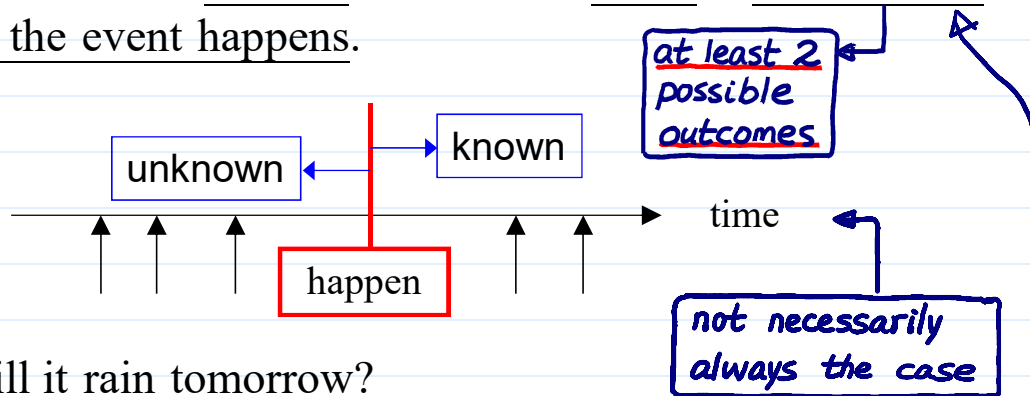


Introduction to Probability

• Uncertainty/Randomness (不確定性/隨機性) in our life

➤ Many events are random in that their result is unknowable before the event happens.



- Will it rain tomorrow?
- How many wins will a player/team achieve this season?
- What numbers will I roll on two dice?
- Q: Is your height/weight measure random?

outcome: random
probability: fixed

c.f.

➤ We often want to assess how likely it is the outcomes of interest occur. Probability is that measurement.

Random vs. Deterministic Patterns

random	混亂 隨機	noise (雜訊)	uncertain result
deterministic	規律 秩序	signal (信號)	predictable result

決定性的 →

100%

Consider the two cases:

➤ Case I (← random pattern?)

1	2	3	4	5	6	7	8	9	10
●	●	●	●	●	●	●	●	●	?
R	R	G	R	G	R	R	R	G	

➤ Case II (← deterministic pattern?)

1	2	3	4	5	6	7	8	9	10
●	●	●	●	●	●	●	●	●	?
R	R	G	R	R	G	R	R	G	

➤ Note. #R : #G = 2 : 1

number

- (Possible) modeling: *the color in the nth trial.*

➤ Case I. $X_1, X_2, \dots, X_n, \dots$ are independent, for $i=1, 2, \dots,$

$$X_i = \begin{cases} \underline{R}, & \text{with prob. } 2/3, \\ \underline{G}, & \text{with prob. } 1/3. \end{cases}$$

random variable (future lecture)

➤ Case II. For $i=3, 4, \dots,$

$$X_i = \begin{cases} \underline{R}, & \text{if } (X_{i-2}, X_{i-1}) \in \{(R, G), (G, R)\}, \\ \underline{G}, & \text{if } (X_{i-2}, X_{i-1}) = \underline{(R, R)}. \end{cases} \quad (*)$$

- Prediction strategy:

➤ Case I: always guess $X_i=R$ (*why?* next slide)

➤ Case II: decide X_i by X_{i-1}, X_{i-2} using $(*)$

➤ Q: why always guess $X_i=R$ for Case 1? →

▪ Let $X_i = \begin{cases} \underline{R}, & \text{with prob. } p, p \in [0, 1] \\ \underline{G}, & \text{with prob. } 1 - p. \end{cases}$

guessing strategy →

$$Y_i = \begin{cases} \underline{R}, & \text{with prob. } q, q \in [0, 1] \\ \underline{G}, & \text{with prob. } 1 - q. \end{cases}$$

To possibly reach 100% accuracy, we need to guess 100% of R and 100(1-p)% of G. Is this a good strategy?

▪ Then,

$$P(X_i = Y_i) = P((X_i, Y_i) \in \{(G, G), (R, R)\})$$

∴ independent assumption →

$$\begin{aligned} &= pq + (1 - p)(1 - q) \\ &= 1 - p + (2p - 1)q \end{aligned}$$

		Y_i	
		R	G
X_i	R	✓	✗
	G	✗	✓

▪ The $P(X_i=Y_i)$ is maximized at

Giving up 100% accuracy and accepting errors in prediction when G (or R) occurs increase overall predictive accuracy.

$$q = \begin{cases} \underline{1}, & \text{if } p > 0.5, \Leftrightarrow 2p - 1 > 0 \leftarrow \text{always guess R} \\ \underline{0}, & \text{if } p < 0.5, \Leftrightarrow 2p - 1 < 0 \leftarrow \text{always guess G} \end{cases}$$

and

What if $p=1/2$?

$$\max_q P(X_i = Y_i) = \begin{cases} p, & \text{if } p > 0.5, \\ 1 - p, & \text{if } p < 0.5. \end{cases}$$

- **Q:** Is Case II really a deterministic pattern?

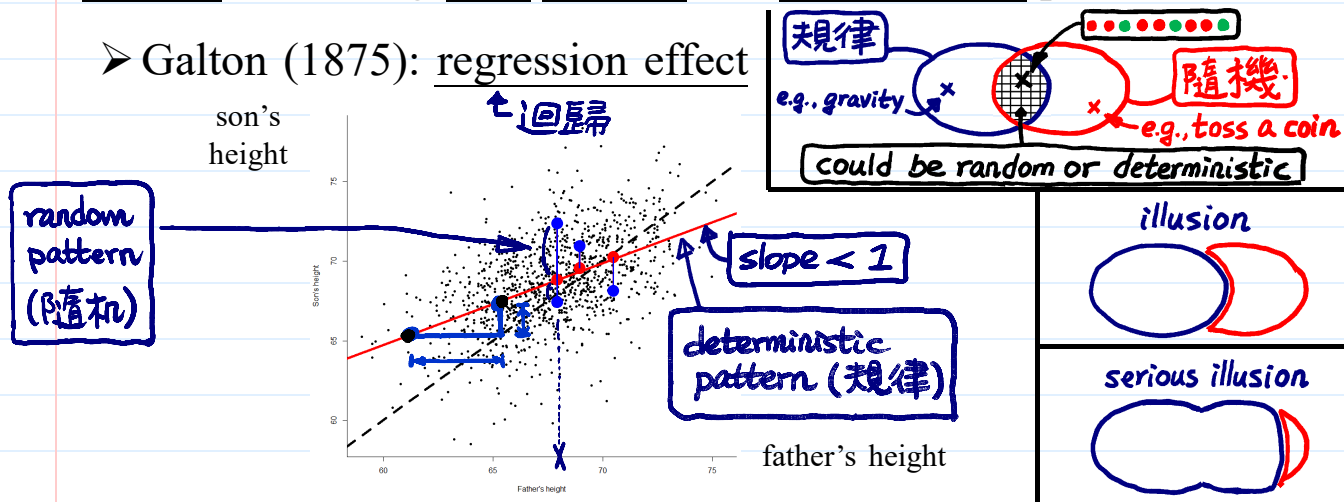
➤ Under the model for Case I, ← *LNp.1-3*

$$P(\underline{RRGRRRGRG}) = \left[\left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right) \right]^3 = \underline{0.325\%}$$

- Random pattern $\xrightarrow{\text{誤判}}$ Deterministic pattern (remedy: replicate)
← 隨機 ← 規律
- Deterministic pattern $\xrightarrow{\text{誤判}}$ Random pattern (remedy: controlled experiment)

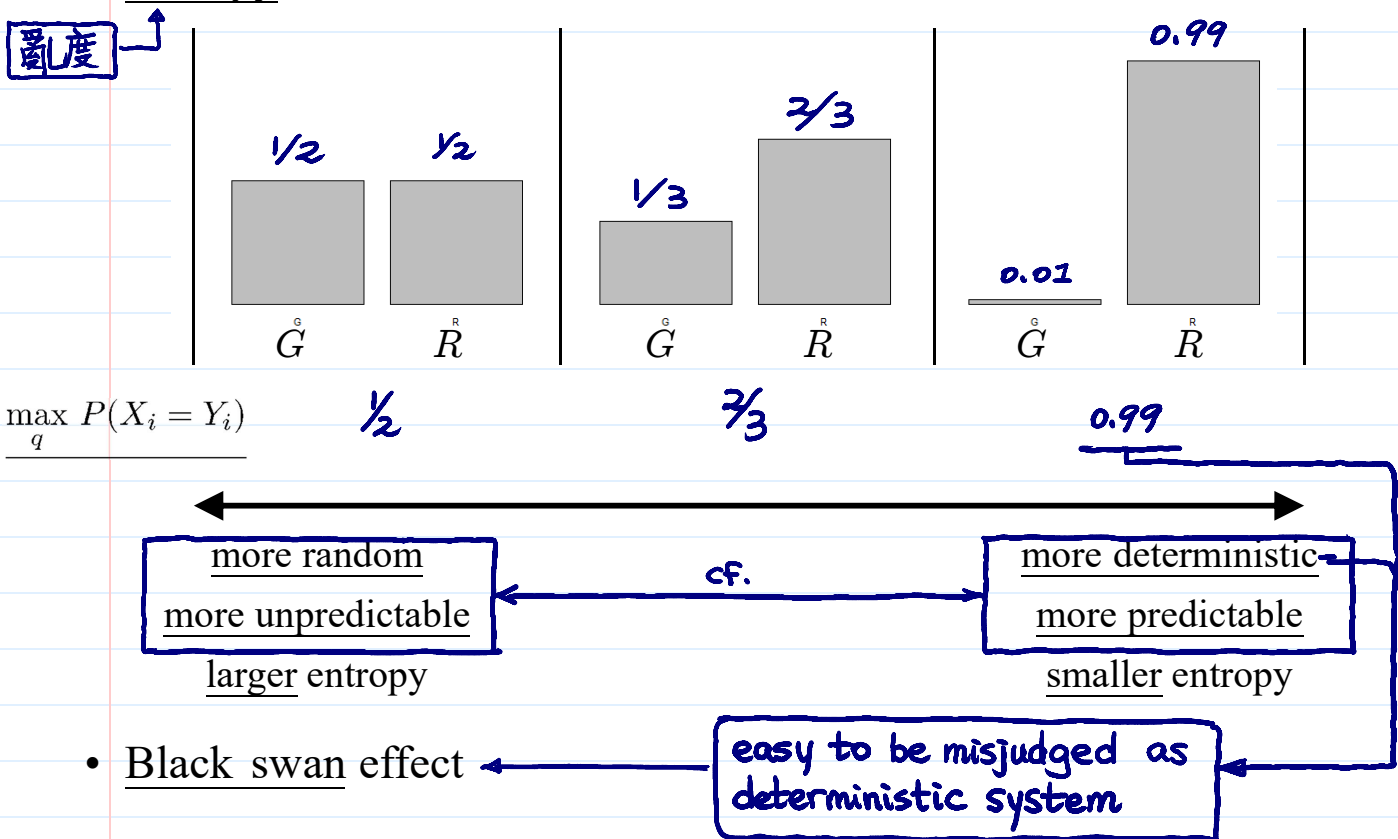
- System containing both random and deterministic patterns

➤ Galton (1875): regression effect



Random = totally unpredictable?

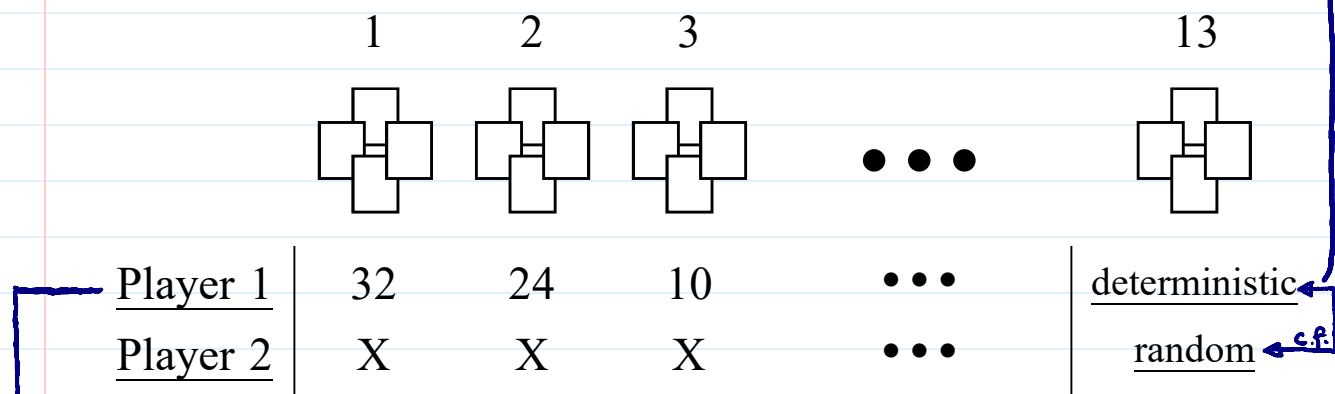
- Entropy: $-[p \log(p) + (1 - p) \log(1 - p)]$



Should everyone have the same probability for an event?

more information
less random

- Example: 52 cards



- Conditional probability ← probability evolves → check the graph in LNp.1-1.
- Subjective (Bayesian) probability:

紅樓夢的作者是曹雪芹嗎?
信者恆信，不信者恆不信

It's the Chance (Probability, Proportion, Frequency), Stupid

- Bill Clinton, 1992, Campaign slogan

It's the Economy, Stupid.

- Examples

- 該買某保險嗎?
- 發生飛機失事事件後，該改成開車嗎?
- 規畫謬誤：蚊子館、該創業嗎?
- 敘述謬誤：偉人（成功）的故事
- 賭徒謬誤：擲筊多次未成，則擲出聖筊機會變大?
- 馬路三寶? 汽車保險金額，男 > 女
- 車禍先問酒駕? 酒駕易肇事，yes，但肇事者多酒駕?
- 屏東人：你怎麼不黑?

肇事酒駕 large ⇨ 酒駕肇事 large

Distinction between Discovery (發現) and Invention (發明)

• Examples

➤ 哥倫布 “發現” 新大陸 ← 原本就有

➤ 愛迪生 “發明” 電燈泡 ← 無中生有

➤ **Q**: 相對論是發明還是發現?

• 機率論是人類 “發明” 來處理生活中的不確定性之理論

• 愛因斯坦: “上帝永遠不會擲骰子”

something is due to chance

not necessarily mean that it's indeterminate

only mean that it is currently indeterminable

e.g.

other approaches? ←

算命、占卜、...

風水、改運、...

祈禱、拜拜、...

Indeterminableness could change with time.

❖ Further Readings: ← optional

✓ Kahneman (2011), *Thinking, Fast and Slow*. (中譯: 快思慢想)

✓ Silver (2012), *The Signal and the Noise*. (中譯: 精準預測)