MATH 2820, 2026, Homework 8

1. Suppose that Y have the pdf:

$$f_Y(y) = \begin{cases} \frac{2(\theta-y)}{\theta^2}, & 0 < y < \theta \\ 0, & \text{otherwise} \end{cases}$$

   (a) Show that $\frac{Y}{\theta}$ is a pivotal quantity.

   (b) Find the cdf of $\frac{Y}{\theta}$.

   (c) Use (a) to find a 90% confidence interval for θ.

2. Suppose that the random variable Y has a Gamma distribution with parameters $\alpha = 2$ and an unknown λ.

   (a) Show that $2\lambda Y$ has a χ^2 distribution with 4 degrees of freedom.

   (b) Use (a) to derive a 95% confidence interval for λ.

   [Hint: Lecture Notes, Ch1-6, p.2-74, item 6 and p.2-77, item 2.]

3. Let $X_1, \ldots, X_n$ be an i.i.d. sample from the Beta distribution with parameters $\beta = 1$ and an unknown α.

   (a) Show that $Y_i = -2\alpha \log X_i$ follows exponential distribution $E(\frac{1}{2})$.

   (b) Show that $-2\alpha \sum_{i=1}^{n} \log X_i$ is a pivotal quantity.

   (c) Use (b) to derive a 95% confidence interval for α.

4. textbook, chapter 8, problem 60 (f)(g).