MATH 2820, 2026, Homework 10

1. textbook, Chapter 9, problem 8. [Hint: Example 7.9 in LNp.4-20.]

2. textbook, Chapter 9, problem 18. [Hint: Neyman-Pearson lemma and Example 7.9 in LNp.4-20.]

3. Let $X_1, \ldots, X_n$ be i.i.d. with pdf f which can be either f_0 or else f_1, where f_0 is Poisson $P(1)$ and f_1 is the Geometric pdf with $p = \frac{1}{2}$. Find the most powerful (MP) test of the hypothesis $H_0 : f = f_0$ vs. the alternative $H_A : f = f_1$ at level of significance 0.05.

   [Hint: apply Neyman-Pearson lemma and construct a *randomized* test]

4. Let $X_1, \ldots, X_n$ be independent random variable with pdf f given by

$$f(x|\theta) = \frac{1}{\theta}\, e^{-\frac{x}{\theta}}, \quad \text{for } x \geq 0,$$

   where $\theta \in \Omega = (0, \infty)$. Derive the uniformly most powerful (UMP) test for testing the hypothesis $H_0 : \theta \geq \theta_0$ vs. the alternative $H_A : \theta < \theta_0$ at level of significance α.

   [Hint: Question 7.10 in LNp.4-24, and notice that $X_1 + \cdots + X_n$ follows a Gamma distribution.]

5. Let X_1, X_2, X_3 be i.i.d. from Binomial $B(1, p)$. Derive the uniformly most powerful unbiased (UMPU) test for testing the hypothesis $H_0 : p = 0.25$ vs. the alternative $H_A : p \neq 0.25$ at level of significance α. Determine the test for $\alpha = 0.05$.

   [Hint: If $Y \sim B(3, 0.25)$, then $P(Y = 0) = 0.42$, $P(Y = 1) = 0.42$, $P(Y = 2) = 0.14$, and $P(Y = 3) = 0.02$]