

MATH 2820, 2025, Homework 8

1. Suppose that Y have the pdf:

$$f_Y(y) = \begin{cases} \frac{2(\theta-y)}{\theta^2}, & 0 < y < \theta \\ 0, & \text{otherwise} \end{cases}$$

- (a) Show that $\frac{Y}{\theta}$ is a pivotal quantity.
 - (b) Find the cdf of $\frac{Y}{\theta}$.
 - (c) Use (a) to find a 90% confidence interval for θ .
2. Suppose that the random variable Y has a Gamma distribution with parameters $\alpha = 2$ and an unknown λ .
- (a) Show that $2\lambda Y$ has a χ^2 distribution with 4 degrees of freedom.
 - (b) Use (a) to derive a 95% confidence interval for λ .
- [Hint: Lecture Notes, Ch1-6, p.74, item 6 and p.77, item 2.]
3. Let X_1, \dots, X_n be an i.i.d. sample from the Beta distribution with parameters $\beta = 1$ and an unknown α .
- (a) Show that $Y_i = -2\alpha \log X_i$ follows exponential distribution $E(\frac{1}{2})$.
 - (b) Show that $-2\alpha \sum_{i=1}^n \log X_i$ is a pivotal quantity.
 - (c) Use (b) to derive a 95% confidence interval for α .
4. textbook, chapter 8, problem 60 (f)(g).