Question

What is Statistics?

統計 ← Data Science

哈利波特	Real Life
占卜學	Statistics
崔老妮	Statisticians
水晶球	Data
未來的資訊	Information

方法 → 占卜學

資訊載體 → ☆ 水晶球

產品 → 未來的資訊

Data ← 數據資料

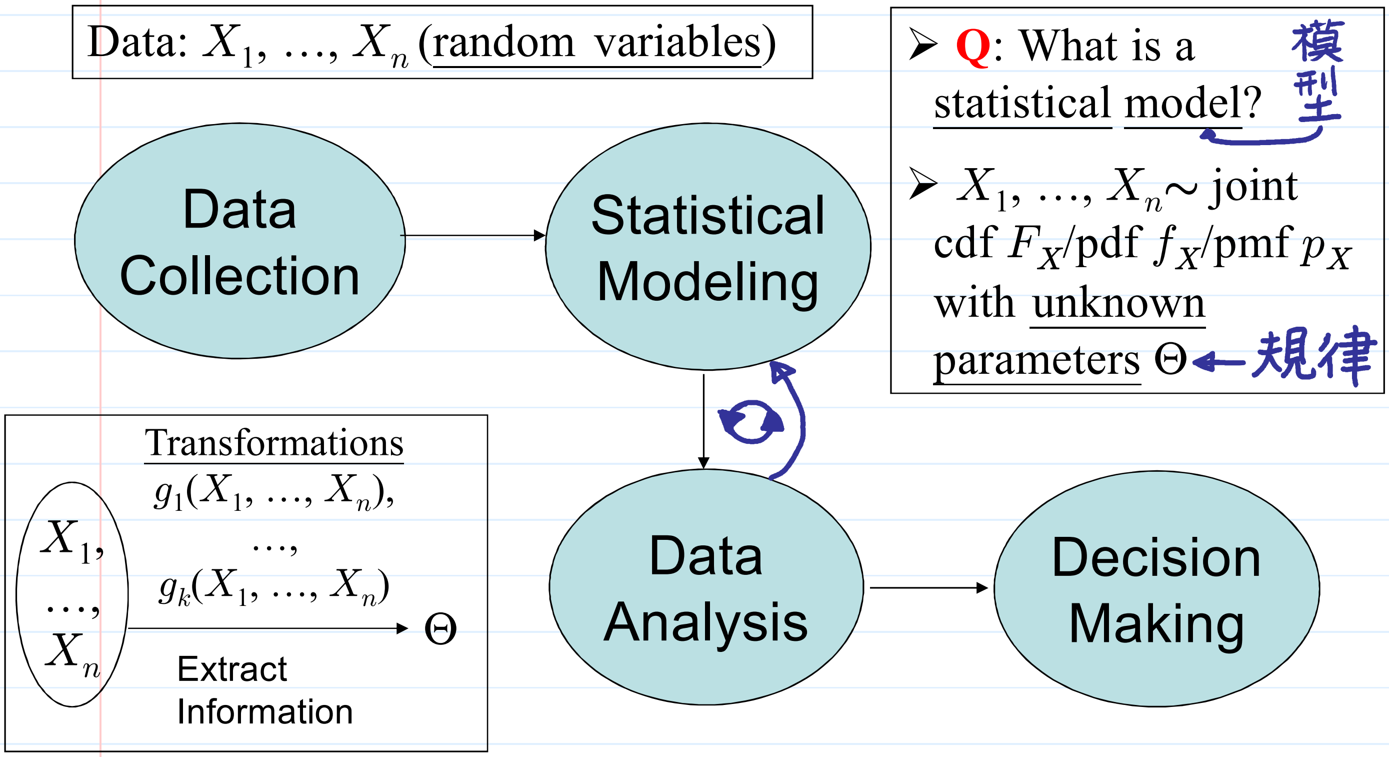
into; see, watch

aim of statistics: provide ***insight*** by means of ***data***

對現象、狀況、系統、…

Basic Procedures of Statistics

- Statistics divides the study of data into *four* steps:

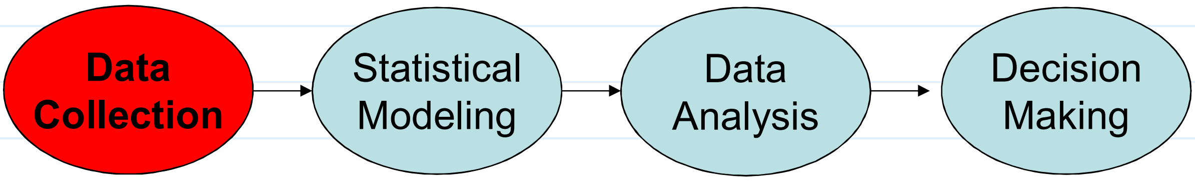


Data: $X_1, \ldots, X_n$ (random variables)

➢ **Q**: What is a statistical model? 模型

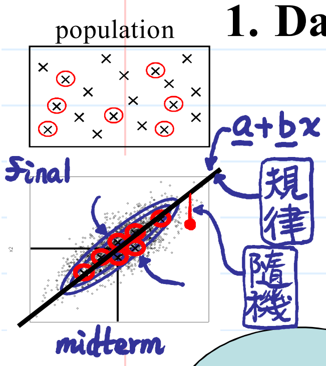
➢ $X_1, \ldots, X_n \sim$ joint cdf F_X/pdf f_X/pmf p_X with unknown parameters Θ ← 規律

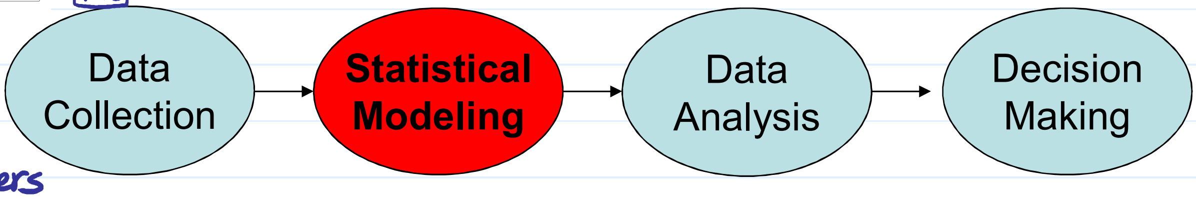
$$X_1, \ldots, X_n \xrightarrow[\text{Extract Information}]{\text{Transformations } g_1(X_1, \ldots, X_n),\ \ldots,\ g_k(X_1, \ldots, X_n)} \Theta$$



1. Data collection: producing *representative* data for drawing correct information ← 有代表性的

- design of experiment ← 實驗設計 ⎤
- survey sampling ← 抽樣調查 ⎦ ← controllable
- observational data ← uncontrollable



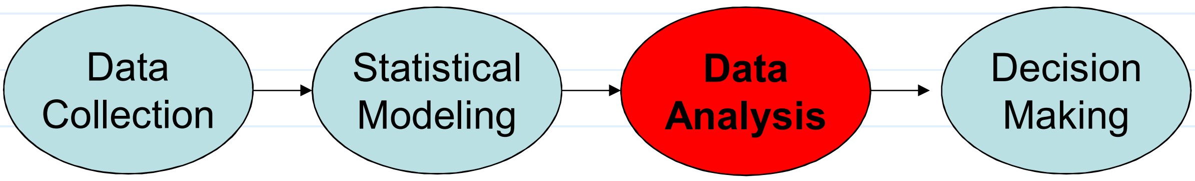


parameters (unknown)

- Systematic part (規律)
- random part (隨機)

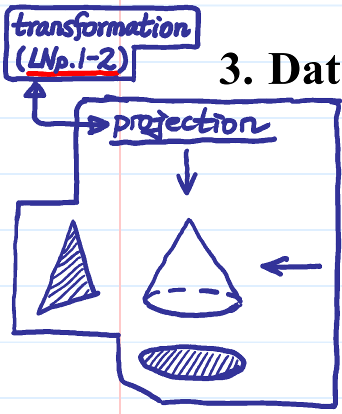
2. Statistical modeling: using the information that we possess to develop *a representation of the underlying system*, which also accounts for *uncertainty* in data ← probability

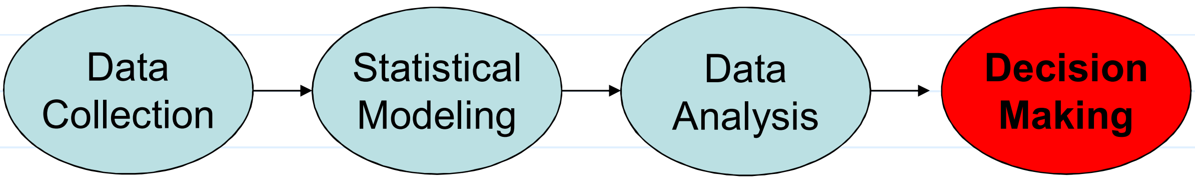
- a statistical model is a description of the joint distribution of data



3. Data analysis: mining information from data ← about parameters (規律)

- graphical methods
- numerical methods
  - ➢ *estimation* ← 估計 — point / interval estimation
  - ➢ *hypothesis testing* ← 假設檢定





4. Decision making: drawing conclusions & answering questions based on results obtained in **3**.

Data collection

Example (heat of fusion of ice, TBp. 423)

(Natrella, 1996) Two methods, A and B, were used in a determination of the latent heat of fusion of ice. The following table gives the change in total heat from ice at $-.72°C$ to water $0°C$ in calories per gram of mass:

Method A	79.98	(80.04	80.02	80.04	80.03	80.03	80.04)	79.97
	(80.05	80.03	80.02	80.00	80.02)			
Method B	(80.02)	79.94	79.98	79.97	79.97	(80.03)	79.95	79.97

The investigators wished to find out:

how much the two methods “differ”?

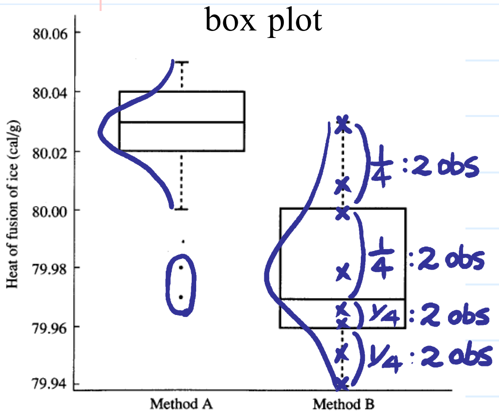
- **Q**: Why not all the values from Method A/B are identical?

  because of uncertainty.

- **Q**: Beyond the uncertainty existing in the data, are there some “certain” information?

  規律, systematic part in data.

Data analysis - graphical method



Q: From the plot, the two methods are different? or not different? and why?

Question

How to model the data and the question, i.e., state them in a mathematical/statistical language?

Statistical modeling

- Let $X_1, \ldots, X_n$ be the n observations from method A
- Let $Y_1, \ldots, Y_m$ be the m observations from method B
- To account for the uncertainty in data,

  regard $X_1, \ldots, X_n$ and $Y_1, \ldots, Y_m$ as random variables.
- Assign distribution to random variables parameters

assumptions method A: $X_1, \ldots, X_n \sim$ i.i.d. Normal(μ_X, σ^2)

method B: $Y_1, \ldots, Y_m \sim$ i.i.d. Normal(μ_Y, σ^2)

$\boxed{\mu_X = \mu_Y?}$

Data analysis - numerical methods

- Estimation: what are the values of μ_X, μ_Y, σ^2?
- Hypothesis testing: $\mu_X = \mu_Y$? true or false? how confident?
  - $\hat{\mu}_X = 80.02$, $\hat{\mu}_Y = 79.98$, $\hat{\sigma}^2$=0.0007178
  - p-value<0.01, H_0: $\mu_X = \mu_Y$ is rejected under significance level 0.99.

- Compare the graphical and numerical methods
  - graphical methods: intuitive perception, vague conclusion

    cf. cf.
  - numerical methods: lack of intuition, accurate conclusion

Decision making

- There is a (statistically significant) difference between the means of the 2 methods: $\mu_X > \mu_Y$
- level of evidence?

- Some other examples of statistical applications
  - Election: survey on voting
  - Lung cancer ⟷ Smoking
  - Moneyball (魔球)
  - Thinking, fast and slow (快思慢想)
  - The signal and the noise (精準預測)
  - Big data
  - Data-based AI
  - …
- Materials to be covered in this course
  - Probability – A Review: Chapters 1~6
  - Estimation: Chapter 8
  - Hypothesis Testing: Chapter 9
  - Decision Theory: Chapter 15 (Rice, 1995, 2nd Edition)
  - Applications:
    - Survey Sampling: Chapter 7
    - Two-Sample Comparison: Chapter 11
    - Analysis of Variance: Chapter 12
    - Some Graphical Methods from Chapter 10

method-ology

applied

Website of my mathematical statistics course

http://www.stat.nthu.edu.tw/~swcheng/Teaching/stat3875/index.php

❖ **Further reading:**

- Lewis (2004), Moneyball (中譯：魔球).
- Kahneman (2011), Thinking, Fast and Slow (中譯：快思慢想).
- Silver (2012), The Signal and the Noise (中譯：精準預測).
- Stigler (2016), The Seven Pillars of Statistical Wisdom.