Statistical Control, Profile Monitoring and Other Recent Developments

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Outline

• Statistical Control Chart 101 6010
• An Example of Statistical Control Chart (Part I)
• Issues and Problems
• Examples of Statistical Profile Monitoring
• Issues and Problems
• Phase I Risk-Adjusted Control Charts (Part II)
• Monitoring Covariance Matrix via Penalized Likelihood Estimation (Part III)
• Wrap-up
Applications of Control Chart

- Manufacturing – automotive, airplane, semiconductor, solar panels, food production, ...
- Non-manufacturing – insurance, banking, customer services, hospitals, total organic carbon analysis, cardiac surgery, ...
Statistical Control Chart Basics

- Quality characteristic, critical dimension (univariate or multivariate, attributes or variables)
- Modeling based on some distribution (say, normal distribution)
- Quality control (process monitoring) is essentially monitoring the distributional parameters (say, mean and variance)
General Approaches

• Taking samples of observations at scheduled time intervals
• For each sample, calculate some statistic intended to estimate the parameter being monitored (say, mean)
• Plot the statistic against sampling sequence on a control chart with appropriate control limits
An Example of Control Chart

Xbar-S Chart of Flow Width

Sample Mean

\[ \bar{X} = 1.5053 \]

\[ \text{UCL} = 1.6932 \]

\[ \text{LCL} = 1.3175 \]

Sample StDev

\[ \bar{S} = 0.1316 \]

\[ \text{UCL} = 0.2749 \]

\[ \text{LCL} = 0 \]
Types of Control Chart

- Shewhart Chart – each statistic is used for only one time period
- Cumulative Sum Chart – statistics across different time periods are accumulated
- Exponentially Weighted Moving Averages Chart - statistics are averaged across different time periods
Phase I Control and Phase II Monitoring

• **Phase I Control** – estimating distributional and charting parameters, and making sure that the process is in control

• **Phase II Monitoring** – monitoring any changes in process (essentially changes in distributional parameters)
One Example (Wafers)
Model Settings

\[ y_t = \mu_t + \varepsilon_t, \quad \text{var}(\varepsilon_t) = \sigma_t^2(\Sigma_t) \]

\[ \mu_t = \begin{cases} 
\mu_0, & t < t_1 \\
\mu_1, & t \geq t_1 
\end{cases} \]

\[ \sigma_t^2(\Sigma_t) = \begin{cases} 
\sigma_0^2(\Sigma_0), & t < t_2 \\
\sigma_1^2(\Sigma_1), & t \geq t_2 
\end{cases} \]
Issues and Problems

- Monitoring mean (variable selection, diagnostics)
- Monitoring variance (variable selection, diagnostics)
- Simultaneous monitoring
- Non-normality
- Dependent observations (for attributes data)
- Individual observations
- Other approaches – change-point, economic design, performance measure
A Different Example (MFC)
Simple Linear Profiles

\[ y = \mu_x + \varepsilon \]

\[ = f(x) + \varepsilon \]

\[ = \beta_0 + \beta_1 x + \varepsilon, \text{ var} (\varepsilon) = \sigma^2 \]

\[ \mu = (\beta_0, \beta_1, \sigma^2)^t \]
Statistical Profile Monitoring

- Semiconductor, calibration, tonnage stamping, torque signals in tapping, force signals in welding, particleboards, etc.
Another Example: DRIE Process
Figure 3: Illustrations of various etching profiles from a DRIE process.
Figure 2. Illustration of modeling the DRIE profile.
General Linear Profiles

\[ y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_{p-1} x_{p-1} + \varepsilon, \quad \text{var}(\varepsilon) = \sigma^2 \]

\[ \mu = (\beta_0, \beta_1, \beta_2, \ldots, \beta_{p-1}, \sigma^2)^t \]
Figure 3. MEWMA chart for the example.
Figure 4. Illustrations of 14 sample profiles.
(The symbol "*" in the exponent of sample number represents an OC profile)
Particleboard Example
Figure 1: Vertical Density Profile (VDP) of 24 Particleboards
Non-Linear Profiles

\[ y = f(x, \beta) + \varepsilon, \quad \text{var}(\varepsilon) = \sigma^2 \]

\[ \mu = \beta \]
Non-Parametric Profiles

\[ y = f(x) + \varepsilon \]

- No parametric assumption of the functional form
- Estimating \( f(x) \)
- \( x \) fixed, random, mixed; correlated responses within each profile and between profiles
Figure 1: NME and nonlinear fits to the VDP data.
Other Approaches

- Wavelets
- PCA
- B-Splines
FIGURE 3. Vertical Density Profiles with Largest (+ + + + +) and Smallest (− − − − −) First Principal-Component Scores. (Solid line represents average profile.)

FIGURE 4. Vertical Density Profiles with Largest (+ + + + +) and Smallest (− − − − −) Second Principal-Component Scores. (Solid line represents average profile.)
Aircraft Construction: Alloy Fastener
## Categorical Response

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<td>60</td>
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<tr>
<td>4300</td>
<td>65</td>
<td>51</td>
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Scatterplot of Logit of Proportion vs Load
Profile for Binary Response

- The response variable is binary

\[
y_i = E(y_i) + \varepsilon_i
\]

\[
E(y_i) = \pi_i = p(y_i = 1) = 1 - p(y_i = 0)
\]

\[
g(\pi_i) = \log \left( \frac{\pi_i}{1 - \pi_i} \right) = \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_p x_{ip}
\]

\[
\mu = (\beta_1, \beta_2, \ldots, \beta_p)^t
\]
Figure 1(c). The $T_I^2$-chart.
Issues and Problems

• Multivariate or multiple categorical response
• Individual observations ($n = 1$, for Phase I simple linear profile monitoring)
• Dependent observations within and between profiles
• Diagnostics
• Performance assessment of different approaches
• Monitoring of shapes and surfaces
A Different Perspective
A Collection of Profiles

10 lines

100 lines

500 lines

5000 lines
New Applications

• Health Care Quality Monitoring
• Public Health Surveillance
A Cardiac Center Data

- A Cardiac Center from UK
- 6994 operations from 1992-1998
- 461 deaths occurred within 30 Days (6.6%)
- Variables include age, gender, surgeon, type of operations, type of procedure, Parsonnet score
- Parsonnet score is a weighted composite score (0-100) based on pre-operative variables
Part II: Risk-Adjusted Phase II Attributes Chart

- The quality characteristic is binary, adjusted for risk by the logistic model on patient-dependent variables (e.g., Parsonnet score)
- EWMA and CUSUM on the risk-adjusted quality characteristic or the log-likelihood ratio scores
- The logistic model is based on 1992-93 data (2,218 patients with 143 deaths, about 6.5% mortality rate)
<table>
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<th>Assigned weight</th>
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<td>Female gender</td>
<td>1</td>
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<tr>
<td>Morbid obesity (≥1.5× ideal weight)</td>
<td>3</td>
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<tr>
<td>Diabetes (unspecified type)</td>
<td>3</td>
</tr>
<tr>
<td>Hypertension (systolic BP &gt; 140 mm Hg)</td>
<td>3</td>
</tr>
<tr>
<td>Ejection fraction (%) (&lt;actual value when available&gt;)</td>
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<tr>
<td>Good (≥50)</td>
<td>0</td>
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<tr>
<td>Fair (30–49)</td>
<td>2</td>
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<tr>
<td>Poor (&lt;30)</td>
<td>4</td>
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<tr>
<td>75–79</td>
<td>12</td>
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<td>≥80</td>
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<td>Reoperation</td>
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<td>First</td>
<td>5</td>
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<td>Second</td>
<td>10</td>
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<tr>
<td>Preoperative IABP</td>
<td>2</td>
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<tr>
<td>Left ventricular aneurysm</td>
<td>5</td>
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<tr>
<td>Emergency surgery following PTCA or catheterization complications</td>
<td>10</td>
</tr>
<tr>
<td>Dialysis dependency (PD or Hemo)</td>
<td>10</td>
</tr>
<tr>
<td>Catastrophic states (e.g., acute structural defect, cardiogenic shock, acute renal failure)*</td>
<td>10–50†</td>
</tr>
<tr>
<td>Other rare circumstances (e.g., paraplegia, pacemaker dependency, congenital HD in adult, severe asthma)*</td>
<td>2–10†</td>
</tr>
<tr>
<td>Valve surgery</td>
<td></td>
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<tr>
<td>Mitral</td>
<td>5</td>
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<td>PA pressure ≥ 60 mm Hg</td>
<td>8</td>
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<tr>
<td>Aortic</td>
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<tr>
<td>Pressure gradient &gt; 120 mm Hg</td>
<td>7</td>
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<tr>
<td>CABG at the time of valve surgery</td>
<td>2</td>
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Standard Phase II CUSUM Chart

\[ X_t = \max(0, X_{t-1} + W_t), t = 1, 2, ..., X_0 = 0 \]

\[ f(y_t | \theta) = p(\theta)^{y_t} (1 - p(\theta))^{1-y_t}, p(\theta_0) = c_0 \]

\[ H_0 : p = c_0 \ \text{vs.} \ H_a : p = c_a \]

\[ W_t = \begin{cases} 
\log \left[ \frac{c_a}{c_0} \right], & \text{if } y_t = 1 \\
\log \left[ \frac{(1-c_a)}{(1-c_0)} \right], & \text{if } y_t = 0 
\end{cases} \]
Phase II Risk-Adjusted CUSUM Chart

\( H_0 : \text{odds ratio} = R_0 \) vs. \( H_a : \text{odds ratio} = R_a \)

\[
W_t = \begin{cases} 
\log \left[ \frac{(1 - p_t + R_0 p_t) R_a}{(1 - p_t + R_a p_t) R_0} \right], & \text{if } y_t = 1 \\
\log \left[ \frac{(1 - p_t + R_0 p_t)}{(1 - p_t + R_A p_t)} \right], & \text{if } y_t = 0 
\end{cases}
\]

\[
\text{logit} (p_t) = \beta_0 + \beta_1 (PS_t), \quad PS_t : \text{ParsonnetScore}
\]
Risk-Adjusted Attributes Chart
(Steiner et al, 2000, Cardiac Surgery)

Figure 1: Trainee Surgeons CUSUM
unadjusted CUSUMs on top, risk adjusted CUSUMs on the bottom
Figure 2: Experienced Surgeon CUSUM
unadjusted CUSUMs on the top, risk adjusted CUSUMs on the bottom
Risk-Adjusted Phase II Variables Chart

- The quality characteristic is continuous, adjusted by a risk profile on the patient-dependent variables.
Figure 2. Log-logistic RAST CUSUM chart applied to the cardiac surgery data from the beginning of 1994 to the end of 1998, with $\rho_1 = 0.2697$, $h = 4.8327$, and $\text{ARL}_0 \approx 10000$. 
Risk-Adjusted Phase I Attributes Chart (Paynabar, Jin and Yeh, 2010)

• The quality characteristic is binary, adjusted for risk by the logistic model on patient-dependent variables (e.g., Parsonnet score), as well as operational categorical covariates such as surgeons

• Charting statistics based on likelihood ratio derived from a change-point model
Cardiac Surgery Example Revisited

• Use 1992-93 as Phase I data
• Three trainee and three experienced surgeons
• 1,112 records with experienced surgeons, possible clusters among experienced surgeons
• RA charts with or without surgeons as covariate are considered
Figure 1: fitted risk-adjusted models for each group and entire data
Figure 2: RA-LRT_{CP1} (top panel) and RA-LRT_{CP2} (bottom panel) control charts of surgical data.
Figure 3: the mortality rate plot for different surgeons and got the model without the surgeon covariate before and after change
One Step Further

• The risk function is assumed to remain the same
• We can treat this as profile monitoring for binary response when \( n = 1 \)
• Risk-adjusted control charts, taking into account of possible changes in profile function (risk function) for pre-operative as well as operational factors
Google Flu Trends

- [http://www.google.org/flutrends](http://www.google.org/flutrends)

\[
\text{logit}(P) = \beta_0 + \beta_t \text{logit}(Q) + \varepsilon
\]

- \(P\) – percentage of ILI physician visits
- \(Q\) – ILI-related query fraction
Part III: Control Charts via Penalized Likelihood Estimation

- Multivariate control charts via penalized likelihood estimation
- Monitoring Mean – Zou and Qiu (2009), Wang and Jiang (2009)
- Multivariate Linear Profile Monitoring – Zou et al (2010)
Monitoring Mean

- Multivariate control charts via penalized likelihood estimation
- Penalized maximum likelihood estimator of the mean vector by adding a penalty function on the number of non-zero elements in the mean vector estimate
\[ H_0 : \mu \in \Omega_0 \ \text{v.s.} H_1 : \mu \in \Omega_1 \]

\[ S^2 (\lambda) = \min_{\mu \in \Omega_1} \left( (y_t - \mu)^T \Sigma^{-1} (y_t - \mu) + \lambda M \right) \]

\[ M = \sum_j I(\| \mu_j \| \neq 0) \]

\[ \Lambda(y_t) = 2 y_t^T \Sigma^{-1} \mu^* - \mu^*^T \Sigma^{-1} \mu^* > UCL \]
Monitoring Covariance Matrix via Penalized Likelihood Estimation

\[ l(X_1, X_2, \ldots, X_n ; \Omega) = tr(\Omega S) - \ln|\Omega| \]

\[ \Omega_{mle} = \arg \min_{\Omega} \{ tr(\Omega S) - \ln|\Omega| \} \]

\[ \Omega = \Sigma^{-1}, \quad S = \frac{1}{n} \sum_{j=1}^{n} (X_j - \overline{X})(X_j - \overline{X})^T \]
\[ l(X_1, X_2, \ldots, X_n ; \Omega) = tr(\Omega S) - \ln |\Omega| + \lambda \| \Omega \|_1 \]

\[ \| A \|_1 = \sum_{j=1}^{p} \sum_{i=1}^{p} |a_{ij}|, A = (a_{ij})_{pxp} \]

\[ \Omega_\lambda = \arg \min_{\Omega > 0} \{ tr(\Omega S) - \ln |\Omega| + \lambda \| \Omega \|_1 \} \]
Penalized Likelihood Ratio Control Chart (PLR-Chart)

\[ H_0 : \Sigma = I_p \quad \text{vs.} \quad H_1 : \Sigma \neq I_p \]

\[ \Lambda = tr(\Omega S) - \ln|\Omega| - tr(S) \]

\[ \Lambda_\lambda = tr(\Omega_\lambda S) - \ln|\Omega_\lambda| - tr(S) \]

The Chart Signals When \( \Lambda_\lambda > UCL_\lambda \)
\[ \Sigma = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}_{5 \times 5} \]
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5 & 1 & .5 & 0 & 0 \\
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0 & 0 & 0 & .5 & 1 \\
\end{pmatrix}_{5\times5}
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An Example: Lapping Process

- Total Thickness Variation (TTV)
- Total Indicator Reading (TIR)
- Site TIR (STIR)
- Warp
- Bow
TTV = MaxThk - MinThk

Reference Line

TIR

Warp

Bow
An In-Control Data Set
An Out-of-Control Data Set
In-Control Covariance

\[ \Sigma_0 = \begin{pmatrix}
1.30 & .46 & .51 \\
.46 & 1.30 & .53 \\
.51 & .53 & 1.30 \\
0 & 1.30 & 0 \\
0 & 0 & 1.30 \\
\end{pmatrix}_{5\times5} \]
Out-of-Control Covariance

$$
\Sigma_{oc} = \begin{pmatrix}
1.30 & .62 & .63 \\
.62 & 1.30 & .68 \\
.63 & .68 & 1.30 \\
0 & 1.30 & -.55 \\
0 & -.55 & 1.30
\end{pmatrix}_{5 \times 5}
$$
The Transformed Variable

\[ \Sigma_{oc} = \begin{pmatrix} .83 & -.03 & .02 \\ -.03 & .81 & 0 \\ .02 & 0 & 1.13 \\ 1.00 & -.43 \\ -.43 & 1.30 \end{pmatrix}_{5\times5} \]
The Conditional Entropy Chart
The Decomposition Chart

Sample

Charting Static

24.81
The Penalized Likelihood Ratio Chart

![Chart showing the penalized likelihood ratio for different samples. The x-axis represents the sample number ranging from 3 to 30, and the y-axis represents the charting statistic ranging from 0.1 to 0.7. The chart includes a horizontal line at 0.5203, indicating the threshold value.](image-url)
Wrap-up

• There are still many open problems in statistical control chart, especially in multivariate quality control, and statistical profile monitoring

• Many potential applications, especially non-manufacturing, remain to be explored
References (Statistical Profile Monitoring and RA Control Charts)


References (Penalized Likelihood Estimation)


• Tibshirani (1996) “Regression Shrinkage and Selection via the Lasso”, *JRSSB*.


• Yuan and Lin (2007) “Model Selection and Estimation in the Gaussian Graphical Model”, *Biometrika*.

"Let's face it--we have no quality and we have no control."
ANY QUESTIONS?

Thank You!!

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