow E(Z) \approx g(\mu_X, Y) \approx \frac{1}{2}$ $g(X, Y) \approx \frac{1}{2}$ $+\frac{1}{2}(X - \frac{1}{2}(Y - \frac{1}{2}(X - \frac{1}{2}(Y - \frac{1}{2}(X - \frac{1}$	Cht-6, p.2-5 or <u>multivariate</u> case, TBp. 165) nivariate random variables $\underline{Z} = \underline{g}(X, Y)$: $\begin{aligned} Y) &\cong g(\mu) + (\underline{X} - \mu_X) \frac{\partial g(\mu)}{\partial x} + (\underline{Y} - \mu_Y) \frac{\partial g(\mu)}{\partial y} \qquad $

made by S.-W. Cheng (NTHU, Taiwan)

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