Probability_HW02_Solution

Throughout this solution we use some notations for convenience:

(i)

Let A be an event in the sample space we are considering.
We use $\#(A)$ to denote the number of elements in A.
For example, if $A = \{2, 3, 4\}$, then $\#(A) = 3$.

(ii)

We use **ANS** to denote the answer.

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$$\begin{gathered}
Let\ x\ be\ the\ final\ score\ of\ the\ HOME\ team,\ y\ be\ the\ final\ score\ of\ the\ AWAY\ team,\ then:\\
The\ sample\ space \equiv \Omega = \{(x, y) : x, y \in \ \{0, 1, ..., 6\},\ x + y \leq 6\}\\
= \{(0,0),(0,1),(0,2),(0,3),(0,4),(0,5),(0,6),\\
(1,0),(1,1),(1,2),(1,3),(1,4),(1,5),\\
(2,0),(2,1),(2,2),(2,3),(2,4),\\
(3,0),(3,1),(3,2),(3,3),\\
(4,0),(4,1),(4,2),\\
(5,0),(5,1),\\
(6,0)\}
\end{gathered}$$

- $A = \{(0,0),(1,1),(2,2),(3,3)\}$
- $B = \{(1,0),(2,0),(3,0),(4,0),(5,0),(6,0),(2,1),(3,1),(4,1),(5,1),(3,2),(4,2)\}$
- $A \cap B = \emptyset,\ the\ empty\ set.$
- $A \cup B = \{(0,0),(1,1),(2,2),(3,3),(1,0),(2,0),(3,0),(4,0),(5,0),(6,0),(2,1),(3,1),(4,1),(5,1),(3,2),(4,2)\}$
- $C = \{(0,1),(0,2),(0,3),(0,4),(0,5),(0,6),(1,1),(1,2),(1,3),(1,4),(1,5),$
  $(2,1),(2,2),(2,3),(2,4),(3,1),(3,2),(3,3),(4,1),(4,2),(5,1)\}$

- $A \cup C^C = \{(0,0),(1,1),(2,2),(3,3),(1,0),(2,0),(3,0),(4,0),(5,0),(6,0)\}$

- $A^C \cap B \cap C \overset{De\ Morgan's\ law}{=} B \cap (A \cup C^C)^C = B \cap (\{(0,0),(1,1),(2,2),(3,3),(1,0),(2,0),(3,0),(4,0),(5,0),(6,0)\})^C$
  $= \{(2,1),(3,1),(3,2),(4,1),(4,2),(5,1)\}$

- $B^C \cup C = \{(0,0),(0,1),(1,1),(2,1),(3,1),(4,1),(5,1),(0,2),(1,2),(2,2),(3,2),(4,2),$
  $(0,3),(1,3),(2,3),(3,3),(0,4),(1,4),(2,4),(0,5),(1,5),(0,6)\}$

  $\Longrightarrow A \cap (B^C \cup C) = \{(0,0),(1,1),(2,2),(3,3)\} = A$

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List all the possible conditions below:

Meals (M)	Sightseeing (S)	Theatre (T)	Percentage (%)
v			31
	v		18
		v	7
v	v		11
	v	v	8
v		v	7
v	v	v	9
			9 (by subtracting)

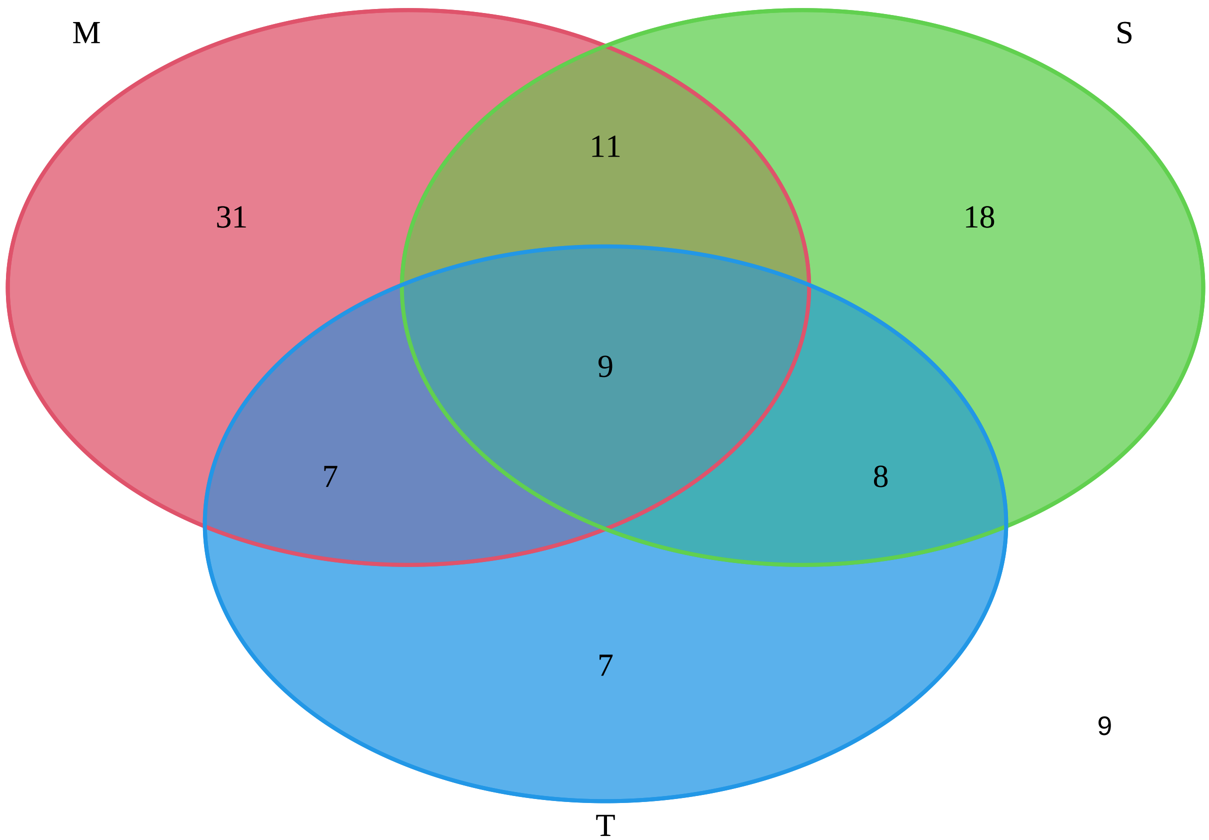
Then we can use this table to draw a Venn diagram (its unit is %):

```r
library("VennDiagram")
```

```
## Loading required package: grid
```

```
## Loading required package: futile.logger
```

```r
venn.plot <- draw.triple.venn(58, 46, 31,
 20, 17, 16, 9, c("M", "S", "T"),col=c(2,3,4),fill=c(2,3,4))
grid.draw(venn.plot)
grid.text("9", x = 0.9, y = 0.15, gp = gpar(fontsize = 10, col = "black"))
```



(a)

$$ANS = Pr\Big(M \cap S \cap T^C\Big) + Pr\Big(M^C \cap S \cap T\Big) + Pr\Big(M \cap S^C \cap T\Big) = 11 + 8 + 7 = 26(\%)$$

(b)

$$ANS = Pr\Big((M \cup S) \cap T^C\Big) = 31 + 11 + 18 = 60(\%)$$

(c)

$$ANS = Pr\Bigg(\Big(M \cup S \cup T\Big)^C\Bigg) = 100 - 31 - 18 - 7 - 11 - 8 - 7 - 9 = 9(\%)$$

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$$\left(The\ following\ facts\ are\ used\ throughout\ (a) \sim (g).\right)$$

$$\#(All\ the\ possible\ outcomes\ of\ simultaneously\ rolling\ 5\ dice) = 6^5,$$
$$since\ each\ of\ these\ 5\ dice\ can\ have\ 6\ different\ outcomes.$$

$$Note\ that\ all\ the\ outcomes\ are\ equally\ likely\ to\ occur$$

$$\Longrightarrow \Omega,\ the\ sample\ space\ has\ the\ "symmetric\ outcomes"\ property.$$
$$\Longrightarrow For\ an\ event\ in\ our\ sample\ space,\ say\ A,\ we\ have\ Pr(A) = \frac{\#(A)}{\#(\Omega)}.$$

(a)

$$Pr\{no\ two\ alike\} = Pr\{all\ the\ dice\ have\ different\ points\}$$

$$SIMPLE\ IDEA:$$
$$The\ 1^{st}\ die\ can\ be\ choose\ from\ \{1, ..., 6\},\ i.e.,\ 6\ options,\ then\ the\ 2^{nd}\ die\ has\ 5\ options,$$
$$the\ 3^{rd}\ die\ has\ 4\ options,\ the\ 4^{th}\ die\ has\ 3\ options, the\ 5^{th}\ die\ has\ 2\ options.$$
$$\Longrightarrow \#(all\ the\ dice\ have\ different\ points) = 6 \times 5 \times 4 \times 3\ \times 2 = 720$$
$$So,\ ANS =\ Pr\{all\ the\ dice\ have\ different\ points\} = \frac{720}{6^5} \approx 0.0926.$$

$$MORE\ RIGOROUS:$$
$$Number\ these\ 5\ dice\ from\ 1\ to\ 5\ and\ let\ their\ values\ be\ N_1, \dots,\ N_5.$$
$$Let\ \Omega_1 = \{1, \dots, 6\},\ \Omega_i = \Omega_{i-1} \backslash N_{i-1},\ for\ i = 2, \dots, 5.$$
$$Then,\ N_i \in \Omega_i,\ \forall\ i\ \in\ \{1, \dots, 5\}.$$
$$We\ observe\ that\ \#(\Omega_i) = (7 - i),\ \forall i \in \{1, 2, 3, 4, 5\}.$$
$$So,\ \#(choosing\ 5\ different\ values\ for\ all\ the\ dice)\ = 6 \times 5 \times 4 \times 3 \times 2\ = 720.$$

$$\#(All\ the\ possible\ outcomes\ of\ simultaneously\ rolling\ 5\ dice) = 6^5,$$
$$since\ each\ of\ these\ 5\ dice\ can\ have\ 6\ different\ outcomes.$$

$$So,\ ANS =\ Pr\{all\ the\ dice\ have\ different\ points\} = \frac{720}{6^5} \approx 0.0926.$$

(b)

$$Since\ there\ is\ exactly\ one\ pair,\ all\ of\ these\ 5\ dice\ are\ of\ 4\ different\ values.$$

$$\begin{cases} \#(Choose\ 4\ different\ values\ from\ \{1,...,6\}) = \binom{6}{4} \\ \#(Choose\ one\ of\ the\ 4\ chosen\ values\ to\ be\ the\ value\ of\ the\ pair) = \binom{4}{1} \\ \#(Distribute\ these\ chosen\ values\ to\ the\ 5\ dice) = \frac{5!}{2!1!1!1!} \end{cases}$$

$$\Longrightarrow By\ the\ multiplication\ rule,\ \#(one\ pair) = \binom{6}{4} \times \binom{4}{1} \times \frac{5!}{2!1!1!1!} = 3600.$$

$$So,\ ANS = \left(\binom{6}{4} \times \binom{4}{1} \times \frac{5!}{2!1!1!1!}\right) \Big/ 6^5 \approx 0.463.$$

(c)

$$Since\ there\ is\ exactly\ two\ pair,\ all\ of\ these\ 5\ dice\ are\ of\ 3\ different\ values.$$

$$\begin{cases} \#(Choose\ 3\ different\ values\ from\ \{1,...,6\}) = \binom{6}{3} \\ \#(Choose\ 2\ of\ the\ 3\ chosen\ values\ to\ be\ the\ value\ of\ the\ 2\ pairs) = \binom{3}{2} \\ \#(Distribute\ these\ chosen\ values\ to\ the\ 5\ dice) = \frac{5!}{2!2!1!} \end{cases}$$

$$\Longrightarrow By\ the\ multiplication\ rule,\ \#(two\ pair) = \binom{6}{3} \times \binom{3}{2} \times \frac{5!}{2!2!1!} = 1800.$$

$$So,\ ANS = \left(\binom{6}{3} \times \binom{3}{2} \times \frac{5!}{2!2!1!}\right) \Big/ 6^5 \approx 0.2315.$$

(d)

$$Since\ there\ are\ 3\ alike,\ all\ of\ these\ 5\ dice\ are\ of\ 3\ different\ values.$$

$$\begin{cases} \#(Choose\ 3\ different\ values\ from\ \{1,...,6\}) = \binom{6}{3} \\ \#(Choose\ 1\ of\ the\ 3\ chosen\ values\ to\ be\ the\ value\ for\ these\ "three\ alike") = \binom{3}{1} \\ \#(Distribute\ these\ chosen\ values\ to\ the\ 5\ dice) = \frac{5!}{3!1!1!} \end{cases}$$

$$\Longrightarrow By\ the\ multiplication\ rule,\ \#(three\ alike) = \binom{6}{3} \times \binom{3}{1} \times \frac{5!}{3!1!1} = 1200.$$

$$So,\ ANS = \left(\binom{6}{3} \times \binom{3}{1} \times \frac{5!}{3!1!}\right) \Big/ 6^5 \approx 0.1543.$$

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(e)

"full house" means 2 of the 5 dice are of a same value, while other 3 dice are all of another different value.

$$By\ the\ definition\ above,\ all\ of\ these\ 5\ dice\ are\ of\ 2\ different\ values.$$

$$\begin{cases} \#(Choose\ 2\ different\ values\ from\ \{1,...,6\}) = \binom{6}{2} \\ \#(Choose\ 1\ of\ the\ 2\ chosen\ values\ to\ be\ the\ value\ for\ those\ "three\ alike") = \binom{2}{1} \\ \#(Distribute\ these\ chosen\ values\ to\ the\ 5\ dice) = \frac{5!}{3!2!} \end{cases}$$

$$\Longrightarrow By\ the\ multiplication\ rule,\ \#(full\ house) = \binom{6}{2} \times \binom{2}{1} \times \frac{5!}{3!2!} = 300.$$

$$So,\ ANS = \left(\binom{6}{2} \times \binom{2}{1} \times \frac{5!}{3!2!}\right) \Big/ 6^5 \approx 0.0386.$$

(f)

$$By\ definition,\ all\ of\ these\ 5\ dice\ are\ of\ 2\ different\ values.$$

$$\begin{cases} \#(Choose\ 2\ different\ values\ from\ \{1,...,6\}) = \binom{6}{2} \\ \#(Choose\ 1\ of\ the\ 2\ chosen\ values\ to\ be\ the\ value\ for\ those\ "four\ alike") = \binom{2}{1} \\ \#(Distribute\ these\ chosen\ values\ to\ the\ 5\ dice) = \frac{5!}{4!1!} \end{cases}$$

$$\Longrightarrow By\ the\ multiplication\ rule,\ \#(four\ alike) = \binom{6}{2} \times \binom{2}{1} \times \frac{5!}{4!1!} = 150.$$

$$So,\ ANS = \left(\binom{6}{2} \times \binom{2}{1} \times \frac{5!}{4!1!}\right) \Big/ 6^5 \approx 0.0193.$$

(g)

$$By\ definition,\ all\ of\ these\ 5\ dice\ are\ of\ 1\ value.$$

$$\#(Choose\ 1\ value\ from\ \{1,...,6\}) = \binom{6}{1}$$

$$\#(Distribute\ these\ chosen\ values\ to\ the\ 5\ dice) = 1$$

$$By\ the\ multiplication\ rule,\ \#(five\ alike) = \binom{6}{1} \times 1 = 6$$

$$\#(All\ the\ possible\ outcomes\ of\ simultaneously\ rolling\ 5\ dice) = 6^5,$$

$$since\ each\ of\ these\ 5\ dice\ can\ have\ 6\ different\ outcomes.$$

$$So,\ ANS = \binom{6}{1} \Big/ 6^5 \approx 0.0008.$$

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Blackjack means "Get an ace + one of {T,J,O,K}", and note there are 16 "T,J,Q,K" in total (each 4 colours).
Denote blackjack by BJ for convenience.
Let A be the event that "you got a BJ".
Let B be the event that "dealer got a BJ".

$$Pr(neither\ you\ nor\ the\ dealer\ is\ dealt\ a\ BJ) = 1 - Pr(A \cup B)$$
$$= 1 - Pr(A) - Pr(B) + Pr(A \cap B)$$
$$= 1 - \frac{\binom{4}{1}\binom{16}{1}\frac{2!}{1!1!}}{52 \times 51} - \frac{\binom{4}{1}\binom{16}{1}\frac{2!}{1!1!}}{52 \times 51} + \frac{\binom{4}{1}\binom{16}{1}\frac{2!}{1!1!}\binom{3}{1}\binom{15}{1}\frac{2!}{1!1!}}{52 \times 51 \times 50 \times 49} \approx 0.9052 = ANS$$

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(a)

$$Pr\{stop\ shooting\ at\ a\ round\}$$
$$= Pr\{bullseye\ twice\} + Pr\{inner\ ring\ twice\} + Pr\{nullseye\ and\ the\ inner\ ring\ each\ once\}$$
$$= (0.1)^2 + (0.5)^2 + 2 \times (0.1) \times (0.5) = 0.36$$

$$Let\ A_n\ be\ the\ event\ that\ she\ have\ not\ stop\ shooting\ at\ the\ n^{th}\ round.$$
$$Since\ each\ round\ is\ independent,\ Pr(A_n) = (1 - 0.36)^n,\ n \geq 1.$$

$$ANS = Pr\{never\ stop\ shooting\} = \lim_{n\to\infty} Pr(A_n) = \lim_{n\to\infty}(1 - 0.36)^n = 0.$$
$$Note\ that\ if\ x \in [0,1],\ then\ \lim_{n\to\infty} x^n = \begin{cases} 1,\ if\ x = 1. \\ 0,\ if\ 0 \leq x < 1. \end{cases}$$

(b)

$$Let\ B_n\ be\ the\ event\ that\ she\ stops\ at\ the\ n^{th}\ round\ and\ the\ bullseye\ are\ hit\ twice.$$

$$Then,\ Pr(B_n) = \underbrace{(1 - 0.36)^{n-1}}_{\substack{didn't\ stop\ until \\ n^{th}\ round}} \times \underbrace{(0.1)^2}_{\substack{bullseye \\ twice}}, \forall n \geq 1.$$

$$So\ Pr\left(stopping\ with\ 2\ bullseye\right) = Pr\left(B_1 \cup B_2 \cup B_3 \cup ...\right)$$
$$\overset{\substack{B_1,B_2,...are \\ disjoint}}{=} \sum_{n=1}^{\infty} Pr(B_n) = \sum_{n=1}^{\infty}(0.1)^2(1 - 0.36)^{n-1} = \frac{(0.1)^2}{0.36} = \frac{1}{36}.$$

$$\left(Note\ that\ \sum_{i=1}^{\infty} a_0 r^{i-1} = \frac{a_0}{1-r},\ \forall\ r\ with\ |r| < 1, \forall a_0 \in\ \mathcal{R}.\right)$$

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$$\Omega : \Big(\overbrace{ball\ 1 \sim ball\ 12}^{(red)},\ \overbrace{ball\ 13 \sim ball\ 28}^{(blue)},\ \overbrace{ball\ 29 \sim ball\ 46}^{(green)} \Big) \xrightarrow[into]{put} \Big\{ \overbrace{\square, \dots, \square}^{7\ positions} \Big\} \Longrightarrow \#(\Omega) = \binom{46}{7}$$

$$\Big(Note\ that\ \Omega\ has\ "symmetric\ outcomes"! \Big)$$

(a)

$$ANS = \frac{\binom{12}{3}\binom{16}{2}\binom{18}{2}}{\binom{12+16+18}{7}} = \frac{3060}{40549} \approx 0.075$$

(b)

$$ANS = 1 - \overbrace{Pr\{0\ red\ is\ withdrawn\}}^{(\star)} - \overbrace{Pr\{1\ red\ is\ withdrawn\}}^{(\star\star)}$$

$$= 1 - \overbrace{\binom{12}{0}\binom{34}{7}\Big/\binom{46}{7}}^{(\star)} - \overbrace{\binom{12}{1}\binom{34}{6}\Big/\binom{46}{7}}^{(\star\star)} \approx 0.5979712$$

(c)

$$ANS = \overbrace{Pr\{7\ red\}}^{(\star)} + \overbrace{Pr\{7\ blue\}}^{(\star\star)} + \overbrace{Pr\{7\ green\}}^{(\star\star\star)}$$

$$= \overbrace{\binom{12}{7}\Big/\binom{46}{7}}^{(\star)} + \overbrace{\binom{16}{7}\Big/\binom{46}{7}}^{(\star\star)} + \overbrace{\binom{18}{7}\Big/\binom{46}{7}}^{(\star\star\star)} \approx 0.000823$$

(d)

Let A denote the event that exactly 3 red balls are withdrawn.

Let B denote the event that exactly 3 blue balls are withdrawn.

$$ANS = Pr\{A \cup B\} = Pr\{A\} + Pr\{B\} - Pr\{A \cap B\}$$

$$= \overbrace{\binom{12}{3}\binom{34}{4}\Big/\binom{46}{7}}^{Pr(A)} + \overbrace{\binom{16}{3}\binom{30}{4}\Big/\binom{46}{7}}^{Pr(B)} - \overbrace{\binom{12}{3}\binom{16}{3}\binom{18}{1}\Big/\binom{46}{7}}^{Pr(A\cap B)} \approx 0.4359$$

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The sample space of this problem is quite similar to that of Problem #35!

Note that Ω, *the sample space, has symmetric outcomes and* $\#(\Omega) = \binom{22}{4}$.

(a)

$$ANS = Pr\{at\ least\ one\ of\ the\ 4\ balls\ chosen\ is\ green\} = 1 - Pr\{all\ the\ chosen\ balls\ are\ NOT\ green\}$$

$$= 1 - \overbrace{\binom{15}{4}\Big/\binom{22}{4}}^{Pr(all\ the\ chosen\ balls\ are\ NOT\ green)} \approx 0.81339$$

(b)

$$ANS = \frac{\binom{4}{1}\binom{5}{1}\binom{6}{1}\binom{7}{1}}{\binom{22}{4}} \approx 0.1148$$

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$$\textit{Suppose we need at least } n \textit{ persons.}$$

$$FYI: \left(\begin{array}{c} \textit{By the Pigeonhole Principle, we know } n \leq 12. \\ \Longrightarrow \textit{An answer greater than 12 must be wrong!} \end{array} \right)$$

$$(\textit{If you do not know what is "Pigeonhole principle", you may search for it online.})$$

$$\textit{Let } A \textit{ denote the event that at least two persons celebrate their birthday at the same month.}$$

$$\textit{Then, } A^C \textit{ is the event that no any 2 persons celebrate their birthday at the same month.}$$

$$\textit{We want}: \ Pr(A) \geq \frac{1}{2} \Longleftrightarrow 1 - Pr(A^C) \geq \frac{1}{2} \overset{(\star)}{\Longleftrightarrow} 1 - \frac{P^{12}_n}{12^n} \geq \frac{1}{2} \Longleftrightarrow \frac{P^{12}_n}{12^n} \leq \frac{1}{2}$$

$$(\star): \textit{The } 1^{st} \textit{person has 12 options, the } 2^{nd} \textit{ has only 11 options left, ...}$$

$$\left(\textit{We can calculate } Pr(A^C) \textit{ by the same method as that used in the "birthday problem", see } LNp3-4.\right)$$

$$\Longrightarrow \textit{We need to find the smallest } n \textit{ such that } \frac{P^{12}_n}{12^n} \leq \frac{1}{2}$$

$$\begin{cases} \textit{When } n=1, \ P^{12}_1 \Big/ 12^1 = 1 \geq \frac{1}{2}. \\ \textit{When } n=2, \ P^{12}_2 \Big/ 12^2 \approx 0.9166 \geq \frac{1}{2}. \\ \textit{When } n=3, \ P^{12}_3 \Big/ 12^3 \approx 0.7638 \geq \frac{1}{2}. \\ \textit{When } n=4, \ P^{12}_4 \Big/ 12^4 \approx 0.5729 \geq \frac{1}{2}. \\ \textit{When } n=5, \ P^{12}_5 \Big/ 12^5 \approx 0.3819 \leq \frac{1}{2}. \end{cases}$$

$$(\textit{In fact, } \frac{P^{12}_n}{12^n} \textit{ is decreasing as } n \textit{ becoming larger, when } 1 \leq n \leq 12.)$$

$$\Longrightarrow ANS: \ \textit{"at least" 5 (persons), under the given assumption.}$$

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$$(If\ you\ do\ not\ know\ how\ to\ play\ "Birdge",\ you\ may\ search\ for\ its\ rule\ online.)$$

$$Let \begin{cases} E_1 = \{the\ bridge\ hand\ is\ void\ in\ spade & (\spadesuit)\} \\ E_2 = \{the\ bridge\ hand\ is\ void\ in\ heart & (\heartsuit)\} \\ E_3 = \{the\ bridge\ hand\ is\ void\ in\ diamond & (\diamondsuit)\} \\ E_4 = \{the\ bridge\ hand\ is\ void\ in\ club & (\clubsuit)\} \end{cases}$$

$$We\ can\ use\ INCLUSION-EXCLUSION\ IDENTITY\ to\ compute\ the\ answer:$$

$$ANS = Pr\left(\bigcup_{i=1}^{4} E_i\right)$$

$$= \underbrace{\sum_{i=1}^{4}}_{(\diamond)} \overbrace{Pr(E_i)}^{(\star)} - \underbrace{\sum_{\substack{i_1<i_2 \\ i_1,i_2\in\{1,2,3,4\}}}}_{(\diamond\diamond)} \overbrace{Pr(E_{i_1}E_{i_2})}^{(\star\star)} + \underbrace{\sum_{\substack{i_1<i_2<i_3 \\ i_1,i_2,i_3\in\{1,2,3,4\}}}}_{(\diamond\diamond\diamond)} \overbrace{Pr(E_{i_1}E_{i_2}E_{i_3})}^{(\star\star\star)} - \overbrace{Pr(E_1E_2E_3E_4)}^{(\star\star\star\star)}$$

$$= \underbrace{4}_{(\diamond)} \times \overbrace{\binom{13}{0}\binom{39}{13}\Big/\binom{52}{13}}^{(\star)} - \underbrace{\binom{4}{2}}_{(\diamond\diamond)} \times \overbrace{\binom{26}{0}\binom{26}{13}\Big/\binom{52}{13}}^{(\star\star)} + \underbrace{\binom{4}{3}}_{(\diamond\diamond\diamond)} \times \overbrace{\binom{39}{0}\binom{13}{13}\Big/\binom{52}{13}}^{(\star\star\star)} - \overbrace{0}^{(\star\star\star\star)} \approx 0.05126206$$

$$\begin{cases} (\star): choose\ 0\ from\ a\ specified\ suit\ and\ choose\ 13\ from\ other\ 3\ suits. \\ (\star\star): choose\ 0\ from\ 2\ specified\ suits\ and\ choose\ 13\ from\ other\ 2\ suits. \\ (\star\star\star): choose\ 0\ from\ 3\ specified\ suits\ and\ choose\ 13\ from\ the\ only\ suit\ left. \\ (\star\star\star\star): This\ is\ impossible\ (why?) \end{cases}$$

$$Note\ we\ have\ abused\ the\ notation\ \sum,\ in\ (\diamond\diamond)\ and\ (\diamond\diamond\diamond)\ for\ simplicity,$$
$$they\ are\ actually\ double\ and\ triple\ summation,\ respectively.$$

$$Note\ the\ answer\ is\ not\ \frac{\binom{4}{1}\binom{39}{13}}{\binom{52}{13}},\ since\ this\ equals\ \sum_{i=1}^{4} Pr(E_i),\ but\ E_1,\ldots,E_4\ are\ not\ disjoint.$$

$$\Longrightarrow The\ case\ E_i \cap E_j\ (i \neq j)\ is\ repeatedly\ counted,\ldots$$

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$$We\ slightly\ abuse\ the\ notation,\ "(A,B)=(x,y)"\ means\ "A\ chooses\ x,\ B\ chooses\ y",\ x,y\in\{a,b,c\}.$$
$$Then,\ we\ have\ the\ following\ results:$$

$$(A=a)\Longrightarrow\begin{cases} Pr\left(B\ wins\ \middle|\ (A,B)=(a,b)\right)=\frac{4}{9} \\ Pr\left(B\ wins\ \middle|\ (A,B)=(a,c)\right)=\frac{5}{9}(\star:B\ chooses\ c)\end{cases}$$

$$(A=b)\Longrightarrow\begin{cases} Pr\left(B\ wins\ \middle|\ (A,B)=(b,a)\right)=\frac{5}{9}(\star:B\ chooses\ a) \\ Pr\left(B\ wins\ \middle|\ (A,B)=(b,c)\right)=\frac{4}{9}\end{cases}$$

$$(A=c)\Longrightarrow\begin{cases} Pr\left(B\ wins\ \middle|\ (A,B)=(c,a)\right)=\frac{4}{9} \\ Pr\left(B\ wins\ \middle|\ (A,B)=(c,b)\right)=\frac{5}{9}(\star:B\ chooses\ b)\end{cases}$$

$$(All\ the\ probabilities\ above\ are\ calculated\ by\ tedious\ enumeration\ like\ below)$$

$$For\ example,\ if\ A\ chooses\ a\ and\ B\ chooses\ b,\ then$$

A=a	1	1	1	5	5	5	9	9	9
B=b	3	4	8	3	4	8	3	4	8
whether B wins	v	v	v			v			

$$\Longrightarrow Pr\left(B\ wins\ \middle|\ (A,B)=(a,b)\right)=\frac{4}{9}$$

$$So,\ no\ matter\ A\ chooses\ which\ spinner,\ B\ can\ choose\ the\ spinner\ given\ by\ (\star)\ such\ that$$
$$Pr\left(B\ wins\ \middle|\ (A,B)\ choose\ as\ (\star)\right)>Pr\left(A\ wins\ \middle|\ (A,B)\ choose\ as\ (\star)\right).$$

$$ANS:\ It\ is\ better\ to\ be\ player\ B!$$

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