Probability_HW07_Solution

Problem 16

$(\star):$ *Assume that the events of annual rainfall exceeding 50 inches each year are mutually independent.*

$Let\ X\ be\ the\ annual\ rainfall\ for\ a\ given\ year.\ Then\ X \sim N(40, 4^2).$
$Let\ p \equiv Pr\Big(X < 50\Big).\ Then\ we\ have:$

$$\begin{aligned} p &= Pr\Big(\frac{X-40}{4} < \frac{50-40}{4}\Big) \\ &= Pr\Big(Z < 2.5\Big) \qquad \Big(By\ normalizing\ X,\ we\ have\ Z \equiv \frac{X-40}{4} \sim N(0,1).\Big) \\ &= \Phi(2.5) = 0.9937903 \end{aligned}$$

$Let\ Y\ be\ the\ number\ of\ years\ starting\ from\ this\ year\ until\ the\ first\ occurrence\ of\ annual\ rainfall\ exceeding\ 50\ inches.$
$Then,\ Y \sim Geometric(1-p),\ and\ the\ desired\ quantity\ is:$

$$\begin{aligned} &Pr\Big(None\ of\ the\ following\ 10\ years\ has\ a\ rainfall\ of\ more\ than\ 50\ inches\Big) \\ &\overset{(\star)}{=} Pr\Big(Y > 10\Big) = \sum_{y=11}^{\infty} p^{y-1}(1-p) = p^{10} = \Big(0.9937903\Big)^{10} \end{aligned}$$

```r
pnorm(2.5)
```

```
## [1] 0.9937903
```

Problem 17

$Let\ X\ be\ the\ salaries.\ Then,\ X \sim N(\mu, \sigma^2),\ and\ we\ have:$

$$\begin{cases} 0.25 = Pr\Big(X < 180{,}000\Big) = Pr\Big(\frac{X-\mu}{\sigma} < \frac{180{,}000-\mu}{\sigma}\Big) = \Phi(\frac{180{,}000-\mu}{\sigma}) \overset{(\star)}{=} \Phi(-0.6744898) \\ 0.25 = Pr\Big(X > 320{,}000\Big) = Pr\Big(\frac{X-\mu}{\sigma} > \frac{320{,}000-\mu}{\sigma}\Big) = 1-\Phi(\frac{320{,}000-\mu}{\sigma}) \overset{(\star)}{=} 1-\Phi(0.6744898) \end{cases}$$

$$\Longrightarrow \begin{cases} \frac{180{,}000-\mu}{\sigma} = -0.6744898 \\ \frac{320{,}000-\mu}{\sigma} = \ \ 0.6744898 \end{cases} \Longrightarrow \mu = \frac{320{,}000+180{,}000}{2} = 250{,}000$$

$$\Longrightarrow \frac{180{,}000-250{,}000}{\sigma} = -0.6744898 \quad \Longrightarrow \sigma = 103782.1$$

$$(\star):\ \Phi(-0.6744898) = 1-\Phi(0.6744898) = 0.25$$

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```r
qnorm(0.25)
```

```
## [1] -0.6744898
```

(a)

$$Pr\Big(X < 200,000\Big) = Pr\Big(\frac{X-\mu}{\sigma} < \frac{200,000-250,000}{103782.1}\Big) \approx \Phi(-0.4817786) \approx 0.3149816$$

```r
pnorm((200000-250000)/103782.1)
```

```
## [1] 0.3149816
```

(b)

$$\begin{aligned} Pr\Big(280,000 < X < 320,000\Big) &= Pr\Big(\frac{280,000-\mu}{\sigma} < \frac{X-\mu}{\sigma} < \frac{320,000-\mu}{\sigma}\Big) \\ &= Pr\Big(\frac{280,000-\mu}{\sigma} < Z < \frac{320,000-\mu}{\sigma}\Big),\ where\ Z \sim N(0,1) \\ &= \Phi\Big(\frac{320,000-\mu}{\sigma}\Big) - \Phi\Big(\frac{280,000-\mu}{\sigma}\Big) \\ &\approx 0.75 - 0.613735 \\ &= 0.136265 \end{aligned}$$

```r
0.75-pnorm((280000-250000)/103782.1)
```

```
## [1] 0.136265
```

Problem 29

Let X be the number of times the stock price goes up (i.e., the price becomes $u\times$ its original value) in the next 1000 periods.
Then, $X \sim Binomial(1000, p)$, and $\Big(1000 - X\Big)$ is the number of times the stock price goes down (i.e., the price becomes d times its original value) in the next 1000 periods.
When the initial stock price is s, after 1000 periods, the price becomes $s\times u^X \times d^{1000-X} = s\times d^{1000}\times(\frac{u}{d})^X$.
So the event of interest is :

$$(u^x d^{1000-x})s \geq 1.3s \iff (1.012^x \times 0.99^{1000-x}) \geq 1.3 \iff 1000log(0.99)+x\Big(log(\frac{1.012}{0.99})\Big) \geq log(1.3) \iff x \geq 470.$$

Because $X \sim Binomial\Big(1000, 0.52\Big) \overset{d}{\approx} N\Big(520, 249.6\Big)$ by the †DeMoivre – Laplace limit theorem (see TBp.219),

$$Pr\Big(X \geq 470\Big) \approx Pr\Big(\frac{X-520}{\sqrt{249.6}} > \frac{470-0.5-520}{\sqrt{249.6}}\Big) \approx Pr(Z > -3.196459) \approx 0.9993044$$

(Note that the "continuity correction" has been taken!)

†***FYI: This result can also be obtained by applying the well-known Central Limit Theorem!***

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```r
1-pnorm((470-0.5-520)/sqrt(249.6))
```

```
## [1] 0.9993044
```

Problem 30

Let $X =$ *the reading of the randomly chosen point.*

Let $Y = \begin{cases} 0, \ if\ this\ point\ is\ in\ the\ white\ section \\ 1, \ if\ this\ point\ is\ in\ the\ black\ section \end{cases}$, *then* $Y \sim Bernoulli(\alpha) \iff p_Y(y) = \begin{cases} 1-\alpha, \ if\ y=0 \\ \alpha, \ if\ y=1 \end{cases}$.

So we have $\begin{cases} \left(X \mid Y=0\right) \sim N\left(4, 2^2\right) \\ \left(X \mid Y=1\right) \sim N\left(6, 3^2\right) \end{cases}$ *and we want to find* α *such that* $Pr\left(Y=0 \middle| X=5\right) = Pr\left(Y=1 \middle| X=5\right) = \frac{1}{2}$.

$$
\begin{aligned}
That\ is,\ \frac{1}{2} &= Pr\left(the\ chosen\ point\ having\ a\ reading\ of\ 5\ is\ from\ the\ black\ section\right) \\
&= Pr\left(Y=1 \mid X=5\right) \\
&= \frac{Pr\left(Y=1, X=5\right)}{Pr\left(X=5\right)} \\
&= \frac{Pr\left(Y=1\right)Pr\left(X=5 \mid Y=1\right)}{Pr\left(Y=1\right)Pr\left(X=5 \mid Y=1\right) + Pr\left(Y=0\right)Pr\left(X=5 \mid Y=0\right)} \\
&= \frac{\alpha \times \frac{1}{2\sqrt{2\pi}} exp\left(-\frac{\left(5-4\right)^2}{2\times 4}\right)}{\alpha \times \frac{1}{2\sqrt{2\pi}} exp\left(-\frac{\left(5-4\right)^2}{2\times 4}\right) + \left(1-\alpha\right) \times \frac{1}{3\sqrt{2\pi}} exp\left(-\frac{\left(5-6\right)^2}{2\times 9}\right)} \\
&\Longrightarrow {}^{\dagger}\alpha \approx 0.41677
\end{aligned}
$$

$\dagger$: *From the above derivation, using* $Pr\left(Y=0 \mid X=5\right) = \frac{1}{2}$ *would also yield the same result.*

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Problem 31

(a)

$$E\Big(|X-a|\Big) = \int_0^A |x-a|\frac{1}{A}dx = \frac{1}{A}\Big(\int_0^a (a-x)dx + \int_a^A (x-a)dx\Big)$$
$$= \frac{1}{A}\left(\Big(ax-\frac{x^2}{2}\Big)\Big|_0^a + \Big(\frac{x^2}{2}-ax\Big)\Big|_a^A\right) = \frac{A}{2} - a + \frac{a^2}{A}$$

$$\textit{To find } \underset{a\in(0,A)}{argmin}\, E\Big(|X-a|\Big),\ \textit{set } \left(\frac{d}{da}E\Big(|X-a|\Big)\right)\Big|_{a^\star} = -1 + \frac{2a^\star}{A} = 0,\ \textit{then } a^\star = \frac{A}{2}.$$

$$\textit{Note that } \left(\frac{d^2}{da^2}E\Big(|X-a|\Big)\right)\Big|_{a^\star} = \frac{2}{A} > 0,\ \textit{so } a = \frac{A}{2} \textit{ is actually the minimizer.}$$

(b)

$$E\Big(|X-a|\Big) = \int_0^\infty |x-a|\lambda e^{-\lambda x}dx = \Big(\int_0^a (a-x)\lambda e^{-\lambda x}dx + \int_a^\infty (x-a)\lambda e^{-\lambda x}dx\Big)$$
$$= \underbrace{\left(\int_0^a a\lambda e^{-\lambda x}dx\right)}_{(1)} - \underbrace{\left(\int_0^a \lambda x e^{-\lambda x}dx\right)}_{(2)} + \underbrace{\left(\int_a^\infty \lambda x e^{-\lambda x}dx\right)}_{(3)} - \underbrace{\left(\int_a^\infty a\lambda e^{-\lambda x}dx\right)}_{(4)}$$
$$= \underbrace{\left(\Big(-ae^{-\lambda x}\Big)\Big|_0^a\right)}_{(1)} - \underbrace{\left(\Big(xe^{-\lambda x}\Big)\Big|_0^a + \int_0^a e^{-\lambda x}dx\right)}_{(2)} + \underbrace{\left(\Big(-xe^{-\lambda x}\Big)\Big|_a^\infty + \int_a^\infty e^{-\lambda x}dx\right)}_{(3)} + \underbrace{\left(\Big(ae^{-\lambda x}\Big)\Big|_a^\infty\right)}_{(4)}$$
$$= \left(\cancel{-ae^{-\lambda a}} + a\right) - \left(\cancel{ae^{-\lambda a}} - \frac{1}{\lambda}e^{-\lambda a} + \frac{1}{\lambda}\right) + \left(\cancel{ae^{-\lambda a}} + \frac{1}{\lambda}e^{-\lambda a}\right) + \left(\cancel{ae^{-\lambda a}}\right)$$
$$= a + \frac{2}{\lambda}e^{-\lambda a} - \frac{1}{\lambda}$$

$$\begin{cases} (1):\ \textit{Direct integration} \\ (2):\ \textit{Do integration by parts with} \begin{pmatrix} u = x & du = dx \\ v = -e^{-\lambda x} & dv = \lambda e^{-\lambda x}dx \end{pmatrix} \textit{ over } \Big\{x\in\mathscr{R}\ \Big|\ 0<x<a\Big\} \\ (3):\ \textit{Do integration by parts with} \begin{pmatrix} u = x & du = dx \\ v = -e^{-\lambda x} & dv = \lambda e^{-\lambda x}dx \end{pmatrix} \textit{ over } \Big\{x\in\mathscr{R}\ \Big|\ a<x\Big\} \\ (4):\ \textit{Direct integration} \end{cases}$$

$$\textit{To find } \underset{a\in(0,A)}{argmin}\, E\Big(|X-a|\Big),\ \textit{set } \left(\frac{d}{da}E\Big(|X-a|\Big)\right)\Big|_{a^\star} = 1 - 2e^{-\lambda a^\star} = 0,\ \textit{then } a^\star = \frac{log(2)}{\lambda}.$$

$$\textit{Note that } \left(\frac{d^2}{da^2}E\Big(|X-a|\Big)\right)\Big|_{a^\star} = 2\lambda e^{-\lambda a^\star} > 0,\ \textit{so } a = \frac{log(2)}{\lambda} \textit{ is actually the minimizer.}$$

Problem 32

Because $X \sim exponential(\lambda = \frac{1}{1.5})$, *the cdf of* X *is* $F_X(x) = 1 - e^{-\frac{2}{3}x}$ *for* $x \geq 0$.

(a)

$$\text{The desired quantity is } Pr\Big(X > 2\Big) = 1 - F_X(2) = e^{-4/3}.$$

(b)

$$\text{The desired quantity is } Pr\Big(X > 2 \,\Big|\, X > 1\Big) = \frac{Pr\Big(X > 2\Big)}{Pr\Big(X > 1\Big)} = \frac{e^{-4/3}}{e^{-2/3}} = e^{-2/3}, \text{ which equals } Pr\Big(X > 1\Big).$$

This shows the memoryless property of an exponential distribution.

Theoretical Exercise 13

(a)

$$\text{Let } X \sim Uniform(a,b), \text{ then the cdf of } X \text{ is } F_X(x) = \begin{cases} \int_a^x \frac{1}{b-a}du & , \text{ if } a < x < b \\ 0 & , \text{ if } x \leq a \end{cases}.$$

Let $m_{(a)}$ *be the median of* X, *so we have* :

$$F_X(m_{(a)}) = \frac{1}{2} = \int_a^{m_{(a)}} \frac{1}{b-a}du = \frac{u}{b-a}\Bigg|_{u=a}^{u=m_{(a)}} = \frac{m_{(a)} - a}{b-a} \Longrightarrow m_{(a)} = \frac{a+b}{2}$$

(b)

Let $m_{(b)}$ *be the median of* X, *where* $X \sim N(\mu, \sigma^2)$.

Then the pdf of X, *say* $f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} exp\Big\{-\frac{(x-\mu)^2}{2\sigma^2}\Big\}$, *is symmetric about* μ,

$$i.e., \ f_X(\mu + \delta) = f_X(\mu - \delta) = \frac{1}{\sqrt{2\pi\sigma^2}} exp\Big\{-\frac{\delta^2}{2\sigma^2}\Big\}, \ \forall \delta \in \mathscr{R} \overset{(\star)}{\Longrightarrow} m_{(b)} = \mu$$

$$(\star): \int_{-\infty}^{\infty} f_X(x)dx = \int_{-\infty}^{\mu} f_X(x)dx + \int_{\mu}^{\infty} f_X(x)dx = 2\int_{-\infty}^{\mu} f_X(x)dx = 1$$

(c)

$$\text{Let } X \sim Exp(\lambda), \text{ then the cdf of } X \text{ is } F_X(x) = \begin{cases} 1 - e^{-\lambda x} & , \text{ if } 0 < x \\ 0 & , \text{ if } x \leq 0 \end{cases}.$$

Let $m_{(c)}$ *be the median of* X, *then* $\frac{1}{2} = 1 - e^{-\lambda m_{(c)}}$.

$$\Longrightarrow log(\frac{1}{2}) = -\lambda m_{(c)} \Longrightarrow m_{(c)} = \frac{log(2)}{\lambda}$$

Theoretical Exercise 19

The pdf of X is $f_X(x) = \lambda e^{-\lambda x},\ x \geq 0,$ *so*

$$\begin{aligned} E[X^k] &= \int_0^\infty x^k f_X(x)dx = \int_0^\infty \lambda x^k e^{-\lambda x}dx \\ &= \frac{\Gamma(k+1)}{\lambda^k} \underbrace{\int_0^\infty \frac{\lambda^{k+1}}{\Gamma(k+1)} x^{\left((k+1)-1\right)} e^{-\lambda x}dx}_{(\star)} \\ &= \frac{\Gamma(k+1)}{\lambda^k} = \frac{k!}{\lambda^k} \end{aligned}$$

$\left(\textit{Note that } (\star) \textit{ equals } 1, \textit{ since its integrand is the pdf of a } \Gamma(k+1,\lambda) \textit{ distribution.}\right)$

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Theoretical Exercise 25

(We solve this problem by (a) CDF method (b) PDF method.)

First note that the transformation $\mathscr{G}: \{x \mid x > \nu\} \longrightarrow \{y \mid y > 0\}$, $x \longmapsto y = \mathscr{G}(x) = \left(\frac{x-\nu}{\alpha}\right)^{\beta}$ *is bijective and monotone,*

and that $x = \mathscr{G}^{-1}(y) = \alpha y^{\frac{1}{\beta}} + \nu$. $(\star)$

(a)

Because $X \sim Weibull(\nu, \alpha, \beta)$, *the cdf of X is* $F_X(x) = \begin{cases} 0 & , \text{ if } x \le \nu \\ 1 - exp\left\{ -\left(\frac{x-\nu}{\alpha}\right)^{\beta} \right\} & , \text{if } x > \nu \end{cases}$. $(\star\star)$

Note that $Y = \mathscr{G}(X) = \left(\frac{X-\nu}{\alpha}\right)^{\beta}$, *so we can derive the cdf of* Y:

$$\left(\begin{aligned} F_Y(y) &= Pr\{Y = \mathscr{G}(X) \le y\} = Pr\left\{\left(\frac{X-\nu}{\alpha}\right)^{\beta} \le y\right\} \overset{(\star)}{=} Pr\left\{X \le \alpha y^{\frac{1}{\beta}} + \nu\right\} \\ &\overset{(\star\star)}{=} 1 - exp\left\{ -\left(\frac{\alpha y^{\frac{1}{\beta}} + \nu - \nu}{\alpha}\right)^{\beta} \right\} = 1 - e^{-y},\ y > 0 \end{aligned} \right) \Longleftrightarrow Y \sim Exp(1)^{\dagger}$$

†: Note that the cdf specifies a distribution.

(b)

Note that $\frac{d\mathscr{G}^{-1}(y)}{dy} = \frac{\alpha}{\beta} y^{\frac{1}{\beta}-1}$, *and that* $\mathscr{G}^{-1}(y) = \alpha y^{\frac{1}{\beta}} + \nu$. $(\star\star\star)$

$X \sim Weibull(\nu, \alpha, \beta)$

$\Longleftrightarrow$ *For* $x \in R_X = \{x \mid x > \nu\}$, $f_X(x) = exp\left\{ -\left(\frac{x-\nu}{\alpha}\right)^{\beta} \right\} \frac{\beta}{\alpha} \left(\frac{x-\nu}{\alpha}\right)^{\beta-1}$

$$\Longleftrightarrow \left(\begin{aligned} \text{For } y \in R_Y = \{y \mid y > 0\},\ f_Y(y) &= \overbrace{f_X(\mathscr{G}^{-1}(y))} \times \overbrace{\left|\frac{d\mathscr{G}^{-1}(y)}{dy}\right|} \\ &\overset{(\star\star\star)}{=} exp\left\{ -\left(\frac{(\alpha y^{\frac{1}{\beta}} + \nu) - \nu}{\alpha}\right)^{\beta} \right\} \frac{\beta}{\alpha} \left(\frac{(\alpha y^{\frac{1}{\beta}} + \nu) - \nu}{\alpha}\right)^{\beta-1} \times \left(\frac{\alpha}{\beta} y^{\frac{1}{\beta}-1}\right) \\ &= e^{-y}, \text{ which is the pdf of exponential}(1). \end{aligned} \right)$$

$\Longleftrightarrow Y \sim Exponential(1)$

Theoretical Exercise 31

$We\ can\ find\ the\ pdf\ f_Y\ of\ Y\ by\ \overbrace{applying\ the\ theorem\ in\ LNp.6-10}^{(\star)}.$

$Let\ f_X(x)\ be\ the\ pdf\ of\ X,\ where\ X \sim N(\mu, \sigma^2).$

$Let\ Y = g(X) \equiv e^X,\ so\ the\ the\ range\ of\ Y\ is\ R_Y = \Big\{ y \ \Big|\ y > 0 \Big\}.$

$We\ then\ have\ g^{-1}(Y) = log(X)\ and\ \left|\frac{dg^{-1}(y)}{dy}\right| = \frac{1}{y}.$

$(\star) : \Big(Note\ that\ \frac{d}{dx}g(x) = e^x > 0\ \forall\ x \in R_X = \mathscr{R},\ so\ g\ is\ differentiable\ and\ strictly\ monotone. \Big)$

$So\ the\ pdf\ of\ y\ is:$

$$f_Y(y) = \begin{cases} f_X(g^{-1}(x))\left|\dfrac{dg^{-1}(y)}{dy}\right| = \dfrac{1}{y\sqrt{2\pi\sigma^2}} exp\Bigg\{ -\dfrac{\big(log(y)-\mu\big)^2}{2\sigma^2} \Bigg\} & ,\ y > 0 \\ 0 & ,\ y \leq 0 \end{cases}.$$

$An\ alternative\ approach\ is\ to\ first\ derive\ the\ cdf\ F_Y\ of\ Y,\ and\ then\ obtain\ the\ pdf\ f_Y\ from\ the\ cdf\ F_Y\ as\ follows.$

$$(\star\star) \begin{cases} F_Y(y) = Pr\Big\{ Y \leq y \Big\} = Pr\Big\{ e^X \leq y \Big\} \\ \qquad = \begin{cases} Pr\Big\{ X < log(y) \Big\} & ,\ if\ y > 0 \\ 0 & ,\ if\ y \leq 0 \end{cases} \\ \qquad = \begin{cases} \int_{-\infty}^{log(y)} f_X(x)dx & ,\ if\ y > 0 \\ 0 & ,\ if\ y \leq 0 \end{cases} \end{cases}$$

$Take\ derivative\ on\ (\star\star)\ with\ respect\ to\ y\ over\ \Big\{ y \ \Big|\ y > 0 \Big\},\ we\ obtain:$

$$f_Y(y) = \frac{d}{dy}F_Y(y) = \frac{1}{y} f_X\Big(log(y)\Big) = \frac{1}{y\sqrt{2\pi\sigma^2}} exp\Bigg\{ -\frac{\big(log(y)-\mu\big)^2}{2\sigma^2} \Bigg\},\ if\ y > 0.$$

$$\Big(Note\ that\ we\ have\ applied\ the\ ^{\dagger}Leibniz\ integral\ rule. \Big)$$

$Also,\ it\ is\ obvious\ that\ f_Y(y) = 0,\ if\ y < 0.$

$Thus,\ the\ pdf\ of\ Y,\ f_Y(y),\ is:$

$$f_Y(y) = \begin{cases} \frac{1}{y\sqrt{2\pi\sigma^2}} exp\Big\{ -\frac{\big(log(y)-\mu\big)^2}{2\sigma^2} \Big\} & ,\ if\ y > 0 \\ 0 & ,\ if\ y \leq 0 \end{cases}$$

$This\ verifies\ our\ answer\ above!$

$\dagger$: ***You may browse this page for details about Leibniz integral rule.***

R

The following is a brief introduction to R language (about pdf, cdf, quantiles, generating random samples).

Some problems above require finding the quantiles of a normal distribution, but looking them up on tables is a very outdated practice. Using R is a more efficient method, below are some basic functions of R:

Let $X \sim N(0,1)$, which is the same as default.

1. pnorm(3)=$Pr(X \leq 3)$=$\Phi(3)$, is the cdf.
2. qnorm(0.2)=$\Phi^{-1}(0.2) = z_{0.2}$.
3. dnorm(0.4)=$f_X(0.4)$ is the pdf of X at 0.4.
4. rnorm(5) generates a sample iid from N(0,1) of size “5”.

For example:

- To have a $N(\mu, \sigma^2)$ distribution, use norm(μ, σ), for example, norm(3,2) is N(3,4).
- To generate a sample iid from N(3,4) of size 100, use rnorm(100,3,2).
- To find $z_{0.05}$, use qnorm(0.05).

The above (p,q,d,r) functions can also be used for other distributions, for example:

$$\left\{\begin{aligned} (What\ we\ want) &\longleftrightarrow (Code) \\ Negative\ Binomial &\longleftrightarrow nbinom \\ Hypergeometric &\longleftrightarrow hyper \\ Exponential &\longleftrightarrow exp \\ Geometric &\longleftrightarrow geom \\ Uniform &\longleftrightarrow unif \\ Binomial &\longleftrightarrow binom \\ Normal &\longleftrightarrow norm \\ Gamma &\longleftrightarrow gamma \\ Posiion &\longleftrightarrow pois \\ Weibull &\longleftrightarrow weibull \\ Cauchy &\longleftrightarrow cauchy \\ Beta &\longleftrightarrow beta \\ \chi^2 &\longleftrightarrow chisq \\ \mathscr{T} &\longleftrightarrow t \\ \mathscr{F} &\longleftrightarrow f \end{aligned}\right\} \Longrightarrow {}^{\dagger}Use\ (p,q,r,d) + RHS,\ like\ rt,\ qbeta,\ dchisq,\ or\ pf \ldots$$

$\dagger:$ *You must enter the required parameters, otherwise R will use the default parameters.*

Resources with hyperlinks:

A resource for those who have not used R, including instructions for downloading R!

An entry-level learning resource!

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