#3 Let x be the score of HOME TEAM. y be the score of AWAY TEAM

Ω be the sample space

$\Omega = \{(x,y) : x, y \in \{0,1,\cdots,6\},\ x+y \le 6\}$

$= \{(0,0),(0,1),(0,2),(0,3),(0,4),(0,5),(0,6),(1,0),(1,1),(1,2),(1,3),(1,4)$
$(1,5),(2,0),(2,1),(2,2),(2,3),(2,4),(3,0),(3,1),(3,2),(3,3),(4,0),(4,1)$
$(4,2),(5,0),(5,1),(6,0)\}$

$A = \{(0,0),(1,1),(2,2),(3,3)\}$

$B = \{(1,0),(2,0),(2,1),(3,0),(3,1),(3,2),(4,0),(4,1),(4,2),(5,0),(5,1)(6,0)\}$

$A \cap B = \phi$

$A \cup B = \{(0,0),(1,0),(1,1),(2,0),(2,1),(2,2),(3,0),(3,1),(3,2),(3,3),(4,0)$
$(4,1),(4,2),(5,0),(5,1)(6,0)\}$

$C = \{(0,1),(0,2),(0,3),(0,4),(0,5),(0,6),(1,1),(1,2),(1,3),(1,4),(1,5),(2,1),$
$(2,2),(2,3),(2,4),(3,1),(3,2),(3,3),(4,1),(4,2),(5,1)\}$

$A \cup C^c = \{(0,0),(1,0),(1,1),(2,0),(2,2),(3,0),(3,3),(4,0),(5,0),(6,0)\}$

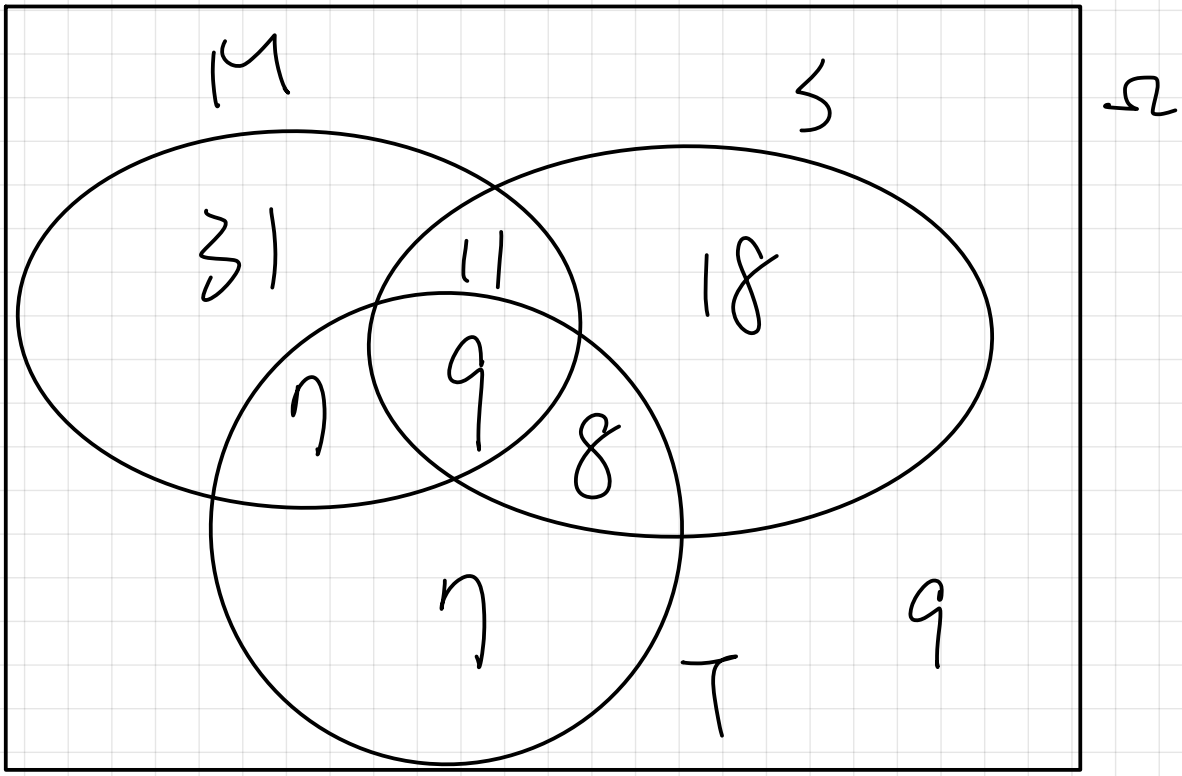
$A^c \cap B \cap C = B \cap A^c \cap C \overset{\text{De Morgan's law}}{=\!=} B \cap (A \cup C^c)^c$

$= \{(2,1),(3,1),(3,2),(4,1),(4,2),(5,1)\}$

$B^c \cup C = \{(0,0),(0,1),(0,2),(0,3),(0,4),(0,5),(0,6),(1,1),(1,2),(1,3),(1,4)$
$(1,5),(2,1),(2,2),(2,3),(2,4),(3,1),(3,2),(3,3),(4,1),(4,2),(5,1)\}$

$A \cap (B^c \cup C) = \{(0,0),(1,1),(2,2),(3,3)\} = A.$

#12



(a) $P(M^c \cap S \cap T) + P(M \cap S^c \cap T) + P(M \cap S \cap T^c) = 8\% + 7\% + 11\% = 26\%$

(b) $P((M \cup S) \cap T^c) = 31\% + 11\% + 18\% = 60\%$

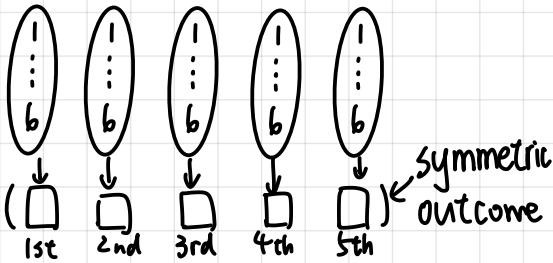
(c) $P((M \cup S \cup T)^c) = 1 - P((M \cup S \cup T))$

$= 100\% - (31\% - 11\% - 18\% - 7\% - 9\% - 8\% - 7\%) = 9\%$

#16.

Ω: sample space $\#(\Omega) = 6 \times 6 \times 6 \times 6 \times 6 = 6^5$

A: event



(a) $\#(A) = 6 \times 5 \times 4 \times 3 \times 2 = 720$

第一個骰子可以投出6面，第二個骰子可以投出除了第一個骰子以外的5面
以此類推。

$P\{\text{no two alike}\} = \frac{720}{6^5} \approx 0.0926$

(b) $\#(A) = \binom{6}{1} \times \binom{5}{3} \times \frac{5!}{2!1!1!1!} = 3600$

先選出1個當成pair的數字，再從剩下5個數字選出3個不重複的數字，最後進行排列

$P\{\text{one pair}\} = \frac{3600}{6^5} \approx 0.463$

#16

(c) $\#(A)=\binom{6}{2}\times\binom{4}{1}\times\frac{5!}{2!2!1!}=1800$

先選出2個當成pair的數字，再從剩下4個數字選出1個數字，最後進行排列

$P\{\text{two pairs}\}=\frac{1800}{6^5}=0.2315$

(d) $\#(A)=\binom{6}{1}\times\binom{5}{2}\times\frac{5!}{3!1!1!}=1200$

先選出1個重複3次的數字，再從剩下5個數字選出2個不重複的數字，最後進行排列

$P\{\text{three alike}\}=\frac{1200}{6^5}\approx 0.1543$

(e) Full house: □□□□□ (3個相同, 2個相同)

$\#(A)=\binom{6}{1}\times\binom{5}{1}\times\frac{5!}{3!2!}=300$

先選出1個重複3次的數字，再從剩下的5個數字選出1個重複2次的數字，最後進行排列

$P\{\text{full house}\}=\frac{300}{6^5}\approx 0.0386.$

(f) $\#(A)=\binom{6}{1}\times\binom{5}{1}\times\frac{5!}{4!1!}=150$

先選出1個重複4次的數字，再從剩下的5個數字選出1個重複1次的數字，最後進行排列

$P\{\text{four alike}\}=\frac{150}{6^5}\approx 0.0193$

(g) $\#(A)=\binom{6}{1}=6$

直接選出1個數字

$P\{\text{five alike}\}=\frac{6}{6^5}\approx 0.0008.$

#20 Blackjack: A + {10, J, Q, K}其中一張 ⇒ 共有$\binom{4}{1}\times\binom{16}{1}=64$種組合

A: 你拿到Blackjack的事件, B: 莊家拿到Blackjack的事件

$P(\text{Neither you nor the dealer is dealt a Blackjack})=P(A^c\cap B^c)=1-P(A\cup B)$

$=1-P(A)-P(B)+P(A\cap B)=1-\frac{64\times 4!}{52\times 51}-\frac{64\times 4!}{52\times 51}-\frac{\binom{4}{1}\binom{16}{1}4!}{52\times 51\times 50\times 49}\times\binom{3}{1}\binom{15}{1}4!\approx 0.9052$

#25

(a) $P\{\text{stop shooting}\}$

$= P\{\text{bullseye twice}\} + P\{\text{inner ring twice}\} + P\{\text{bullseye and inner ring each one}\}$

$= 0.1^2 + 0.5^2 + 2 \times 0.1 \times 0.5 = 0.36$

A_n: event that she haven't stop shooting at the n_{th} round.

$P(A_n) = (1-0.36)^n \ \forall n \in \mathbb{N}$, since each round is independent.

$P\{\text{Never stop shooting}\} = \lim_{n\to\infty} P(A_n) = \lim_{n\to\infty} (1-0.36)^n = 0$

(b) Let B_n be the event that she stops at the n_{th} round and the bullseye are hit twice

$P(B_n) = (1-0.36)^{n-1} \times (0.1)^2, \ n \geq 1$

$P\{\text{stopping with two bullseye}\} = P(B_1 \cup B_2 \cup \cdots) \underset{B_n\text{'s are disjoint}}{=} \sum_{i=1}^{\infty} P(B_i) = \sum_{i=1}^{\infty} (0.1)^2 \cdot (1-0.36)^{i-1}$

$= (0.1)^2 \cdot \frac{1}{1-(1-0.36)} = \frac{0.01}{0.36} = \frac{1}{36}$

35.

Ω: ball 1, ..., ball 12 ← red; ball 13, ..., ball 28 ← blue; ball 29, ..., ball 46 ← green

$\{ \square, \square, \square, \square, \square, \square, \square \}$ ← symmetric outcome

(a)

$$P(3 \text{ red}, 2 \text{ blue}, 2 \text{ green}) = \frac{\binom{12}{3}\binom{16}{2}\binom{18}{2}}{\binom{12+16+18}{7}} = \frac{3060}{40549} \approx 0.075$$

(b)

$$P(\text{at lease 2 red}) = 1 - P(\text{no red}) - P(1 \text{ red})$$

$$= 1 - \frac{\binom{12}{0}\binom{34}{7}}{\binom{46}{7}} - \frac{\binom{12}{1}\binom{34}{6}}{\binom{46}{7}} \approx 0.5979712$$

(c)

$$P(7 \text{ red}) + P(7 \text{ blue}) + P(7 \text{ green})$$

$$= \frac{\binom{12}{7}}{\binom{46}{7}} + \frac{\binom{16}{7}}{\binom{46}{7}} + \frac{\binom{18}{7}}{\binom{46}{7}} \approx 0.000823$$

(d)

$A = \{ \geq 3 \text{ red} \}$

$B = \{ 3 \text{ blue} \}$

求 $P(A \cup B)$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{\binom{12}{3}\binom{34}{4}}{\binom{46}{7}} + \frac{\binom{16}{3}\binom{30}{4}}{\binom{46}{7}} - \frac{\binom{12}{3}\binom{16}{3}\binom{18}{1}}{\binom{46}{7}} \approx 0.4359$$

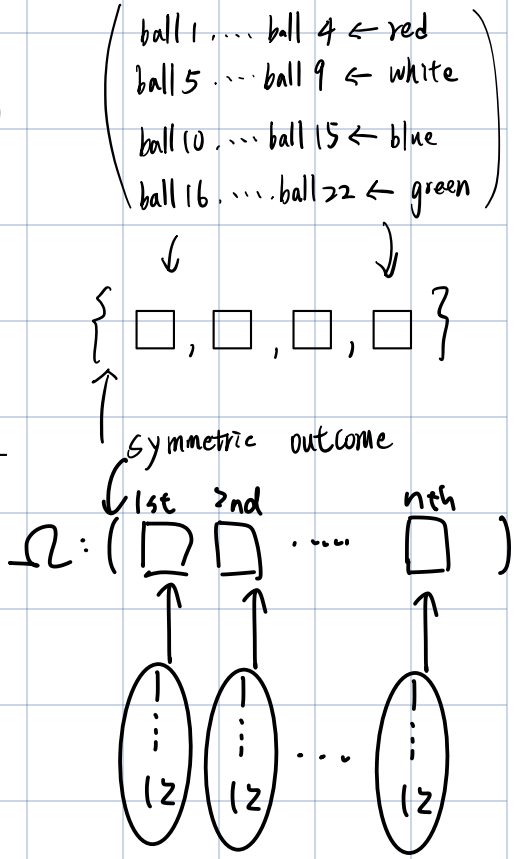
40.

(a) $P(\text{至少1球是綠色}) = 1 - P(\text{沒有球是綠色})$

$$= 1 - \frac{\binom{15}{4}}{\binom{22}{4}} = \frac{170}{209}$$

(b) $P(\text{每個顏色都有抽到}) = \frac{\binom{4}{1}\binom{5}{1}\binom{6}{1}\binom{7}{1}}{\binom{22}{4}} = \frac{24}{209}$

Ω:



46. 設房間內有 n 個人 $(n \le 12)$

設 A_n = {至少2個人生日同月份}

A_n^c = {所有人生日都不同月份}

求 $P(A_n) \ge \frac{1}{2} \iff 1 - P(A_n^c) \ge \frac{1}{2} \iff P(A_n^c) \le \frac{1}{2}$

$$P(A_n^c) = \frac{\#A_n^c}{\#\Omega} = \frac{(12)_n}{12^n}$$

$$= \frac{12!}{(12-n)!\,12^n}$$

$n=1$ $P(A_n^c) = 1 \ge \frac{1}{2}$

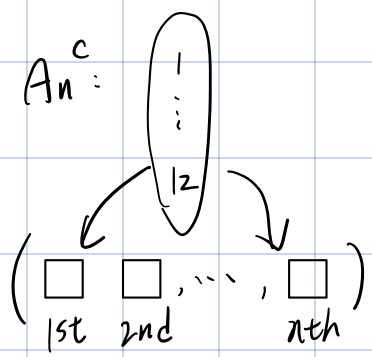
$n=2$ $P(A_n^c) = 0.9166 \ge \frac{1}{2}$

$n=3$ $P(A_n^c) = 0.7638 \ge \frac{1}{2}$

$n=4$ $P(A_n^c) = 0.5729 \ge \frac{1}{2}$

$n=5$ $P(A_n^c) = 0.3819 \le \frac{1}{2}$

A_n^c:



∴ at least 5 people have to be in a room

54.

設 S_1, S_2, S_3, S_4 是4種花色

$P(\text{void in at least 1 suit})$

$= P(\text{void in } S_1 \cup \text{void in } S_2 \cup \text{void in } S_3 \cup \text{void in } S_4)$

(by prop 4.4)

$= \sum_{i=1}^{4} P(\text{void in } S_i) - \sum_{i<j} P(\text{void in } S_i \cap \text{void in } S_j)$

$+ \sum_{i<j<k} P(\text{void in } S_i \cap \text{void in } S_j \cap \text{void in } S_k)$

$- P(\text{void in } S_1 \cap \text{void in } S_2 \cap \text{void in } S_3 \cap \text{void in } S_4)$

$= \binom{4}{1} \cdot \frac{\binom{39}{13}}{\binom{52}{13}} - \binom{4}{2} \frac{\binom{26}{13}}{\binom{52}{13}} + \binom{4}{3} \frac{\binom{13}{13}}{\binom{52}{13}} - 0 = \frac{16213649\,09}{3175067\,7980} \doteq 0.05106552$

↑ 不可能4種花色都沒有

$\frac{\binom{4}{1}\binom{39}{13}}{\binom{52}{13}}$ equals $\sum_{i=1}^{4} Pr(E_i)$, but $E_1, \cdots, E_4$ are not disjoint

The case $E_i \cap E_j$ $(i \neq j)$ is repeatedly counted

56

$a = \{1, 5, 9\}$ $b = \{3, 4, 8\}$ $C = \{2, 6, 7\}$

① a 和 b

a \ b	3	4	8
1	b	b	(b)
5	a	a	b
9	a	a	(a)

(b) 代表 $b > a$

(a) 代表 $a > b$

$P(a > b) = \frac{5}{9}$

$P(a < b) = \frac{4}{9}$

② b 和 c

b\c	2	6	7
3	b	c	c
4	b	c	c
8	b	b	b

$P(b>c) = \frac{5}{9}$

$P(c>b) = \frac{4}{9}$

③ c 和 a

c\a	1	5	9
2	c	a	a
6	c	c	a
7	c	c	a

$P(c>a) = \frac{5}{9}$

$P(a>c) = \frac{4}{9}$

$A=a$ {
$P(B \text{ wins} \mid (A,B)=(a,b)) = \frac{4}{9}$
$P(B \text{ wins} \mid (A,B)=(a,c)) = \frac{5}{9}$ ✓ ← 代表 $> \frac{1}{2}$
}

$A=b$ {
$P(B \text{ wins} \mid (A,B)=(b,a)) = \frac{5}{9}$ ✓
$P(B \text{ wins} \mid (A,B)=(b,c)) = \frac{4}{9}$
}

$A=c$ {
$P(B \text{ wins} \mid (A,B)=(c,a)) = \frac{4}{9}$
$P(B \text{ wins} \mid (A,B)=(c,b)) = \frac{5}{9}$ ✓
}

所以不管 A 選什麼 B 都可以選到勝率更高的 ($> \frac{1}{2}$)

∴ B 後選比較好