

4.

4 instruments
 (1) $(\square, \square, \square, \square)$ 4 people $\Rightarrow 4! = 24$.
 John Jim Jay Jack other two instruments

(2) $(\square\square, \square\square)$ $\Rightarrow 2! \times 2! = 4$.
 Jay Jack John Jim

10.

(a) $(\square\square \dots \square\square)$ $\Rightarrow 8! = 40320$.

(b) $(\square\square \dots \square)$ $\Rightarrow 7! \times 2! = 10080$
 $\square = AB \text{ or } BA$

(c) $\text{順序: 男 女} \dots \text{女} \quad \text{女 男} \dots \text{男}$
 $\text{男} \text{女} \dots \text{男} \text{女} + \text{女} \text{男} \dots \text{女} \text{男} = (|\square\square\square\square|, |\square\square\square\square|) + (|\square\square\square\square|, |\square\square\square\square|) = 4! \cdot 4! + 4! \cdot 4! = 1152$

(d) $(|\square\square \dots \square|, |\square\square\square\square|) \Rightarrow 4! \cdot 5! = 2880$.

(e) 4 couples, each couples sit together $\Rightarrow (\square\square\square\square, |\square\square\square\square|) \Rightarrow 4! \times (2!)^4 = 384$.
 1st 2nd 3rd 4th seat 1,2 seat 3,4 seat 5,6 seat 7,8

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$(\{\square\square\square\square\square\square\}, \{\square\square\square\square\square\square\}, \{\square\square\square\square\square\square\}) \text{ fixed} \Rightarrow \binom{12}{5} \times \binom{10}{5} \times 5! = 23950080$.
 $\text{男} \text{女} \text{男} \text{女} \text{男} \text{女} \text{男} \text{女} \text{男} \text{女} \text{男} \text{女}$
 $1 \leq i_1 < i_2 < i_3 < i_4 < i_5 < i_6 \leq 12$
 $1 \leq j_1 < j_2 < j_3 < j_4 < j_5 < j_6 \leq 10$

22.

(a) 1. 2人挑1個: $\binom{2}{1} \times \binom{6}{4} = 30$, $(\square, \{\square\square\square\square\})$
 2. 2人都沒挑: $\binom{6}{5} = 6$, $\{\square\square\square\square\square\square\}$
 $\Rightarrow 30 + 6 = 36$.

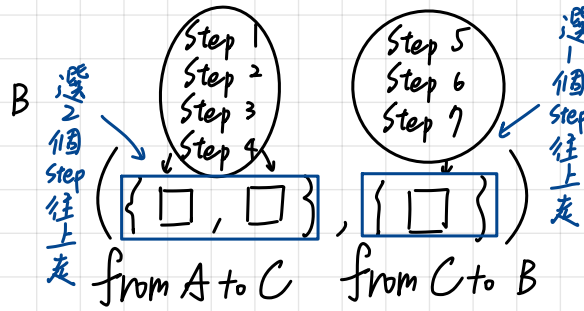
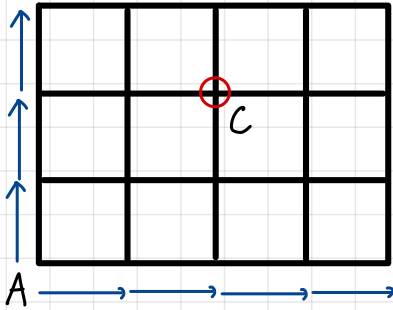
(b) 1. 2人都挑: $\binom{6}{2} = 20$, $\{A, B, \square\square\square\square\}$
 2. 2人都沒挑: $\binom{6}{5} = 6$, $\{\square\square\square\square\square\square\}$
 $\Rightarrow 20 + 6 = 26$

23. $A \rightarrow B$ needs $4 \rightarrow 3 \uparrow \Rightarrow$ 不置相異物排列: $\frac{7!}{4!3!} = 35$. $\leftarrow \uparrow \rightarrow \uparrow \uparrow \rightarrow \rightarrow \rightarrow$

另解 將這題想成用7個移動的step移動到終點, 取三個step為向上走: $\binom{7}{3} = 35$ $\{\square\square\square\}$

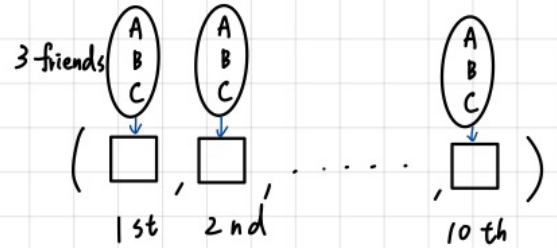
step 1, step 2, ..., step 7

24.

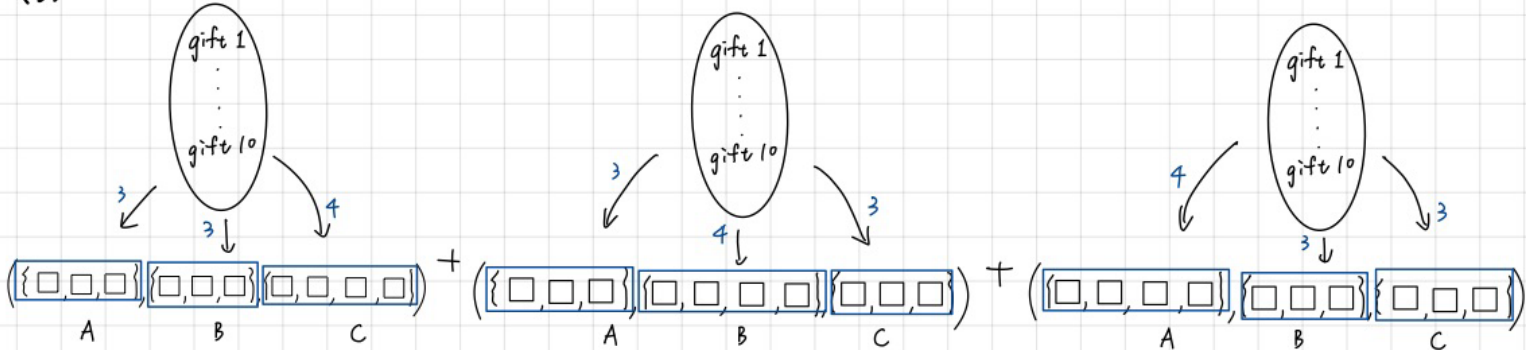


from A to C from C to B
 $C_2^4 \times C_1^3 = 18$ 種方法

31. (a) 有 $3 \times 3 \times \dots \times 3 = 3^{10}$ 種分法
 10 gifts



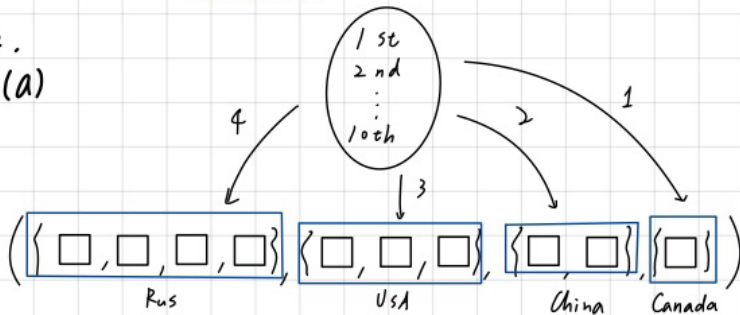
(b)



$\Rightarrow \frac{10!}{3!3!4!} \times 3$

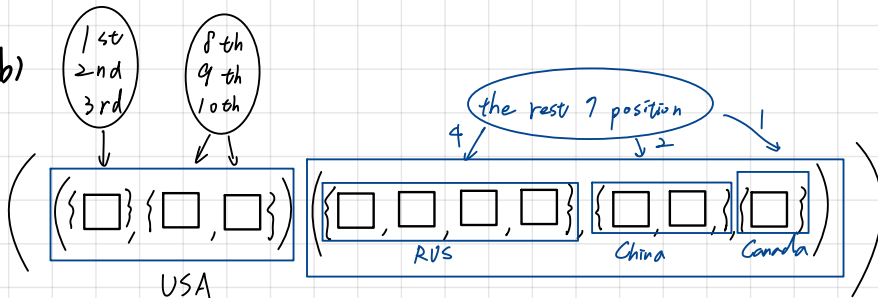
32.

(a)



$\Rightarrow \frac{10!}{4!3!2!1!}$ 種結果

(b)



$\Rightarrow C_1^3 \cdot C_2^3 \cdot \frac{7!}{4!2!1!}$ 種結果

整數解個數 $\Rightarrow H_g^4 = C_g''$ 種分法

$$a+b+c+d=4 \quad a \geq 0, b \geq 0, c \geq 0, d \geq 0.$$

整數解個數 $\Rightarrow H_4^1 = C_4^1$ 種分法

36. 令 x_1, x_2, x_3, x_4 分別為第 1~4 項投資金額 (單位: 千元)

(a) $x_1 + x_2 + x_3 + x_4 = 20, x_1 \geq 2, x_2 \geq 2, x_3 \geq 3, x_4 \geq 4$

$$\Rightarrow y_1 + y_2 + y_3 + y_4 = (x_1 - 2) + (x_2 - 2) + (x_3 - 3) + (x_4 - 4) = 9, \quad y_1 \geq 0, y_2 \geq 0, y_3 \geq 0, y_4 \geq 0$$

整數解個數 $\Rightarrow H_9^4 = C_9^{12}$ 種方法

(b) ① $x_1 + x_2 + x_3 = 20, x_1 \geq 2, x_2 \geq 2, x_3 \geq 3$

$$\Rightarrow y_1 + y_2 + y_3 = (x_1 - 2) + (x_2 - 2) + (x_3 - 3) = 13, \quad y_1 \geq 0, y_2 \geq 0, y_3 \geq 0, y_4 \geq 0$$

整數解個數 $\Rightarrow H_{13}^3 = C_{13}^{15} = C_2^{15}$ 種方法

② $x_1 + x_2 + x_4 = 20, x_1 \geq 2, x_2 \geq 2, x_4 \geq 4$

$$\Rightarrow y_1 + y_2 + y_4 = (x_1 - 2) + (x_2 - 2) + (x_4 - 4) = 12, \quad y_1 \geq 0, y_2 \geq 0, y_4 \geq 0$$

整數解個數 $\Rightarrow H_{12}^3 = C_2^{14}$ 種方法

③ $x_1 + x_3 + x_4 = 20, x_1 \geq 2, x_3 \geq 3, x_4 \geq 4$

$$\Rightarrow y_1 + y_3 + y_4 = (x_1 - 2) + (x_3 - 3) + (x_4 - 4) = 11, \quad y_1 \geq 0, y_3 \geq 0, y_4 \geq 0$$

整數解個數 $\Rightarrow H_{11}^3 = C_2^{13}$ 種方法

④ $x_2 + x_3 + x_4 = 20, x_2 \geq 2, x_3 \geq 3, x_4 \geq 4$

$$\Rightarrow y_2 + y_3 + y_4 = (x_2 - 2) + (x_3 - 3) + (x_4 - 4) = 11, \quad y_2 \geq 0, y_3 \geq 0, y_4 \geq 0$$

整數解個數 $\Rightarrow H_{11}^3 = C_2^{13}$ 種方法

⑤ by part (a) $\Rightarrow H_q^4 = C_q^{12}$ 種方法

⇒ 投資至少3項: $C_2^{13} + C_2^{13} + C_2^{14} + C_2^{15} + C_3^{12}$ 種方法