

樣本空間

Sample Space and Events

can chosen to be larger than all possible outcomes p. 3-1

- Sample Space Ω : the set of all possible outcomes in a random phenomenon. Examples:

known

Final outcome is not 100% predictable

discrete → countable

1 Sex of a newborn child: $\Omega = \{\text{girl, boy}\}$

2 The order of finish in a race among the 7 horses 1, 2, ..., 7:

finite set

$\Omega = \{\text{all } 7! \text{ Permutations of } 1, 2, 3, 4, 5, 6, 7\}$

a list

3 Flipping two coins: $\Omega = \{(H, H), (H, T), (T, H), (T, T)\}$

infinite set

4 Number of phone calls received in a year: $\Omega = \{0, 1, 2, 3, \dots\}$

countably infinite

5 Lifetime (in hours) of a transistor: $\Omega = [0, \infty)$

continuous → uncountable

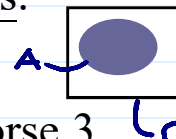
might be difficult to identify in some cases

uncountably infinite

- Event: Any (measurable) subset of Ω is an event. Examples:

事件

1. $A = \{\text{girl}\}$: the event - child is a girl.



2. $A = \{\text{all outcomes in } \Omega \text{ starting with a 3}\}$: the event - horse 3 wins the race.

• $A = \{(H, H), (H, T)\}$: the event - head appears on the 1st coin. p. 3-2

• $A = \{0, 1, \dots, 500\}$: the event - no more than 500 calls received

• $A = [0, 5]$: the event - transistor does not last longer than 5 hours.

power set → 2^Ω : collection of all subsets in Ω | $\Omega = \{\omega_1, \dots, \omega_n\}$

➤ an event occurs $\Leftrightarrow$ outcome $\in$ the event (subset)



➤ Q: How many different events if $\#\Omega = n < \infty$?

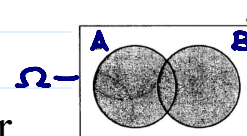
$2^\Omega = \{\emptyset, \{\omega_1\}, \dots, \{\omega_n\}, \{\omega_1, \omega_2\}, \dots, \Omega\}$

• Set Operations of Events

Ans. $\#2^\Omega = 2^n$

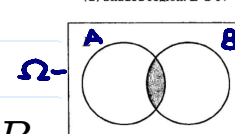
LN p. 2-7.

➤ Union. $C = A \cup B \Rightarrow C$: either A or B occurs



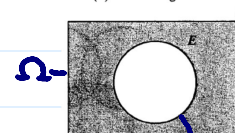
(a) Shaded region: $E \cup F$.

➤ Intersection. $C = A \cap B \Rightarrow C$: both A and B occur



(b) Shaded region: EF .

➤ Complement. $C = A^c \Rightarrow C$: A does not occur



(c) Shaded region: E^c .

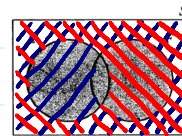
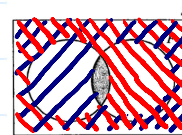
➤ Mutually exclusive (disjoint). $A \cap B = \emptyset \Rightarrow A$ and B have no outcomes in common.

including countably infinite many

➤ Definitions of union and intersection for more than 2 events can be defined in a similar manner

Some Simple Rules of Set Operations

- Commutative Laws. $A \cup B = B \cup A$ and $A \cap B = B \cap A$
- Associative Laws. $(A \cup B) \cup C = A \cup (B \cup C)$
 $(A \cap B) \cap C = A \cap (B \cap C)$.
- Distributive Laws. $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$
 $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$
- DeMorgan's Laws.

(a) Shaded region: $E \cup F$.(b) Shaded region: $E \cap F$.

required,
not optional

$$\left(\bigcup_{i=1}^n A_i\right)^c = \bigcap_{i=1}^n A_i^c \quad \text{and} \quad \left(\bigcap_{i=1}^n A_i\right)^c = \bigcup_{i=1}^n A_i^c.$$

❖ Reading: textbook, Sec 2.2

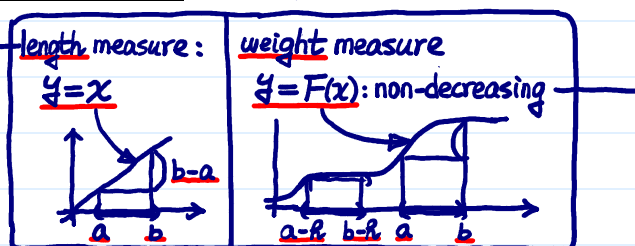
by induction
(exercise)

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~~$\Omega \rightarrow [0, 1]$~~

機率測度 → Probability Measure → a function: $2^\Omega \rightarrow [0, 1]$

- The Classical Approach $\int g(x) dx$
- Sample Space Ω is a finite set
- Probability: For an event A ,



probability space
 $(\Omega, \mathcal{F}, P)$
collection of events

$$P(A) = \frac{\#A}{\#\Omega}$$

This explains why combinatorial Thm plays an important role in probability.

$$\int g(x) dF(x)$$