樣本空間

Sample Space and Events

can chosen to be larger than all possible outcomes

- *Sample Space* Ω: the set of all possible outcomes in a random phenomenon. Examples:

known

final outcome is not 100% predictable

discrete → countable

1. Sex of a newborn child: Ω = {girl, boy}
2. The order of finish in a race among the 7 horses 1, 2, …, 7:

   finite set

   Ω = { all 7! Permutations of (1, 2, 3, 4, 5, 6, 7)}

   a list
3. Flipping two coins: Ω = {(H, H), (H, T), (T, H), (T, T)}

   infinite set
4. Number of phone calls received in a year: Ω = {0, 1, 2, 3, …}

   countably infinite
5. Lifetime (in hours) of a transistor: $\Omega = [0, \infty)$

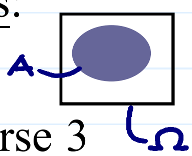
   uncountably infinite

continuous → uncountable

might be difficult to identify in some cases

- *Event*: Any (measurable) subset of Ω is an event. Examples:

事件



1. A={girl}: the event - child is a girl.
2. A={all outcomes in Ω starting with a 3}: the event - horse 3 wins the race.

- A={(H, H), (H, T)}: the event - head appears on the 1st coin.
- A={0, 1, …, 500}: the event - no more than 500 calls received
- A=[0, 5]: the event - transistor does not last longer than 5 hours.

power set → 2^Ω: collection of all subsets in Ω | $\Omega=\{\omega_1, \cdots, \omega_n\}$ 1 0 … 1 0

➢ an event occurs ⇔ outcome ∈ the event (subset)

➢ **Q**: How many different events if $\#\Omega=n<\infty$?

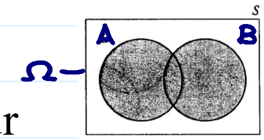
$2^\Omega=\{\phi, \{\omega_1\}, \cdots, \{\omega_n\}, \{\omega_1,\omega_2\}, \cdots, \Omega\}$

Ans. $\#2^\Omega=2^n$

LNp 2-7

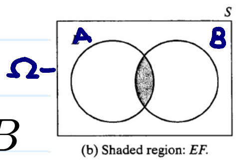
- Set Operations of Events

➢ Union. $C = A \cup B \Rightarrow C$: either A or B occurs



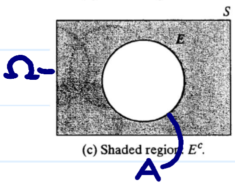
(a) Shaded region: $E \cup F$.

➢ Intersection. $C = A \cap B \Rightarrow C$: both A and B occur



(b) Shaded region: EF.

➢ Complement. $C = A^c \Rightarrow C$: A does not occur



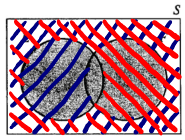
(c) Shaded region: E^c.

➢ Mutually exclusive (disjoint). $A \cap B = \emptyset \Rightarrow A$ and B have no outcomes in common.

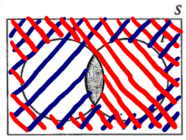
including countably infinite many

➢ Definitions of union and intersection for more than 2 events can be defined in a similar manner

- Some Simple Rules of Set Operations
  - Commutative Laws. $A \cup B = B \cup A$ and $A \cap B = B \cap A$
  - Associative Laws. $(A \cup B) \cup C = A \cup (B \cup C)$
    $(A \cap B) \cap C = A \cap (B \cap C)$.
  - Distributive Laws. $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$
    $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$
  - DeMorgan's Laws.



(a) Shaded region: $E \cup F$.



(b) Shaded region: EF.

required, not optional

$$(\cup_{i=1}^n A_i)^c = \cap_{i=1}^n A_i^c \quad \text{and} \quad (\cap_{i=1}^n A_i)^c = \cup_{i=1}^n A_i^c.$$

$\boxed{\text{by induction}}$ (exercise)

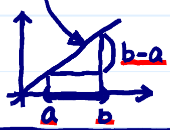
Reading: textbook, Sec 2.2

~~$\Omega \to [0,1]$~~

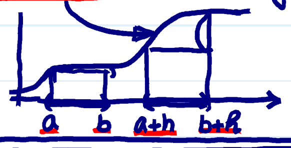
機率測度 → Probability Measure → a function: $2^\Omega \to [0,1]$

- The Classical Approach
  - Sample Space Ω is a *finite* set
  - Probability: For an event A,

length measure: $y = x$



weight measure
$y = F(x)$ non-decreasing



probability space
$(\Omega, \mathcal{F}, P)$
collection of events

$$P(A) = \frac{\#A}{\#\Omega}$$

This explains why combinatorial Thm plays an important role in probability.